National Debt, Savings, and Real Interest Rates in a Neoclassical Model with Endogenous Labour Supply and Knowledge-Based Growth

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National debt, savings, and real interest rates in a neoclassical model with endogenous labour supply and knowledge-based growth

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Abstract. In this paper the interest rate effect of government deficit financing in neoclassical model with endogenous labour supply and knowledge-based growth is examined. It is shown that government deficits do not raise interest rates if consumption and leisure are good substitutes and there is a large spillover effect in knowledge creation. JEL Classification: E13, E60, H60

Dette nationale, épargnes et taux d'intérêt réels dans un modèle néo-classique où la croissance est fondée sur la connaissance et l'offre de travail endogène. Ce mémoire examine les effets sur le taux d'intérêt du financement d'un déficit gouvernemental dans un modèle néo-classique où la croissance est fondée sur la connaissance et l'offre de travail endogène. On montre que les déficits gouvernementaux ne font pas croître les taux d'intérêt si la consommation et le loisir sont des biens substituts et qu'il y a des effets externes importants qui découlent de la production de la connaissance.

1. Introduction

A widely cited neoclassical proposition is that debt financing of government spending reduces private investment and raises real interest rates. The intuition is as follows. For a finitely lived economic agent, an one dollar debt-for-tax swap increases his lifetime income and current consumption. If there is no corresponding change in savings of other currently alive agents (note that in a two-period overlapping-generations model only the young save), then the total personal savings of the economy (including the government debt) is increased by a fraction of a dollar but private investment is decreased, since the vehicle of saving (government debt or private investment) makes no difference to investors. Given diminishing returns to capital, the reduction in the supply of investment increases the marginal

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productivity of capital and therefore the equilibrium interest rates. This intuition is formally illustrated by the seminal paper of Diamond (1965).

The central implication of the Diamond model, that government deficits raise long-term interest rates, has been the subject of numerous empirical studies. The results are mixed. Some authors (e.g., Feldstein 1986; Hoelscher 1986; Miller and Russek 1991; Cebula and Belton 1993) conclude that government deficits indeed raise long-term interest rates. While others (e.g., Evans 1985, 1987; Plosser 1982, 1987; Mascaro and Meltzer 1983; Boothe and Reid 1989) find that government deficits have insignificant and sometimes negative effects on long-term interest rates.

The objective of the present paper is to demonstrate that if we relax a key assumption of Diamond model that capital is the only variable input in production, then government debt does not necessarily increase real interest rates in the neoclassical model. We extend the Diamond model by allowing for two elastically supplied inputs, capital as well as labour.1 Furthermore, we assume that the economy experiences knowledge-based growth. The source of growth in knowledge is learning-by-doing in the spirit of Arrow (1962) and Lucas (1988). In the economy considered in the present paper, non-excludable knowledge is created as a by-product of each generation’s labour activities and benefits future generations. Kohn and Marion (1992) call this knowledge spillover an ‘intertemporal externality.’2

The main result of the research is that the effect of government deficits on interest rates depends on the spillover of knowledge and the elasticity of labour supply. In the Diamond model, a debt-for-tax swap only reduces the supply of capital. In the present paper, a debt-for-tax swap reduces the supply of capital as well as the demand for capital. If consumption and leisure are good substitutes, then shifting tax burden to the future not only reduces current savings and the supply of capital stock in the near future but also reduces labour supply of the near future. The impact of the reduction in future labour supply on real interest rates may partially offset that of the reduction in capital stock. Moreover, current labour supply is reduced by government deficits and slows dow accumulation of knowledge. While the reduction in physical capital makes equilibrium interest rate higher, the reduction in knowledge spillover and future labour supply pushes the interest rate lower. It is possible that the effect of the government debt on knowledge spillover and labour supply overcomes the effect on the accumulation of physical capital. The empirical implication of the model is that the interest rate effect of government deficits, which the empirical studies estimate, may be small in magnitude and even have the opposite sign than that predicted by the Diamond model.

1 Neoclassical growth models with variable labour supply and with human capital accumulation have been used in the analysis of government fiscal policies (e.g., Auerbach, Kotlikoff, and Skinner 1983; Stokey and Rebelo 1995). But these studies are focused on the impact of distortionary taxes rather than the effect of government deficits on interest rates.

