Static Consumer and Firm Behavior

Economics 4353 - Intermediate Macroeconomics

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Models and Assumptions

- Our approach will be to build and test models of the macroeconomy.

- A model is a simplified, mathematical description of reality designed to yield insight and generate testable predictions.

- Successful models combine simplicity and explanatory power.

- All models rely on assumptions.

- An assumption is good when it helps build a model that makes accurate predictions and accounts for observations well.
Consumer Preferences

- Starting point: **representative consumer, static** environment.

- Preferences over consumption $C$ of a basket of goods and leisure $l$ given by $U(C, l)$. Refer to $(C, l)$ as a **consumption bundle**.

- Some properties of preferences:
  1. **Monotonicity** (increasing utility): If $C_2 > C_1$ and $l_2 > l_1$, then $U(C_2, l_2) > U(C_1, l_1)$.
     
     “More is better.”

  2. **Convexity** (quasi-concave utility): $U(C_\theta, l_\theta) \geq \min\{U(C_1, l_1), U(C_2, l_2)\}$, where $C_\theta = (1 - \theta)C_1 + \theta C_2$ and $l_\theta = (1 - \theta)l_1 + \theta l_2$.

     “Averages are better than extremes.”

  3. **Normality**: Consumption and leisure are normal goods.
Consumer Preferences

- An **indifference curve** contains bundles that give equal utility.

- The **marginal rate of substitution** of leisure for consumption, \( MRS_{l,C} \) is

  \[
  MRS_{l,C} = \frac{\partial U(C,l)}{\partial l} \bigg/ \frac{\partial U(C,l)}{\partial C} = \frac{U_l(C,l)}{U_C(C,l)}
  \]

- “How much consumption are you willing to give up to obtain 1 more unit of leisure?”

- Convexity \( \Rightarrow \) **diminishing MRS**.
Assume that consumers exhibit **competitive behavior**, i.e. they are price-takers.

Cashless economy (**barter**) assumption: exchange labor for consumption.

Time constraint: \( l + N^s = h \).

Budget constraint: \( C = wN^s + \pi - T \)

\[ \Rightarrow C = w(h - l) + \pi - T \]
Consumer Optimization

- Consumers optimize by maximizing utility subject to their budget constraint:

\[
\max_{C, l} U(C, l) \text{ such that } C = w(h-l) + \pi - T
\]

- Interior solution:

\[
\frac{U_l(C, l)}{U_C(C, l)} = w \text{ and } C = w(h-l) + \pi - T
\]

  - \( MRS_{l,C} = \text{slope of budget constraint} \)

- Corner solution:

\[
l = h, \ C = \pi - T
\]
Consumer Optimization: Detailed Solution

The steps to solving the consumer’s problem are as follows:

1. Write down the consumer problem,

   \[
   \max_{C, I} \quad u(C, I) \\
   \text{subject to} \\
   C = w(h - l) + \pi - T
   \]

2. Write the Lagrangian,

   \[
   L = U(C, I) + \gamma [w(h - l) + \pi - T - C]
   \]

3. Set up the solution conditions,

   First-Order Conditions: \[
   \begin{align*}
   \frac{\partial L}{\partial C} &= 0 \\
   \frac{\partial L}{\partial I} &= 0
   \end{align*}
   \]

   Constraints: \[
   w(h - l) + \pi - T - C = 0
   \]
Consumer Optimization: Detailed Solution

Detailed solution steps, continued:

4. **Calculate** the solution conditions,

First-Order Conditions:

\[
\begin{align*}
\frac{\partial L}{\partial C} &= 0 = U_C(C, l) - \gamma \\
\frac{\partial L}{\partial l} &= 0 = U_I(C, l) - \gamma w
\end{align*}
\]

Constraints:
\[
w(h - l) + \pi - T - C = 0
\]

5. Combine the two first-order conditions to eliminate the multiplier \(\gamma\),

**Solution Conditions**

First-Order Condition:
\[
w = \frac{U_I(C, l)}{U_C(C, l)}
\]

Constraint:
\[
C = w(h - l) + \pi - T
\]
Consumer Optimization: Detailed Solution

Detailed solution steps, continued:

6. If given a specific function $U(C, l)$, isolate either $C$ or $l$ in the first-order condition, substitute into the budget constraint, and solve.

