Perspectives on Inference for Restricted Stochastic Dominance

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Prologue

Defining “Better”
“Better”? 

Two distributions (of earnings, productivity, ...)

Which is “better”? 
“Better”?

Two distributions (of earnings, productivity, ...)

Which is “better”?

► Would you prefer to (live there, buy this, use that, ...?)
“Better”?  

2 PDFs
“Better”?

2 CDFs
I prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for my $u(\cdot)$
Expected Utility

I prefer $Y \succeq Z \iff \mathbb{E}[u(Y)] \geq \mathbb{E}[u(Z)]$ for my $u(\cdot)$

You prefer $Y \succeq Z \iff \mathbb{E}[u(Y)] \geq \mathbb{E}[u(Z)]$ for your $u(\cdot)$
Expected Utility

I prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for my $u(\cdot)$
You prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for your $u(\cdot)$
All prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for all $u(\cdot)$
Expected Utility

I prefer $Y \succeq Z \iff \mathbb{E}[u(Y)] \geq \mathbb{E}[u(Z)]$ for my $u(\cdot)$

You prefer $Y \succeq Z \iff \mathbb{E}[u(Y)] \geq \mathbb{E}[u(Z)]$ for your $u(\cdot)$

All prefer $Y \succeq Z \iff \mathbb{E}[u(Y)] \geq \mathbb{E}[u(Z)]$ for all $u(\cdot)$

Most prefer $Y \succeq Z \iff \mathbb{E}[u(Y)] \geq \mathbb{E}[u(Z)]$ for $u \in \mathcal{U}$

All: first-order stochastic dominance (SD$_1$)

Most: utility restricted stochastic dominance (SD$_U$) (thanks to Tim Armstrong)

includes second-order SD (etc.)
Expected Utility

I prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for my $u(\cdot)$

You prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for your $u(\cdot)$

All prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for all $u(\cdot)$

Most prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for $u \in U$

All: first-order stochastic dominance (SD$_1$)

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You prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for your $u(\cdot)$
All prefer $Y \succeq Z \iff E[u(Y)] \geq E[u(Z)]$ for all $u(\cdot)$
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CDFs (Atkinson, 1987, §1)

Poverty line: $v$
Headcount poverty: $F_Y(v)$ and $F_Z(v)$
CDFs (Atkinson, 1987, §1)

Poverty line: $v$

Headcount poverty: $F_Y(v)$ and $F_Z(v)$

Me: $Y \succeq Z \iff F_Y(v) \leq F_Z(v)$ for my $v$

You: $Y \succeq Z \iff F_Y(v) \leq F_Z(v)$ for your $v$

All: $Y \succeq Z \iff F_Y(v) \leq F_Z(v)$ for all $v$

Most: $Y \succeq Z \iff F_Y(v) \leq F_Z(v)$ for $v \in \mathcal{V}$
CDFs (Atkinson, 1987, §1)

Poverty line: \( v \)

Headcount poverty: \( F_Y(v) \) and \( F_Z(v) \)

Me: \( Y \succeq Z \iff F_Y(v) \leq F_Z(v) \) for my \( v \)

You: \( Y \succeq Z \iff F_Y(v) \leq F_Z(v) \) for your \( v \)

All: \( Y \succeq Z \iff F_Y(v) \leq F_Z(v) \) for all \( v \)

Most: \( Y \succeq Z \iff F_Y(v) \leq F_Z(v) \) for \( v \in \mathcal{V} \)

All: first-order stochastic dominance (SD\(_1\))

Most: CDF restricted stochastic dominance (SD\(_\mathcal{V}\))

Condition I of Atkinson (1987, p. 751)
Brief Tangent: Economic Inequality

Literature on measuring inequality, comparing distributions

Similar issue (me/you/all/most), like

- $\epsilon$ of Atkinson (1970, p. 257)
- $\alpha$ of Cowell and Flachaire (2017, §4.3)
Inference

Learning from Data
Literature: Testing

Two features in common:

- Single $H_0$: all-or-nothing
- CDF-based

Barrett and Donald (2003), many others

Good for testing economic theory that implies $SD_1$ $H_0$: $Y$ non-$SD_1 Z$ ($H_1$: $Y$ $SD_1 Z$)