2 Lucas (1988) and Romer (1986) argue that cross-country empirical evidence is consistent with the view that positive externality in knowledge plays a crucial role in economic growth. Kohn and Marion (1992) show that intertemporal externality of knowledge has important implications for whether a country should open its capital market to the world.
Numerical examples in the present paper illustrate quantitative importance of elastic labour supply and knowledge spillover on the interest rate effect of government deficits. Furthermore, the examples show non-trivial dynamics of interest rates. The latter result suggests that steady-state analysis may not offer adequate guidance for empirical studies.

The paper is organized as follows. In section 2 the model is set up. In section 3 competitive equilibrium savings and interest rates under alternative deficit policies are characterized. Numerical examples are contained in section 4. In section 5 the empirical literature of interest rate tests is discussed. Finally, the paper concludes in section 6.

2. The Model

The economy under consideration is an extended Diamond's overlapping-generations set-up. The economy starts at period 0. At any given period $t > 0$, the economy consists of two generations of agents of constant population, young and old, each having a life span of two periods. The government spends $G$ in period 0 and has no spending thereafter. The one-time spending may be thought as war-related government military purchases or as a lump-sum transfer to the period 0 old. The government considers the following alternative plans of lump-sum taxes to finance the spending:

**Policy One:** Collect the lump-sum tax solely from the generation born at period 0.

**Policy Two:** Collect the lump-sum tax solely from the generation born at a given period $T (T \geq 1)$.

In other words, Policy Two shifts the tax burden to a future generation. We assume that the amount of government spending $G$ is small enough so that neither policy results in corner solutions. The question under examination is whether equilibrium interest rates are necessarily higher under Policy Two than under Policy One.

Assume that a representative agent of each generation works when he is young and saves for retirement. Let $C_{t,1}$ and $C_{t+1,2}$ be the consumption of the first and second period of life of an agent born at period $t$, and $1 - L_t$ be the leisure of the young. The utility function is assumed to be

$$\ln [\theta C_{t,1}^\eta + (1 - \theta)(1 - L_t)^\eta]^\frac{1}{\eta} + \beta \ln C_{t+1,2},$$

where $\theta \in [0, 1], \eta \leq 1$, and $\beta \in (0, 1)$. Here $\eta$ measures the substitutability between consumption and leisure. $\eta = 1$ corresponding to perfect substitution

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3 Diamond (1965) considers balanced growth path, while in this paper we examine the dynamics of equilibrium savings and interest rates. On the other hand, no parametric assumptions on the utility and production functions are needed in the Diamond paper, but they are needed in this paper.

4 Since in this economy there is no growth in population and technology, it is not possible to postpone permanently the taxation unless the net interest rate is less than zero.
and \( \eta = -\infty \) corresponds to no substitutability. When \( \eta = 0 \) the utility function becomes \( \theta \ln C_{t,1} + (1 - \theta) \ln (1 - L_t) + \beta \ln C_{t+1,2} \).

There are several reasons to focus on this particular utility function rather than a more general one. This utility function enables us to prove the uniqueness of competitive equilibrium, and to characterize the equilibrium allocations and prices. Unlike the Diamond model, where the labour supply is inelastic and the dynamics of the equilibrium can be traced by a single difference equation concerning capital stock or interest rate, here the dynamics generally involve two difference equations pertaining to both capital stock and labour. The present utility function allows for unambiguous solutions yet the utility function is general enough to yield some insights. In particular, it reveals an important role the parameter \( \eta \) plays in determining the effect of deficit policy on interest rates.

As for the production function, the output at period \( t \) is given by

\[
Y_t = K_t^\alpha (H_tL_t)^{1-\alpha}, \quad \alpha \in (0,1).
\] (2)

\( K_t \) is the private investment made by the generation born at \( t - 1 \) and is given at period \( t \). Depreciation of capital is assumed to be total. To simplify notation, capital stock will not be distinguished from private investment. In (2), \( H_t \) is a measure of current state of knowledge, \( L_t \) is labour service, and \( H_tL_t \) is effective labour. \( H_t \) is taken as given, while \( L_t \) is chosen by the generation born at \( t \). Following the convention of the growth theory literature, we assume that the accumulation of knowledge \( H_t \) depends on the economic activities and the existing stock of knowledge. Specifically, the evolution of knowledge is given by

\[
H_{t+1} = AL_t^\gamma H_t^\psi,
\] (3)

where \( A, \gamma, \psi > 0 \), and \( H_0 \) is given.

Equation (2) suggests that existing knowledge contributes to current production, but the current young generation does not compensate the old for this spillover. By the same token, the current young generation cannot be compensated for its role in creation of new knowledge and know-how. This positive intertemporal externality is an essential element in the endogenous growth theory of Kohn and Marion (1992).

