MATH 8702 TOPICS IN APPLIED MATH: NONLINEAR DISPERSIVE EQUATIONS II

One of the most important moments in the history of fluid dynamics was the chance discovery of the Great Wave of Translation. As he rode past on horseback, J. Russel observed a wave consisting of a single isolated crest traveling down the Edinburgh canal without changing shape. According to the contemporary understanding of water waves, this should not have been possible: waves were modeled using Burgers' equation, and hence were expected steepen and then break (develop a shock). By contrast, this wave wasn't developing any singularities, it was simply translating with a fixed speed. On the other hand, linear models of water waves (such as Airy's equation) could not account for the localized form of the wave.

Coming to terms with this behavior led to the discovery of (nonlinear) dispersion. Certain equations have the property that, a solution localized in frequency will tend to propagate in space with a velocity determined by that frequency. This is the magical ingredient keeping Russell's wave from breaking: the dispersion acts to spread the wave, as different frequency components move at different speeds. In fact, this phenomenon is observed in a surprisingly diverse array of physical settings. Most famously, Schrödinger's equation — which is of fundamental importance to quantum mechanics — is in some ways the canonical dispersive PDE.

As one might expect, the study of dispersive PDEs has benefited greatly from the many recent advances in harmonic analysis. It is currently a very active and rapidly developing field with a number of important open problems. The objective of this course will be to get a familiarity with the "big ideas", main machinery, and current state-of-the-art.

This is a continuation of the course from last semester. We will continue to explore the book by T. Tao. Topics to be covered include the KdV equation, energy critical semilinear dispersive equations, and wave maps.

Like last semester, this will be a working seminar course. I will give lectures introducing the major topics, as well as fleshing out background material as necessary, but each student will be asked to prepare presentations from the text (or supplemental sources).

Textbook. The main reference for the course will be *Nonlinear Dispersive Equations: Local and Global Analysis* by T. Tao. It can be obtained for free online from the author's website.

Prerequisites. A year of graduate analysis and at least one semester of PDEs (equivalent to 8420, 8421, 8445). It is not strictly necessary, but having seen the material in PDE II (8846), Harmonic Analysis (8630), and Functional Analysis (8628) will be very helpful.

Structure of the course. This is an advanced seminar course, so much of the responsibility for learning the material will rest on you. There will be no traditional homework assignments or examinations. Instead, you are expected to volunteer to present sections of the course material. As this is a "working seminar", you should also read the material that others present *before the lecture* so that we can work together to fill in any details. This is a collaborative enterprise, not just passive viewing experience.

Disabilities. If you need accommodations because of a disability, if you have emergency medical information to share with me, or if you need special arrangements in case the building must be evacuated, please inform me immediately. Please see me privately after class, or at my office. To request academic accommodations (for example, a note taker), students must also register with Disability Services (http://web.missouri.edu/~accesscm), AO38 Brady Commons, 882-4696 or 882-8054 TTY. It is the campus office responsible for reviewing documentation provided by students requesting academic accommodations, and for accommodations planning in cooperation with students and instructors, as needed and consistent with course requirements. Another resource, MU's Adaptive Computing Technology Center (http://iatservices.missouri.edu/adaptive), 884-2828, is available to provide computing assistance to students with disabilities. For more information about the rights of people with disabilities, please see ada.missouri.edu or call 884-7278.

Academic Honesty. Academic honesty is fundamental to the activities and principles of a University. Any effort to gain an advantage not given to all students is dishonest whether or not the effort is successful. When in doubt about plagiarism or collaboration, consult the course instructor. The academic community regards academic dishonesty as an extremely serious matter, with serious consequences that range from probation to expulsion. If at any time you have questions about this policy, please ask.

Complaints. If you have communication (or other problems) with your instructor, you can report them to Professor Stephen Montgomery-Smith (Director of Graduate Studies) either by phone (882-4540) or by e-mail (`stephen@missouri.edu`).