Davidson and Duclos (2013)

Want stronger evidence for $SD_1$ (analog: $H_0$: $\beta = 0$)

Actually $SD_{[v_1, v_2]}$
Two features in common:

- Single $H_0$: all-or-nothing
- CDF-based

$H_0: Y \ SD_1 \ Z$ (or $SD_2, \ldots$)

- 1-sided Kolmogorov–Smirnov
- Barrett and Donald (2003), many others
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Literature: Testing

Two features in common:

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$H_0$: $Y \ SD_1 \ Z$ (or $SD_2, \ldots$)

- 1-sided Kolmogorov–Smirnov
- Barrett and Donald (2003), many others
- Good for testing economic theory that implies $SD_1$

$H_0$: $Y \ nonSD_1 \ Z$ ($H_1: Y \ SD_1 \ Z$)

- Davidson and Duclos (2013)
- Want stronger evidence for $SD_1$ (analog: $H_0: \beta = 0$)
- Actually $SD_{[v_1,v_2]}$
New Perspectives

Single $H_0$ Multiple testing

- Goldman and Kaplan (2018)
- $H_{0v}: F_Y(v) \geq F_Z(v)$ for each $v \in \mathbb{R}$
- Learn about $\mathcal{V} \equiv \{v : F_Y(v) < F_Z(v)\}$ ($Y \text{ SD}_\mathcal{V} Z$)
New Perspectives

Single $H_0$ Multiple testing

- Goldman and Kaplan (2018)
- $H_{0v}: F_Y(v) \geq F_Z(v)$ for each $v \in \mathbb{R}$
- Learn about $\mathcal{V} \equiv \{v : F_Y(v) < F_Z(v)\}$ ($Y$ SD$\mathcal{V}$ $Z$)

CDF-based Utility-based

- Draft circulated for this talk
- $H_{0u}: \mathbb{E}[u(Y)] \leq \mathbb{E}[u(Z)]$ for each $u \in \mathcal{U}$
- Learn about $\mathcal{D} \equiv \{u : \mathbb{E}[u(Y)] > \mathbb{E}[u(Z)]\}$ ($Y$ SD$\mathcal{D}$ $Z$)
Examples (**distcomp** in Stata)
Examples (distcomp RDD)
Examples (distcomp experiment)
Examples (CDF)

Monthly earnings (1980 USD)

Empirical CDF

GK reject
Non-urban
Urban

Empirical CDF

0.0 0.2 0.4 0.6 0.8 1.0

Monthly earnings (1980 USD)

0 500 1000 1500 2000 2500 3000
Examples (utility)

inner 95% CS for higher expected utility (urban > non–urban)
Multiple Testing Goal

Multiple testing procedure (MTP)

- Test $H_{0v}: F_Y(v) \geq F_Z(v)$ for each $v \in \mathbb{R}$
- $\mathcal{V} \equiv \{v : F_Y(v) < F_Z(v)\}$
- $\mathcal{V}^C = \{v : H_{0v} \text{ is true}\}$
Multiple Testing Goal

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- Test $H_{0v}: F_Y(v) \geq F_Z(v)$ for each $v \in \mathbb{R}$
- $\mathcal{V} \equiv \{v : F_Y(v) < F_Z(v)\}$
- $\mathcal{V}^C = \{v : H_{0v} \text{ is true}\}$

Familywise error rate (FWER)

- FWER $\equiv P(\text{reject any true } H_{0v})$
- “Weak control”: FWER $\leq \alpha$ if $\mathcal{V}^C = \mathbb{R}$ (all $H_{0v}$ true)
- “Strong control”: FWER $\leq \alpha$ regardless
Multiple Testing Goal

Multiple testing procedure (MTP)
- Test $H_{0v}: F_Y(v) \geq F_Z(v)$ for each $v \in \mathbb{R}$
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Familywise error rate (FWER)
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- “Strong control”: FWER $\leq \alpha$ regardless

Expected utility version
- Test $H_{0u}: E[u(Y)] \leq E[u(Z)]$ for each $u \in \mathcal{U}$
- Strong control of FWER
MTP vs. All-or-Nothing Test

If $H_0: Y SD_1 Z$ rejected:

- MTP shows where/why (which $v$ or $u$)
MTP vs. All-or-Nothing Test

If $H_0: Y \ SD_1 \ Z$ rejected:
  ▶ MTP shows where/why (which $v$ or $u$)

If $H_0: Y \ SD_1 \ Z$ not rejected:
  ▶ MTP shows evidence favoring $Y \ SD_1 \ Z$ vs. just uncertainty
  ▶ “Reject $H_0: Z \ SD_1 \ Y$” is a crude version of this idea
  ▶ Non-rejection may be type II error if small sample, etc.
Confidence Sets (CDF)

\[ \mathcal{V} \equiv \{ v : F_Y(v) < F_Z(v) \} \]

“Inner” CS: \( 1 - \alpha \leq P(\hat{\mathcal{V}} \subseteq \mathcal{V}) \)

- Invert MTP of \( H_{0v} : F_Y(v) \geq F_Z(v) \) (\( H_{0v} : v \notin \mathcal{V} \))
- \( \hat{\mathcal{V}} = \{ v : H_{0v} \text{ rejected} \} \)
- \( P(\hat{\mathcal{V}} \subseteq \mathcal{V}) = P(\text{reject only false } H_{0v}) = 1 - \text{FWER} \geq 1 - \alpha \)
Confidence Sets (CDF)

\[ V \equiv \{ v : F_Y(v) < F_Z(v) \} \]

“Inner” CS: \( 1 - \alpha \leq P(\hat{V} \subseteq V) \)

- Invert MTP of \( H_{0v} : F_Y(v) \geq F_Z(v) \) \( (H_{0v} : v \notin V) \)
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“Outer” CS: \( 1 - \alpha \leq P(\hat{V} \supseteq V) \)

- Invert MTP of \( H_{0v} : F_Y(v) < F_Z(v) \) \( (H_{0v} : v \in V) \)
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“2-sided” CS: \( 1 - \alpha \leq \Pr(\hat{\mathcal{V}}_1 \subseteq \mathcal{V} \subseteq \hat{\mathcal{V}}_2) \)

- Combine \( 1 - \alpha/2 \) inner \& outer (Bonferroni)
Confidence Sets (Expected Utility)

Same arguments but with $\mathcal{D}$ instead of $\mathcal{V}$

$\mathcal{D} \equiv \{ u(\cdot) : E[u(Y)] > E[u(Z)] \}$

“Inner” CS: $1 - \alpha \leq P(\hat{\mathcal{D}} \subseteq \mathcal{D})$

- Invert MTP of $H_{0u}: E[u(Y)] \leq E[u(Z)]$ \hspace{1em} ($H_{0u}: u \notin \mathcal{D}$)
- $\hat{\mathcal{D}} = \{u : H_{0u} \text{ rejected}\}$
- $P(\hat{\mathcal{D}} \subseteq \mathcal{D}) = P(\text{reject only false } H_{0u}) = 1 - \text{FWER} \geq 1 - \alpha$

“Outer” CS: $1 - \alpha \leq P(\hat{\mathcal{D}} \supseteq \mathcal{D})$

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CS/MTP from Uniform Confidence Band

Uniform confidence band for $\Delta(\cdot) \implies$ CS/MTP

$\Delta(v) \equiv F_Z(v) - F_Y(v)$ \hspace{1cm} $\mathcal{V} = \{ v : \Delta(v) > 0 \}$

$\Delta(u) \equiv E[u(Y)] - E[u(Z)]$ \hspace{1cm} $\mathcal{D} = \{ u : \Delta(u) > 0 \}$

- Inner CS: values where lower band above zero
- Outer CS: values where upper band above zero
- MTP: equivalent to CS like before

Availability

- CDF diff: asymptotic band, but finite-sample CS/MTP

Information vs. comprehension

- EU band more informative, CS/MTP easier to comprehend
CS/MTP from Uniform Confidence Band