A competitive firm chooses capital and labour so that equilibrium real interest rate

\[
r_t = \alpha(K_t/H_tL_t)^{\alpha-1},
\] (4)

and real wage

\[
w_t = (1 - \alpha)H_t^\alpha(K_t/L_t)^\alpha.
\] (5)

5 For arbitrary utility functions, uniqueness of competitive equilibrium in overlapping generations models is not guaranteed. See Galor and Ryder (1989) for sufficient conditions of uniqueness of equilibrium with production and inelastic labour supply.
In period 0, the wage and interest rates relevant to the young generation are \(w_0\) and \(r_1\), respectively. Under Policy One, the period-0-born young generation bears the tax \(G\) and has budget constraints\(^6\)

\[
C_{0,1} + K_1 + G = w_0L_0, \text{ and } C_{1,2} = r_1K_1. \quad (6a)
\]

The budget constraints for later generations are

\[
C_{t,1} + K_{t+1} = w_tL_t, \text{ and } C_{t+1,2} = r_{t+1}K_{t+1}, \quad t > 0. \quad (6b)
\]

Under Policy Two, all variables are marked by ‘\(^\sim\)’ (tilde) to distinguish them from their counterparts under Policy One. Under Policy Two, tax is collected only from the generation born at period \(T\) (\(T \geq 1\)). It follows that the budget constraints for the period \(t(0 \leq t < T)\) born are

\[
\tilde{C}_{t,1} + \tilde{K}_{t+1} + \tilde{B}_{t+1} = \tilde{w}_t\tilde{L}_t, \text{ and } \tilde{C}_{t+1,2} = \tilde{r}_{t+1}(\tilde{K}_{t+1} + \tilde{B}_{t+1}). \quad (7a)
\]

Under Policy Two, the government budget constraints are \(B_1 = G\), and \(B_{t+1} = r_tB_t\) for \(t < T\), and \(B_t = 0\) for \(t > T\).

The budget constraints for the period \(T\) born are

\[
\tilde{C}_{T,1} + \tilde{K}_{T+1} + \tilde{r}_T B_T = \tilde{w}_T\tilde{L}_T, \text{ and } \tilde{C}_{T+1,2} = \tilde{r}_{T+1}\tilde{K}_{T+1}. \quad (7b)
\]

The budget constraints for all future generations born after period \(T\) are

\[
\tilde{C}_{t,1} + \tilde{K}_{t+1} = \tilde{w}_t\tilde{L}_t, \text{ and } \tilde{C}_{t+1,2} = \tilde{r}_{t+1}\tilde{K}_{t+1}, \quad t > T. \quad (7c)
\]

The variables of particular interest are \(r_t\) and \(\tilde{r}_t\). It is often taken for granted that neoclassical growth models imply \(K_t > k_t\) and \(r_t < r_t\) (for all \(t > 0\)). In the present model these inequalities do not always hold.

2.1. A special case corresponding to the Diamond model

The Diamond model of inelastic labour supply corresponds to the case \(\theta = 1, H_0 = 1, A+1, \gamma = 0,\) and \(\psi = 1\). In this case \(L_t = 1\) and \(H_t = 1\) for all \(t\). According to (4), \(r_t < \tilde{r}_t\) if and only if \(K_t > \tilde{K}_t\) for \(t > 0\). In the following we first show that \(K_1 > \tilde{K}_1\).

The optimal saving decisions by the period 0 born under the alternative policies imply that \(C_{1,0} = K_1/\beta\), and \(\tilde{C}_{1,0} = (\tilde{K}_1 + G)/\beta\). Taking these into the corresponding budget constraints yields \(K_1 = (w_0 - G)/(1 + 1/\beta)\) and \(\tilde{K}_1 = w_0/(1 + 1/\beta) - G\).

From (5), \(w_0\) is determined by \(K_0\) and hence is exogenous. Therefore, \(K_1 > \tilde{K}_1\).

\(^6\) Here the tax is imposed at period 0. But as long as the current value of the tax burden falls on a given generation, whether the tax is collected at the first or second period of the agent’s life does not make any difference.

\(^7\) The policy analysis in this paper is different from Diamond’s comparative static study of changes in the level of constant government debt. The current setting is chosen because it makes clear how the nominal tax burden is shifted, whereas under the assumption of constant debt the tax burden on a given generation depends on the equilibrium net interest rate at the time.
It follows that $r_1 < \tilde{r}_1$. It is easy to show by iteration that $w_t > \tilde{w}_t, K_t > \tilde{K}_t$, and $r_t < \tilde{r}_t$ for all $t > 0$.