Example: $U(C, l) = \sqrt{C} + \sqrt{l} \Rightarrow$ solution conditions are given by

First-Order Condition: $w = \frac{\sqrt{C}}{\sqrt{l}}$

Constraint: $C = w(h - l) + \pi - T$

From the F.O.C., we isolate $C$ to get $C = w^2l$ and substitute into the budget constraint, giving $w^2l = w(h - l) + \pi - T \Rightarrow l = \frac{wh + \pi - T}{w(w+1)}$. 
Consumer Optimization: Detailed Solution

Detailed solution steps, continued:

7. Lastly, substitute the variable you solved for to find the variable you originally isolated.

From before, solve for $C$ by plugging the solution for $l$ into $C = w^2l$:

\[ l = \frac{wh + \pi - T}{w(w + 1)} \]

\[ C = \frac{w(wh + \pi - T)}{w + 1} \]

- Remember that the goal is to find the consumer’s choices $C$ and $l$ in terms of variables the consumer takes as given, $w$, $\pi$, and $T$.

- If given a generic utility function $U(C, l)$, the two solution conditions are as far as you can go (step 5). Otherwise, complete steps 6 and 7.
Response to a Change in Taxes or Dividends

- How do consumers respond to changes in $\pi$ or $T$?
- Mathematically, what are $\frac{\partial C}{\partial \pi}$ and $\frac{\partial l}{\partial \pi}$?
- Using comparative statics, we get

$$\frac{\partial C}{\partial \pi} = - \frac{\partial C}{\partial T} = \frac{wuU_{CI} - U_{II}}{\Delta} > 0$$
$$\frac{\partial l}{\partial \pi} = - \frac{\partial l}{\partial T} = \frac{U_{CI} - wU_{CC}}{\Delta} > 0$$

where $\Delta \equiv -U_{II} + 2wuU_{CI} - w^2U_{CC} > 0$ because of quasi-concave utility.

- Income effect only.
Response to a Change in the Real Wage

- How do consumers respond to changes in $w$?

- Mathematically, what are $\frac{\partial C}{\partial w}$ and $\frac{\partial l}{\partial w}$?

- Using comparative statics, we get

$$\frac{\partial C}{\partial w} = \frac{wU_C + (h - l)(wU_{Cl} - U_{ll})}{\Delta} > 0$$

$$\frac{\partial l}{\partial w} = \frac{-U_C + (h - l)(U_{Cl} - wU_{CC})}{\Delta}$$

- Income and substitution effects: consumption increases, but labor supply ambiguous because $\left.\frac{\partial l}{\partial w}\right|_{subst} = \frac{-U_C}{\Delta} < 0$. 
Optimal labor supply is \( N^s(w, \pi, T) = h - l(w, \pi, T) \).

When the substitution effect dominates, we get the familiar upward sloping labor supply curve.
Comparative Statics: Detailed Solution

- Comparative statics tell us how choices adjust to a change in a variable that is given, e.g. how $C$ responds to a change in $w$.

- Mathematically, we are looking for $\frac{\partial C}{\partial w}$, $\frac{\partial l}{\partial w}$, $\frac{\partial C}{\partial \pi}$, $\frac{\partial l}{\partial \pi}$, etc.

- If we are able to derive an exact equation for $C$ and $l$, we directly take the above derivatives and we are done.

- If given a generic $U(C, l)$, then to find $C$ and $l$, we can only go as far as the solution conditions,

  First-Order Condition: $w = \frac{U_l(C, l)}{U_C(C, l)}$

  Constraint: $C = w(h - l) + \pi - T$
Comparative Statics: Detailed Solution

The steps to doing comparative statics for general $U(C, I)$ are as follows:

1. Re-write the solution conditions to get two equations equaling zero,

$$U_l(C, I) - wU_C(C, I) = 0$$
$$w(h - I) + \pi - T - C = 0$$

These conditions hold for all $w, \pi, and T$ because $C = C(w, \pi, T)$ and $I = I(w, \pi, T)$ always adjust to changes in $w, \pi, and T$ to ensure the solution conditions are still satisfied.