Uniform confidence band for $\Delta(\cdot) \implies$ CS/MTP

- $\Delta(v) \equiv F_Z(v) - F_Y(v)$ \hspace{1cm} $\mathcal{V} = \{v : \Delta(v) > 0\}$
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Utility-based CS/MTP

inner 95% CS for higher expected utility (urban > non-urban)
Details

Theoretical & Otherwise
CDF: KS vs. Probability Integral Transform

Kolmogorov–Smirnov MTP/CS

- Reject $H_{0v}$ when $\hat{F}_Y(v) - \hat{F}_Z(v)$ exceeds KS critical value
- Prop. 3 of Goldman and Kaplan (2018)

KS: well-known low tail power

- \texttt{ks.test(c(1:15/21,10^6+1:5),punif)}
  
  D = 0.25, p-value = 0.1376
CDF: KS vs. Probability Integral Transform

Kolmogorov–Smirnov MTP/CS

- Reject $H_{0v}$ when $\hat{F}_Y(v) - \hat{F}_Z(v)$ exceeds KS critical value
- Prop. 3 of Goldman and Kaplan (2018)

KS: well-known low tail power

- `ks.test(c(1:15/21,10^6+1:5),punif)`
  
  D = 0.25, p-value = 0.1376

If cts, $F_Y(Y_i) \sim \text{Unif}(0, 1)$

- Retain finite-sample properties
- Power more even than KS across distribution
- Goldman and Kaplan (2018): two-sample MTP, RDD, computation
CDF: KS vs. Probability Integral Transform

![Graph showing CDF comparison between KS and Probability Integral Transform](image)

- The x-axis represents time (t)
- The y-axis represents the transformed values (B(t))
- The graph compares the KS and Probability Integral Transform methods across various time points.

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CDF: KS vs. Probability Integral Transform

![Plot showing CDF comparison]

- **Dirichlet**
- **KS**
- **Weighted KS**
CDF: KS vs. Probability Integral Transform

Pointwise type I error, nx=ny=40, Fx=Fy=Unif(0,1)

X (or Y)
Rejection probability

Dirichlet KS
CDF: KS vs. Probability Integral Transform

![Graph showing CDF comparison between Dirichlet and KS distributions.](image)
Expected Utility: Asymptotics

\[ \Delta(v) \equiv F_Z(v) - F_Y(v) = E[1 \{ Z \leq v \}] - E[1 \{ Y \leq v \}] \]

\[ \{ f_v(\cdot) : f_v(t) = 1 \{ t \leq v \}, v \in \mathbb{R} \} \text{ is Donsker} \]

\[ \hat{\Delta}(\cdot) : \text{Gaussian limit} \]

\[ \Delta(u) \equiv E[u(Y)] - E[u(Z)] \]

\[ \hat{\Delta}(\cdot) : \text{Gaussian limit and bootstrap consistency if Donsker } \mathcal{U} \]
Expected Utility: Asymptotics

\[ \Delta(v) \equiv F_Z(v) - F_Y(v) = E[1\{Z \leq v\}] - E[1\{Y \leq v\}] \]

- \{f_v(\cdot) : f_v(t) = 1\{t \leq v\}, v \in \mathbb{R}\} is Donsker
- \hat{\Delta}(\cdot): Gaussian limit

\[ \Delta(u) \equiv E[u(Y)] - E[u(Z)] \]

- \hat{\Delta}(\cdot): Gaussian limit and bootstrap consistency if Donsker \( \mathcal{U} \)

Cor. 3.1 of van der Vaart (1996): \( \mathcal{U} \) Donsker if

- non-decreasing
- bounded from below (or above)
- \( 2 + \delta \) moments of envelope function
Expected Utility: MTP

\[ H_{0u}: \Delta(u) \equiv E[u(Y)] - E[u(Z)] \leq 0, \text{ each } u \in U \]

Compute pointwise \( t \)-statistics \( \hat{T}_u = \frac{\Delta(u)}{\hat{SE}_u} \)

Bootstrap cv: \( 1 - \alpha \) quantile of \( \sup_{u \in U} \hat{T}_u \mid \text{all } \Delta(u) = 0 \)

FWER = \( P(\text{reject any true}) \leq P(\sup_{u} \hat{T}_u > \text{cv}) \rightarrow \alpha \)
Expected Utility: MTP

\[ H_{0u} : \Delta(u) \equiv E[u(Y)] - E[u(Z)] \leq 0, \text{ each } u \in \mathcal{U} \]