3. Characterizing competitive equilibrium under alternative deficit policies

Now we consider the case $0 < \theta < 1$ where labour service is no longer fixed at unity. In this section, we first prove that there is always a unique (interior) equilibrium under either of the two policies and then compare the equilibrium interest rates.

At period $t$, under Policy One the marginal conditions for optimal choices of savings and leisure are

\[
\theta C_{t,1}^{\eta - 1} / [\theta C_{t,1}^{\eta} + (1 - \theta)(1 - L_t)^{\eta}] = \beta / K_{t+1} \tag{8}
\]

and

\[
w_t \theta C_{t,1}^{\eta - 1} = (1 - \theta)(1 - L_t)^{\eta - 1} \text{ for all } t. \tag{9}
\]

Under Policy Two, (9) still holds for all $t$. For generations born before and after period $T$ (the period when the tax is collected) the corresponding Euler equations for optimal savings are different:

\[
\theta C_{t,1}^{\eta - 1} / [\theta C_{t,1}^{\eta} + (1 - \theta)(1 - L_t)^{\eta}] = 1/(K_{t+1} + B_{t+1}) \text{ for } t < T \tag{8'}
\]

\[
\theta C_{t,1}^{\eta - 1} / [\theta C_{t,1}^{\eta} + (1 - \theta)(1 - L_t)^{\eta}] = 1/K_{t+1} \text{ for } t \geq T. \tag{8''}
\]

An equilibrium under Policy One is a sequence \( \{C_{t,1}, C_{t+1,2}, K_{t+1}, L_t, H_{t+1}, w_t, r_{t+1}\}_{t=0}^{\infty} \) that solves (6a), (6b), (8), and (9), with $K_0$ and $H_0$ given. Similarly, an equilibrium under Policy Two is a sequence \( \{\tilde{C}_{t,1}, \tilde{C}_{t+1,2}, \tilde{K}_{t+1}, \tilde{L}_t, \tilde{H}_{t+1}, \tilde{w}_t, \tilde{r}_{t+1}\}_{t=0}^{\infty} \) that solves (7a), (7b), (7c), (8'), (8''), and (9) with $K_0$ and $H_0$ given. Before we compare equilibrium savings and interest rates under the alternative government policies, we first establish the existence and uniqueness of the equilibria.

**Proposition 1.** Under either Policy One or Policy Two, there is a unique equilibrium.

For a proof see the appendix.

Proposition 2 further characterizes the economy in the initial period under alternative deficit policies.

**Proposition 2.** Under Policy One, the initial-period labour service, knowledge creation, and savings are larger than those under Policy Two. That is, $L_0 > \tilde{L}_0, H_1 > \tilde{H}_1$, and $K_1 > \tilde{K}_1$.

For a proof see the appendix.
Proposition 2 indicates that, at least in the initial period, delaying taxation reduces employment and savings of the current young. The interest rate effect in the initial period with variable labour supply is not as obvious as that in the Diamond model because the interest rate is a function of the capital-effective labour ratio, which involves the labour service of the next generation. We need to solve for \( L_i \) and \( \tilde{L}_i \) in order to compare \( r_i \) with \( \tilde{r}_i \).

Consider the interest rate of the initial period. From (4), the ratio of interest rates under the two deficit policies is given by

\[
\frac{r_i}{\tilde{r}_i} = \left[ \frac{H_i}{\tilde{H}_i} \left( \frac{L_i}{\tilde{L}_i} \right) \left( \frac{K_i}{\tilde{K}_i} \right) \right]^{1-\alpha}.
\]

Deficits raise interest rates in period 0 if \( r_1/\tilde{r}_1 < 1 \). Proposition 2 states that \( H_1 < \tilde{H}_1 \) and \( K_1 < \tilde{K}_1 \). The effect of deficit on capital and knowledge accumulation has the opposite effect on the productivity of capital and real interest rates. The reduction on savings tends to raise interest rates, but the reduction in knowledge accumulation tends to reduce interest rates. To compare \( \tilde{r}_1 \) with \( r_1 \), we also need to calculate \( \tilde{L}_1 \) and \( L_1 \). It is not obvious whether \( \tilde{L}_1 \) is larger or smaller than \( L_1 \). From (10), if \( \tilde{L}_1 < L_1 \), then it is less likely that deficits raise interest rates. We know from the proof of proposition 2 that deficit financing reduces the consumption of future generations, owing to lower capital and knowledge input in production. Given that the period 1 consumption and labour productivity are lower under Policy Two, if leisure is a good substitute for consumption, then leisure should be higher. That is, if \( \eta \) is positive, then \( \tilde{L}_1 < L_1 \) is more likely to be true. The intuition is formalized in the following proposition.