Because the solution conditions equal zero for all $w, \pi, and T$, we know their derivatives with respect to $w, \pi, and T$ equal zero,

$$\frac{\partial [U_l(C, I) - wU_C(C, I)]}{\partial \pi} = 0, \quad \frac{\partial [U_l(C, I) - wU_C(C, I)]}{\partial w} = 0, \text{ etc.}$$
$$\frac{\partial [w(h - I) + \pi - T - C]}{\partial \pi} = 0, \quad \frac{\partial [w(h - I) + \pi - T - C]}{\partial w} = 0, \text{ etc.}$$
Comparative Statics: Detailed Solution

Detailed comparative statics steps, continued:

1. Calculate the derivative of each solution condition with respect to \( w, \pi, \) or \( T \) (any variable the consumer \textbf{does not} choose) and equate it to zero, where \( C = C(w, \pi, T) \) and \( l = l(w, \pi, T) \). For example,

\[
\begin{align*}
\frac{\partial [U_l(C, l) - wU_C(C, l)]}{\partial w} &= 0 = U_{lC} \frac{\partial C}{\partial w} + U_{ll} \frac{\partial l}{\partial w} - \left[ w \left( U_{CC} \frac{\partial C}{\partial w} + U_{Cl} \frac{\partial l}{\partial w} \right) + U_C \right] \\
\frac{\partial [w(h - l) + \pi - T - C]}{\partial w} &= 0 = h - \left[ w \frac{\partial l}{\partial w} + l \right] - \frac{\partial C}{\partial w} \\
&= \frac{\partial [wl]}{\partial w} \text{(product rule)}
\end{align*}
\]

2. Isolate either \( \partial C \) or \( \partial l \) in the second equation, substitute into the first equation, and solve for the variable you did not isolate.

3. Substitute back to solve for the variable you isolated.
Comparative Statics: Detailed Solution

- Doing comparative statics with a fully general $U(C, l)$ can get rather messy but is not as bad with a slightly less general function.
  - Examples: $U(C, l) = u(C) + v(l)$ or $U(C, l) = u(C + v(l))$ with $u' > 0$, $v' > 0$, $u'' < 0$, and $v'' < 0$.

- Without a specific $U(C, l)$, the best we can usually do is to sign the derivatives of $C$ and $l$ (i.e. $< 0$, $> 0$, or ambiguous).

- Example: $U(C, l) = u(C) + v(l) \Rightarrow$ solution conditions are given by
  
  \[ v'(l) - wu'(C) = 0 \]
  \[ w(h - l) + \pi - T - C = 0 \]
Comparative Statics: Detailed Solution

To find $\frac{\partial C}{\partial \pi}$ and $\frac{\partial l}{\partial \pi}$, first differentiate through both conditions by $\pi$,

\[
\begin{align*}
\frac{\partial [v'(l) - wu'(C)]}{\partial \pi} &= 0 = v''(l) \frac{\partial l}{\partial \pi} - wu''(C) \frac{\partial C}{\partial \pi} \\
\frac{\partial [w(h - l) + \pi - T - C]}{\partial \pi} &= 0 = -w \frac{\partial l}{\partial \pi} + 1 - \frac{\partial C}{\partial \pi}
\end{align*}
\]

Isolate $\frac{\partial C}{\partial \pi}$ in the second equation, plug into the first equation to find $\frac{\partial l}{\partial \pi}$, and substitute $\frac{\partial l}{\partial \pi}$ back to find $\frac{\partial C}{\partial \pi}$,

\[
\begin{align*}
\frac{\partial C}{\partial \pi} = 1 - w \frac{\partial l}{\partial \pi} \Rightarrow 0 &= v''(l) \frac{\partial l}{\partial \pi} - wu''(C) \left(1 - w \frac{\partial l}{\partial \pi}\right) \\
\text{from second equation}
\end{align*}
\]

\[
\Rightarrow \frac{\partial l}{\partial \pi} = \begin{dcases}
\frac{wu''(C)}{v''(l) + w^2u''(C)} & <0 \\
\end{dcases} > 0 \quad \text{and} \quad \frac{\partial C}{\partial \pi} = \begin{dcases}
\frac{v''(l)}{v''(l) + w^2u''(C)} & <0 \\
\end{dcases} > 0
\]
Comparative Statics: Detailed Solution