Compute pointwise t-statistics \( \hat{T}_u = \frac{\hat{\Delta}(u)}{\hat{SE}_u} \)

Bootstrap cv: \( 1 - \alpha \) quantile of \( \sup_{u \in \mathcal{U}} \hat{T}_u \mid \text{all } \Delta(u) = 0 \)

FWER = \( P(\text{reject any true}) \leq P(\sup_u \hat{T}_u > cv) \to \alpha \)

Stepdown (Holm, 1979)

\[ \leq \text{maybe very conservative if many } \Delta(u) > 0 \]
\[ \Rightarrow \text{Re-compute bootstrap cv using only non-rejected } u \]
\[ \Rightarrow \text{Iterate: bounded by oracle test using true } \{u : H_{0u} \text{ true}\} \]

Can also pre-test to remove \( \hat{\Delta}(u) \ll 0, \text{ etc.} \)
Expected Utility: CS

Invert MTP to get CS
Simulation

Performance of New Methods
Setup

\[ Y_i \overset{iid}{\sim} \log N(0, 1) + 0.1, \ i = 1, \ldots, n \]

\[ Z_i \overset{iid}{\sim} \log N(\mu, \sigma) + 0.1, \ i = 1, \ldots, n \]

\( \mathcal{U} \): CRRA w/ risk aversion \( \theta \in [0, 3] \)

Band for \( \Delta(u) = \mathbb{E}[u(Y)] - \mathbb{E}[u(Z)] \)

► Equivalently: \( \Delta(\theta) \) on \( \theta \in [0, 3] \)

CSs for \( \mathcal{D} \equiv \{ u(\cdot) : \mathbb{E}[u(Y)] \geq \mathbb{E}[u(Z)] \} \)

► Equivalently: \( \mathcal{D} \) is subset of \( \theta \in [0, 3] \)
Results: $n = 40$

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\mu$</th>
<th>${\theta : u_\theta \in D}$</th>
<th>Coverage $(1 - \alpha = 0.9)$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>band</td>
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<tr>
<td>0.7</td>
<td>-0.3</td>
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<td>[0.0, 1.1]</td>
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<td>0.3</td>
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<td>-0.3</td>
<td>[0.0, 3.0]</td>
<td>0.920</td>
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<tr>
<td>1.0</td>
<td>0.0</td>
<td>[ ]</td>
<td>0.938</td>
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<td>1.0</td>
<td>0.3</td>
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<td>0.3</td>
<td>[2.5, 3.0]</td>
<td>0.861</td>
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</table>
Results: $n = 100$

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<tbody>
<tr>
<td>0.7</td>
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<td>[0.0, 2.8]</td>
<td>0.907 0.968 0.975 0.993</td>
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<td>0.929 0.965 0.965 1.000</td>
</tr>
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<td>1.0</td>
<td>0.3</td>
<td>[ ]</td>
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<tr>
<td>1.3</td>
<td>-0.3</td>
<td>[0.2, 3.0]</td>
<td>0.901 0.974 0.979 0.995</td>
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<td>1.3</td>
<td>0.3</td>
<td>[2.5, 3.0]</td>
<td>0.887 0.964 0.992 0.972</td>
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</table>
Results: $n = 250$

<table>
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<td>0.0</td>
<td>[0.0, 1.1]</td>
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<td>0.893 0.998 0.998 1.000</td>
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<tr>
<td>1.0</td>
<td>-0.3</td>
<td>[0.0, 3.0]</td>
<td>0.920 1.000 1.000 1.000</td>
</tr>
<tr>
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<tr>
<td>1.3</td>
<td>-0.3</td>
<td>[0.2, 3.0]</td>
<td>0.927 0.976 0.978 0.998</td>
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<td>[2.5, 3.0]</td>
<td>0.892 0.974 0.994 0.980</td>
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Epilogue

Past & Future
Conclusion

“Better”: restricted stochastic dominance based on
▶ CDF
▶ Expected utility

Inference on set of:
▶ values with lower CDF
▶ utility functions with higher expected utility

Future:
▶ non-iid, improve power, implement richer utility family
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▶ restricted stochastic monotonicity
▶ other ideas?

Thank you / further questions & comments welcome
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References I


References II


References III
