1. a. Prove that if A and B are invertible then the product AB is invertible.
   b. Prove that if the product of two square matrices AB is invertible then both A and B must be invertible. (Cannot use determinants.)

2. Determine if the following matrices are invertible. If an inverse exists, find it.

   \[
   A = \begin{bmatrix}
   2 & 0 & -1 \\
   -3 & 3 & 4 \\
   2 & 1 & 0
   \end{bmatrix},
   B = \begin{bmatrix}
   2 & 1 & 5 & 7 \\
   0 & 2 & 1 & 3 \\
   0 & 0 & 3 & 1 \\
   0 & 0 & 0 & 8
   \end{bmatrix},
   C = \begin{bmatrix}
   3 & -1 & 1 \\
   1 & 4 & 3 \\
   5 & -6 & -1
   \end{bmatrix}
   \]

3. Find all possible values of x that will make the following matrix singular.

   \[
   \begin{bmatrix}
   2 & 3 & -1 \\
   x & -3 & 2 \\
   0 & 1 & x
   \end{bmatrix}
   \]

4. Which of the following are true? If true, give your reasoning. If not true, give an example to show that it may not be true.
   i. Inverse of a triangular matrix, if it exists, is also triangular.
   ii. Inverse of a diagonal matrix, if it exists, is also diagonal.
   iii. Inverse of a symmetric matrix, if it exists, is also symmetric.
   iv. If an invertible matrix has integer entries, then its inverse also has integer entries.