- To find $\frac{\partial C}{\partial w}$ and $\frac{\partial l}{\partial w}$, first differentiate through both conditions by $w$,

$$\frac{\partial [v'(l) - wu'(C)]}{\partial w} = 0 = v''(l) \frac{\partial l}{\partial w} - \left[ wu''(C) \frac{\partial C}{\partial w} + u'(C) \right]$$

$$\frac{\partial [w(h - l) + \pi - T - C]}{\partial w} = 0 = h - \left[ w \frac{\partial l}{\partial w} + l \right] - \frac{\partial C}{\partial w}$$

- Isolate $\frac{\partial C}{\partial w}$ in the second equation, plug into the first equation to find $\frac{\partial l}{\partial w}$, and substitute $\frac{\partial l}{\partial w}$ back to find $\frac{\partial C}{\partial w}$,

$$\frac{\partial C}{\partial w} = h - l - w \frac{\partial l}{\partial w} \Rightarrow 0 = v''(l) \frac{\partial l}{\partial w} - \left[ wu''(C) \left( h - l - w \frac{\partial l}{\partial w} \right) + u'(C) \right]$$

$$\Rightarrow \frac{\partial l}{\partial w} = \frac{u'(C) + wu''(C)(h - l)}{v''(l) + w^2 u''(C)} \text{ (ambiguous sign) and } \frac{\partial C}{\partial w} = \frac{-wu'(C) + v''(l)(h - l)}{v''(l) + w^2 u''(C)} > 0$$
Firms

- Starting point: a representative firm with production technology 
  \( Y = zF(K, N) \).
  - Inputs are capital \( K \) and labor \( N \) with total factor productivity \( z \).

- Production satisfies:
  1. **Constant returns to scale**: 
     \( F(xK, xN) = xF(K, N) \) for all \( x > 0 \).
  2. **Monotonicity**: \( F_K > 0 \) and \( F_N > 0 \).
  3. **Decreasing marginal product**: 
     \( F_{KK} < 0 \) and \( F_{NN} < 0 \).
  4. **Complementarity**: \( F_{KL} > 0 \).
Marginal Product and Technological Improvements

- **Marginal Product of Labor (MP\textsubscript{L}):**
  - \( F(K^*, N^d) \)
  - Slope = \( MP\textsubscript{L} \)

- **Technological Improvements:**
  - Increase in \( F(K^*, N^d) \)
  - \( z_1 F(K^*, N^d) \)
  - \( z_2 F(K^*, N^d) \)
  - \( MP^1_{L} \)
  - \( MP^2_{L} \)
Profit Maximization

- In the short run \( K \) is fixed, but firms choose \( N \) to solve

\[
\max_N zF(K, N) - wN
\]

- The optimality condition is \( zF_N(K, N^d) = w \) where \( N^d \) is optimal labor.

- How does \( N^d \) respond to changes in \( w, z, \) and \( K \)?

\[
\frac{\partial N^d}{\partial w} = \frac{1}{zF_{NN}} < 0
\]
\[
\frac{\partial N^d}{\partial z} = \frac{-F_N}{zF_{NN}} > 0
\]
\[
\frac{\partial N^d}{\partial K} = \frac{-F_{KN}}{F_{NN}} > 0
\]
Cobb-Douglas Production

- Consider the Cobb-Douglas production function \( Y = zK^\alpha N^{1-\alpha} \).

- Short-run profit maximizing labor choice is \( N^d = \left( \frac{(1-\alpha)z}{w} \right)^{1/\alpha} K \).

- Total wage bill divided by output is \( \frac{wN^d}{Y} = 1 - \alpha \Rightarrow \) constant labor share of income.

- Labor share of income in U.S. has been approximately constant with \( 1 - \alpha = 0.64 \).
Solow Residuals and Labor Productivity

- Can use measures of output $Y$, capital $K$, and labor $N$ to extract Solow residuals $z = \frac{Y}{K^{0.36}N^{0.64}}$.

- Total factor productivity different from average labor productivity, $\frac{Y}{N} = z \left( \frac{K}{N} \right)^{0.36}$.
Productivity in the 2008-2009 Recession

- Given the severity of the 2008-2009 recession, average labor productivity declined surprisingly little.

- Potential causes:
  1. Recession originated more in housing and financial services.
  2. Long term sectoral shifts in employment.