**Proposition 3.** If \( T > 1 \) (i.e., tax is not collected from the generation born in period 1), then \( L_1 < (\geq, >)\tilde{L}_1 \) if \( \eta < (\geq, >)0 \).

For a proof see the appendix.

4. Numerical examples

In this section we examine several numerical examples that show different patterns of equilibrium interests under alternative deficit policies. Without losing generality, in all examples we assume \( T \) is 2. That is, under Policy One the tax is collected from the generation born in period 0, and under Policy Two the tax is collected from the generation born in period 2. Experiments show that letting \( T \) be larger or less than 2 does not change the nature of the examples. In these examples we set the share of capital in the production function \( \alpha \) to be 0.3. The discount factor \( \beta \) is set at 0.36. If each period in the model is twenty years, then the value of \( \beta \) implies an annual discount rate of 0.95. We select \( \theta \) so that steady-state labour service is about one-third of time endowment, as observed by Ghez and Becker (1975) for the U.S. economy. Prescott (1986) argues that the long-term statistics for the U.S. economy are consistent with the notion that elasticity of substitution between leisure and consumption is close to unity (i.e., \( \eta \) is close to zero). Many other
Savings and interest rates under alternative policies with parameter values $T = 2, \eta = 0, \psi = 1, \lambda = 1, \gamma = 0, \theta = 1, G = 0.024, K_0 = 0.09, H_0 = 1$

<table>
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<th>Period $t$</th>
<th>$K_{t+1}/\bar{K}_{t+1}$</th>
<th>$r_{t+1}/\bar{r}_{t+1}$</th>
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<td>1.000</td>
</tr>
</tbody>
</table>

Example 1

This is a special case of Diamond model, the case with inelastic labour supply and no knowledge spillover. Table 1 reports the ratio of variables under Policy One and Policy Two. The latter equilibrium is represented by the ‘~’ sign (tilde).

Here the labour supply is fixed at unity. The pattern of the dynamics is that savings are reduced and interest rates are raised significantly in every period by government deficits. Under the deficit finance policy, taxes are collected from the generation born in period 2 to finance the initial government spending of 5 per cent steady-state output. Compared with equilibrium of no government deficits, under the deficit finance policy savings are only about one-fourth as much in period 2 and the interest rate is almost tripled.

In the following examples we allow for elastic labour supply and knowledge spillover. The dynamic effects of government deficits on savings and interest rates exhibit patterns sharply different from Example 1.

Example 2

This example exhibits the pattern that $K_t > \bar{K}_t$ and $H_t > \bar{H}_t$ for all $t$, but $r_t > \bar{r}_t$. That is, although in every period capital stock and knowledge stock are reduced...
by government borrowing, the real interest rate in the initial period is lowered by government deficits. The initial government spending $G$ is about 5 per cent of the steady-state output. The numerical results are given in Table 2.

The lower interest rate under deficit policy is explained as follows. Under Policy One, the disposable income of generation born at period 0 is reduced, leading to lower consumption and higher labour supply. Higher labour input leads to higher knowledge accumulation. In this example, $\eta$ is positive. According to proposition 3, in this case the period 1 equilibrium labour service is higher under Policy One than under Policy Two. It follows from equation (10) that when the deficits make big enough difference in knowledge spillover and labour supply, then even the savings are reduced and the interest rate can still be higher. Table 2 also shows that the interest rate effect of government deficits is quantitatively small, given that the initial government spending is 5 per cent of steady-state output. Such a small impact may not be easy to detect, regardless of whether deficits raise rate in the initial period.

Example 2 shows that deficits do not necessarily raise interest rates even when they reduce savings. But do deficits always reduce savings? The following example suggests the answer is no.

Example 3
This example exhibits the pattern that $K_t < \bar{K}_t$ and $H_t < \bar{H}_t$ for some $t > 1$. The parameter settings are similar to Example 2, except that the consumption and leisure are not as strong substitutes as in the previous example. As in the earlier examples, the initial government spending is about 5 per cent of steady state output.

