MATH 8702 TOPICS IN APPLIED MATH: WATER WAVES

Instructor	Samuel Walsh (`walshsa@missouri.edu`)
Lecture	MWF 2:00PM–2:50PM in MSB 110
Office hours	WR 4:30–5:30 PM in MSB 307
Website	`http://faculty.missouri.edu/~walshsa/index.html`

Overview. The material for this course is divided into two parts. The first is a general introduction to the incompressible Euler equations — the system of PDEs that is most commonly used to model the motion of water. We will begin by building up a physical motivation, and then highlight some "big ideas" in the mathematical literature. A few things to be discussed are: the classical Eulerian and Lagrangian formulations of the Euler equations; Arnold's geometric formulation; Zakharov's Hamiltonian formulation of the water wave problem; and applications of these concepts to the question of well-posedness.

The second part of the course will be a more focused study of traveling water waves, specifically their existence theory. A traveling wave (or steady wave), is a wave of permanent configuration: it does not break, it evolves simply by moving at a constant velocity without changing shape. This is truly bizarre behavior for the solution of a nonlinear PDE, yet traveling waves are observed in many physical systems; they are even known to play an important role in the dynamics of the ocean and atmosphere. Mathematical investigations of steady water waves extend back over a century, but a number of substantial breakthroughs have occurred only in the past decade. In addition to their physical significance, these results are mathematically quite beautiful: they draw on deep ideas from applied mathematics, analysis, dynamical systems, and PDEs. Indeed, we will make use of (global) bifurcation theory, topological degree theory, Schauder estimates, Fredholm theory, and more. Prior exposure to any of these will be helpful but not necessary; the course is meant to be as self-contained as possible.

Textbook. There is no official book for the class, but a good reference (particularly for the latter half) is the recent monograph by A. Constantin [Con11], which essentially collects a number of papers on the subject with additional exposition. It is a very useful resource for future researchers in the field. For the merely curious, it can be more-or-less replicated (freely) by referencing journals (primarily, for this course, [CS04]).

Grading. Instead of a written final exam, you will read and present a seminal paper from the mathematical literature on water waves. Beginning around mid-semester, each week we will devote Friday's lecture to one of these presentations. In principle, you can select any relevant paper that interests you (so long as there is no overlap with other students), but a good list to choose from is the following: [BKM84, CE98, CS00, CEW07, Ebi88, CS93, AFT82, GT97, Ser62, Bea77, BT11, Tao09, EEW11]. These are important/interesting results that are also not unreasonably long.

Disabilities. If you anticipate barriers related to the format or requirements of this course, if you have emergency medical information to share with me, or if you need to make arrangements in case the building must be evacuated, please let me know as soon as possible. If disability related accommodations are necessary (for example, a note taker, extended time on exams, captioning), please register with the Disability Center (http://disabilitycenter.missouri.edu), S5 Memorial Union, (573) 882-4696, and then notify me of your eligibility for reasonable accommodations. For other MU resources for persons with disabilities, click on "Disability Resources" on the MU homepage.

Academic Honesty. Academic honesty is fundamental to the activities and principles of a University. Any effort to gain an advantage not given to all students is dishonest whether or not the effort is successful. When in doubt about plagiarism or collaboration, consult the course instructor. The academic community regards academic dishonesty as an extremely serious matter, with serious consequences that range from probation to expulsion. If at any time you have questions about this policy, please ask.

Complaints. If you have communication (or other problems) with your instructor, you can report them to Professor Montgomery-Smith (Director of Graduate Studies) either by phone at (573) 882-4540, or by e-mail (`stephen@missouri.edu`).

References

[AFT82] C. J. Amick, L. E. Fraenkel, and J. F. Toland. On the Stokes conjecture for the wave of extreme form. *Acta Math.*, 148:193–214, 1982.

[Bea77] J. Thomas Beale. The existence of solitary water waves. *Comm. Pure Appl. Math.*, 30(4):373–389, 1977.

[BKM84] J. T. Beale, T. Kato, and A. Majda. Remarks on the breakdown of smooth solutions for the 3-D Euler equations. *Comm. Math. Phys.*, 94(1):61–66, 1984.

[BT11] G. R. Burton and J. F. Toland. Surface waves on steady perfect-fluid flows with vorticity. *Comm. Pure Appl. Math.*, 64(7):975–1007, 2011.

[CE98] Adrian Constantin and Joachim Escher. Wave breaking for nonlinear nonlocal shallow water equations. *Acta Math.*, 181(2):229–243, 1998.

[CEW07] Adrian Constantin, Matts Ehrnström, and Erik Wahlén. Symmetry of steady periodic gravity water waves with vorticity. *Duke Math. J.*, 140:591–603, 2007.

[Con11] Adrian Constantin. *Nonlinear water waves with applications to wave-current interactions and tsunamis*, volume 81 of *CBMS-NSF Regional Conference Series in Applied Mathematics*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2011.

[CS93] W. Craig and C. Sulem. Numerical simulation of gravity waves. *J. Comput. Phys.*, 108(1):73–83, 1993.

[CS00] Adrian Constantin and Walter A. Strauss. Stability of peakons. *Comm. Pure Appl. Math.*, 53(5):603–610, 2000.

[CS04] A. Constantin and W. Strauss. Exact steady periodic water waves with vorticity. *Comm. Pure Appl. Math.*, 57(4):481–527, 2004.

[Ebi88] David G. Ebin. Ill-posedness of the Rayleigh-Taylor and Helmholtz problems for incompressible fluids. *Comm. Partial Differential Equations*, 13(10):1265–1295, 1988.

[EEW11] Mats Ehrnström, Joachim Escher, and Erik Wahlén. Steady water waves with multiple critical layers. *SIAM J. Math. Anal.*, 43(3):1436–1456, 2011.

[GT97] Mark D. Groves and John F. Toland. On variational formulations for steady water waves. *Arch. Rational Mech. Anal.*, 137(3):203–226, 1997.

[Ser62] James Serrin. On the interior regularity of weak solutions of the Navier-Stokes equations. *Arch. Rational Mech. Anal.*, 9:187–195, 1962.

[Tao09] Terence Tao. Why are solitons stable? *Bull. Amer. Math. Soc. (N.S.)*, 46(1):1–33, 2009.