1. a. If you have to show a function \( f \) is one-one, begin with \( f(x) = f(y) \). Proceed logically from there using whatever is known or given. Then arrive at \( x = y \). Now you can conclude that \( f \) is one-one.

   b. If you have to prove a function \( f \) is onto, start with \( f(x) = y \). Solve for \( x \).

2. To show that \( f \) is well defined, start with \( x = y \) and proceed from there to arrive at \( f(x) = f(y) \). Then you can conclude that \( f \) is well defined.

3. To show a set \( A \) is infinite, show that any subset of \( A \) with \( n \) elements is not all of \( A \) or that \( A \) contains more than \( n \) elements for any \( n \) or exhibit a specific one-one function from the set of natural numbers in \( A \) (it does not have to be onto). If you can show that \( A \) contains another subset which is already known to be infinite, then \( A \) is infinite.

4. In order to use induction to prove something:
   a. There must be a varying positive integer in the statement.
   b. You need to know the proof for at least one particular positive integer, say 1.
   c. You need to be able to reduce the proof for \( n \) to that of \( n-1 \) by some argument specific to the problem.

5. If you have to check that a number \( n \) is a multiple of a number \( k \), it is a good idea to use division algorithm to write, \( n = ka + r \). Then knowing that \( r = 0 \) or \( 0 \leq r < k \) will be helpful.

6. To prove a subset \( H \) of \( G \) is a subgroup, you must do these three things.
   i. Prove that \( e \in H \).
   ii. Let \( x, y \in H \). Then show that \( x * y \in H \).
   iii. Let \( x \in H \). Show that \( x^{-1} \in H \).

7. To prove that a group \( G \) is abelian or commutative. Let \( x, y \in G \) be any two arbitrary elements in \( G \). Show that \( x * y = y * x \). Warning: Just knowing some two elements in \( G \) commute is not enough to conclude the group is abelian.

8. To show a group is not abelian, all you need to do is to find and specify two elements in it that do not commute.

9. To show that a finite group \( G \) is cyclic, all you need to do is to produce one element in \( G \) that has the same order as that of \( G \).

10. To compute the order of an element \( g \) in a group \( G \), compute \( g, g^2, \ldots \), till you get the identity. The first time you hit identity, you get the order of the element \( g \).

11. If \( g \) in \( G \) has infinite order, then \( g^n \neq e \) for any positive integer \( n \).

12. To find all subgroups of a group \( G \), first write down all the cyclic groups, then the groups generated by two elements and so on. If you put an element \( g \) in a subgroup, remember, the entire cyclic group generated by \( g \) must be in that subgroup. So, pick an element \( h \) not in the cyclic subgroup \( \langle g \rangle \) to get a group generated by two elements.

13. Some simple ways to see that a group \( G \) is not isomorphic to \( H \), are
   a) they have different orders
   b) One is abelian and the other is not.
   c) One is cyclic and the other is not.
   d) for some number \( t \), the number of elements of order \( t \) in \( G \) is different from that of \( H \).

14. To determine all abelian groups up to isomorphism of order \( n \), use the structure theorem for abelian groups.

15. Determine all groups up to isomorphism of order \( n \).
   For small \( n \), use Lagrange’s theorem to do this problem.
   First use structure theorem to classify all abelian groups.
   Then use Lagrange’s theorem and normal subgroups etc, if necessary to classify the rest. For small \( n \), Lagrange’s theorem will suffice.

16. To show two groups are isomorphic, write down a homomorphism and show that it is one-one and onto.

17. To show that a subgroup \( H \) is normal in \( G \):
   Let \( h \in H, g \in G \) be arbitrary. Show that \( ghg^{-1} \in H \).
or Show that there is only one subgroup in $G$ of order $H$.

or show that $gH = Hg$ for all $g \in G$

18. To determine all homomorphisms from $G$ to $H$: Use the fact that $o(f(x))$ divides the order of $x$ for every $x$ in $G$.

19. To give examples of many groups of order $n$, factor $n$, and use the fact $|G_1 \times G_2| = |G_1||G_2|$ and $G_1 \times G_2$ is abelian if and only if both $G_1$ and $G_2$ are and $G_1 \times G_2$ is cyclic if and only if $G_1$ and $G_2$ are cyclic of relatively prime orders.

20. To construct a homomorphism $\phi : G \to H$ where

(i) $G$ is cyclic $G = \langle g \rangle$ you just need to send $g$ to any element in $H$ whose order is a factor of $o(g) = |G|$

(ii) $G$ is the dihedral group $D_{2n} = \langle r, f, r^n = f^2 = e, fr = r^{n-1}f \rangle$, you send $\phi(r)$ to anything which is a factor of $n$ and $f$ to any element in $H$ of order 2 or $e$, and then check if $\phi(f)\phi(r) = \phi(r)^{n-1}\phi(f)$. If it checks, then it is a homomorphism. If it does not, it is not.