Note that although proposition 2 shows that in the initial period saving and
knowledge are reduced by government deficits, Table 3 shows that they may not be reduced in every period. The reason is that the timing of increases in labour service creates complicated dynamics in knowledge stock, hence complicated dynamics of demand for capital. Under Policy One, in period 0 labour supply increases above the steady-state level, owing to the wealth effect of taxation. In the following period the knowledge stock also rises above its steady-state level and starts to decline afterwards. Under Policy Two, the surge in labour supply occurs in period 2. The rise in knowledge stock occurs in period 3, when knowledge stock under Policy One already converges to the steady-state level. After period 3 the relatively high knowledge stock under Policy Two raises the demand for capital and eventually makes capital stock larger than it is under Policy One.

The examples above show that the magnitude of the interest rate effect of government deficits depends on both the elasticity of labour supply as well as the intertemporal spillover of knowledge. The following proposition shows, however, that in absence of the intertemporal externality government deficits always raise interest rates. In other words, allowing for elastic labour supply alone does not qualitatively change the conventional neoclassical conclusion.

**PROPOSITION 4.** Government deficits always reduce savings and raise interest rates if consumption and leisure are not perfect substitutes and there is no knowledge-based growth. That is, \( K_t > \bar{K}_t \) and \( r_t < \bar{r}_t \) for all \( t \geq 1 \) if \( \eta < 1 \) and \( H_t \) is constant.

For a proof see the appendix.

**Example 4**

This example is different from the previous ones in that it considers an economy of sustained growth. Note that the source of sustained growth must come from growth in the stock of knowledge. On a balanced path of growth, labour supply
TABLE 4
Labour supply, savings, knowledge spillover, and interest rates under alternative policies with parameter values $T = 2, \eta = 0, \psi = 1, A = 10, \gamma = 2.0, \theta = 0.15, G = 0.013, K_0 = 0.1, H_0 = 1$

<table>
<thead>
<tr>
<th>Period $t$</th>
<th>$L_t / \bar{L}_t$</th>
<th>$K_{t+1} / \bar{K}_{t+1}$</th>
<th>$H_{t+1} / \bar{H}_{t+1}$</th>
<th>$r_{t+1} / \bar{r}_{t+1}$</th>
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TABLE 5
The growth rates of savings and knowledge in example 4

<table>
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<tr>
<th>Period $t$</th>
<th>$K_{t+1} / K_t$</th>
<th>$\bar{K}_{t+1} / \bar{K}_t$</th>
<th>$H_{t+1} / H_t$</th>
<th>$\bar{H}_{t+1} / \bar{H}_t$</th>
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</thead>
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$L_t$ is constant over time. From equation (3) $H_{t+1} = AL^\gamma H_t^\psi$, one can calculate the growth rate of knowledge $H_{t+1} / H_t = (AL^\gamma)H_t^{\psi-1}$. If knowledge grows at a constant rate, it must be true that $\psi = 1$ and $AL^\gamma > 1$. The parameter values are set in table 4, so that balanced growth eventually occurs. In this example, government deficits reduce savings and knowledge accumulation in every period, but reduce the interest rate in the initial period. The government spending is 5 per cent of the initial period output. In contrast to the previous examples, government deficits have a permanent effect on savings and knowledge spillover. The growth rates of capital and knowledge stock in example 4 are given in table 5.

In summary, the examples show that allowing for elastic labour and knowledge spillover makes a significant impact on how government deficits affects equilibrium interest rates. The example of the Diamond model with a fixed labour supply predicts that government deficits result in significant declines in real interest rates.
In examples with elastic labour and knowledge spillover, government deficits result in only mild increases and even decreases in interest rates. Furthermore, government deficits affect not only the current interest rate but also the dynamics of interest rates. These results are useful in interpreting the results of empirical studies on the interest rate effect of government deficits.

5. Discussion of related empirical studies

The conventional neoclassical proposition that deficits raise interest rates has been extensively used in empirical studies for tests of competing theories. As an alternative to the neoclassical paradigm, Barro (1974) argues that if the currently alive agents are concerned with the welfare of future generations, then shifting the tax burden to the future does not change the effective lifetime budget of the current generation. It follows that the debt-for-tax swap has no impact on resource allocations and interest rates – a proposition known as Ricardian equivalence. Cukierman and Meltzer (1989) show that if the bequest motive is not operational for some economic agents, then under majority rule government debt is not neutral. The fundamental disagreement between the neoclassical and Ricardian theories is whether the altruistic links among generations are operative, which ultimately becomes an empirical issue.

Many empirical tests focus on whether government deficits and debt increase interest rates. But not all interest rate regressions are equally powerful in testing Ricardian equivalence versus the neoclassical alternative. In what follows, we consider only empirical studies on long-term interest rates. This is because although short-term interest rates can be useful in testing deficit neutrality against some alternatives, they are not appropriate for the purpose of examining the effect of intergenerational distribution of tax burden.

The findings on how government deficits affect long-term interest rates are potentially influenced by several factors, for example, the benchmark models that specify how interest rates are determined and the data sample. Widely used models for interest tests include (a) reduced-form regressions derived from structural settings, such as the Keynesian model (e.g., Mascaro and Meltzer 1983; Evans 1985; Monadjemi and Kearney 1991; Cebula and Belton 1993), and equilibrium of market of loanable funds (e.g., Feldstein 1986; Hoelscher 1986); and (b) non-structural vector autoregression (VAR) models (e.g., Plosser 1982, 1987; Booth and Reid 1989; Miller and Russek 1991). Most studies use annual or quarterly post-war U.S. data, with the exception of Evans (1985; wartime U.S. data), Barro (1987; U.K. data), and Booth and Reid (1989; Canadian data).

The studies produce mixed results. Barro (1987), Mascaro and Meltzer (1983), Evans (1985, 1987), Plosser (1982, 1987), and Boothe and Reid (1989) conclude that government deficits in the United States do not significantly raise long-term interest rates. In fact, Evans (1985, 1987) and Booth and Reid (1989) find deficits sometimes have a negative effect on interest rates. Meanwhile, other researchers do find significant and positive effects of deficits on interest rates (e.g., Hoelscher
The inclusiveness of the literature is highlighted by the survey articles of Bernheim (1987) and Seater (1993), which draw opposite conclusions from similar collections of empirical studies.

It appears to be difficult to attribute the conflicting findings to a single factor. In the studies cited above, quite different results are reported from similar reduced-form models in the post-war period. There are many possible reasons for questioning the power of interest rate tests. One of the important factors not explicitly modelled in many studies is the international flow of capital. Among the studies that control the effect of capital inflows, Monadjemi and Kearney (1991) and Cebula and Belton (1993) conclude that government deficits reduce savings but do not raise interest rates because of the foreign capital inflow to the United States. The international capital flows reduce the power of interest rate tests. Another factor that reduces the power of interest rate tests is the endogeneity of government deficits. Miller and Roberds (1992) illustrate the importance and the difficulty of identifying of deficit policy. Different identifying restrictions used in various studies can result in different conclusions.

Furthermore, according to Poterba and Summers (1987) and Bohn (1991), until 1980s most of the U.S. government debts were retired within the lifespan of one generation. This suggests that there are not enough historical data to construct formal tests of the neoclassical theory.

In this paper questions are raised on the validity of interest rate tests of Ricardian equivalence against the neoclassical alternative. It is suggested that besides the lack of data and the questions on the power of empirical tests, there are theoretical reasons for weak correlation between long-term interest rates and government deficits. This research indicates that magnitude of the interest rate effect of government deficits depends on the substitutability between consumption and leisure as well as on the degrees of knowledge spillover across generations. The interest rate effect of government deficits may be positive but too small to be detected empirically. If there is a large spillover effect in knowledge creation, the government deficits may actually reduce the interest rate. The conventional practice of treating the interest rate neutrality of deficits as an indication of Ricardian equivalence cannot be presumed without qualification.

8 Contradictory results are found in studies using Keynesian models as well as VAR models. Although there is no clear pattern of subsample differences, Cebula and Belton (1993) show the deficit-long-term interest rate correlation is weaker after the 1980s.
9 Boothe and Reid (1989), however, conclude that government deficits do not raise interest rates in Canada even when the capital inflow is controlled.
11 Berhneim (1987) examines intermediate-term (six-year and twelve-year) average data across countries and finds that government deficits reduce savings.
6. Concluding remarks

In the neoclassical model of Diamond (1965), capital is the only variable input. In such an environment government deficits always reduce savings and raise interest rates. In this paper it is demonstrated that, in a neoclassical growth model with elastic labour supply and intergenerational spillover knowledge government deficits may not always raise real interest rates. Furthermore, numerical examples indicate that the dynamics of the interest rate effect are generally complicated. Caution is called for in conducting and interpreting empirical studies on the interest rate effect of government deficits.

Appendix

Proof of proposition 1

We prove the uniqueness of the equilibrium by construction. First, consider the equilibrium under Policy One at period 0. Combining conditions (8) and (9), then using the budget constraint to substitute out $K_1$ and equation (5) to substitute out $w_0$ yields the following equation for $L_0$:

$$\Omega(L_0, K_0, H_0) = \Gamma(L_0, K_0, H_0, G),$$

(A1)

where

$$\Omega(L, K, H) = (1+\beta)((1-\alpha)\theta/(1-\theta))^{1/(1-\eta)}(1-L)H^{(1-\alpha)\eta/(1-\eta)}L^\alpha/(1-\alpha)^{1/(1-\alpha)}K^{\alpha/(1-\alpha)}$$

$$+ \beta(1-\alpha)(1-L)L^{-\alpha}$$

(A2)

$$\Gamma(L, K, H, G) = (1-\alpha)L^{1-\alpha} - GK^{-\alpha}H^{\alpha-1}.$$  

(A3)

It is easy to verify that $\partial\Omega(L, K, H)/\partial L < 0$ and $\partial\Gamma(L, K, H, T)/\partial L > 0$. It is also easy to verify that $\Omega(0, K, H) = +\infty$, $\Omega(1, K, H) = 0$, $\Gamma(0, K, H, G) < 0$, and if the tax is feasible, $\Gamma(1, K, H, G) > 0$. It follows from the facts that $\Omega(.,K, H)$ and $\Gamma(.,K, H, G)$ are continuous that for any given $(K_0, H_0, G)$ there is a unique $0 < L_0 < 1$ that satisfies $\Omega(L_0, K_0, H_0) = \Gamma(L_0, K_0, H_0, G)$. For $t > 0$, $L_t$ is given by the unique solution to $\Omega(L_t, K_t, H_t) = \Gamma(L_t, K_t, H_t, 0)$. The dynamics of the equilibrium path is given by equation (3): $H_{t+1} = AL_t^2H_t^{\theta}$, $C_{t,1} = (1-L_t)[\theta w_t/(1-\theta)]^{1/(1-\theta)}$, and $K_{t+1} = w_t L_t - C_{t,1}$, where $w_t$ is given by (5).

Now consider the equilibrium under Policy Two. (8') and (9) imply that at period 0 the labour service under Policy Two is determined by

$$\Omega(\bar{L}_0, K_0, H_0) = \Gamma(\bar{L}_0, K_0, H_0, 0).$$  

(A1')

The same argument used for Policy One can be used here to prove that there is a unique $\bar{L}_0 \in (0,1)$ that solves (A1'). Similarly, for $0 < t < T$, $\bar{L}_t$ is given by the unique solution to $\Omega(\bar{L}_t, \bar{K}_t, \bar{H}_t) = \Gamma(\bar{L}_t, \bar{K}_t, \bar{H}_t, 0)$. The dynamics of the
equilibrium path is given by equation (3), \( \bar{C}_{t,1} = (1 - \bar{L}_t)[\theta \bar{w}_t/(1 - \theta)]^{1/(1 - \eta)} \), \( \bar{K}_{t+1} = \bar{w}_t \bar{L}_t - \bar{C}_{t,1} - B_{t+1} \) (where \( \bar{w}_t \) is given by (5)), \( B_1 = G \), and \( B_{t+1} = \bar{r}_t B_t \) for \( t < T \) with \( \bar{r}_t \) given by (4). At period \( T \), the equilibrium labour service is determined by \( \Omega(\bar{L}_T, \bar{K}_T, \bar{H}_T) = \Gamma(\bar{L}_T, \bar{K}_T, \bar{H}_T, \bar{r}_T B_T) \). \( \bar{C}_{T,1} = (1 - \bar{L}_T)[\theta \bar{w}_T/(1 - \theta)]^{1/(1 - \eta)} \), and \( \bar{K}_{t+1} = \bar{w}_T \bar{L}_t - \bar{C}_{t,1} - \bar{r}_T B_t, \bar{r}_T \) and \( \bar{w}_T \) are given by (4) and (5), respectively. For \( t > T \), \( \bar{L}_t \) is given by the unique solution to \( \Omega(\bar{L}_t, \bar{K}_t, \bar{H}_t) = \Gamma(\bar{L}_t, \bar{K}_t, \bar{H}_t, 0) \). The dynamics of the equilibrium path is given by equation (3), \( \bar{C}_{t,1} = (1 - \bar{L}_t)[\theta \bar{w}_t/(1 - \theta)]^{1/(1 - \eta)} \), and \( \bar{K}_{t+1} = \bar{w}_t \bar{L}_t - \bar{C}_{t,1} \). Again, \( \bar{w}_t \) is given by (5).

**Proof of proposition 2**

Now we compare the equilibrium labour service \( \bar{L}_0 \) and \( \tilde{L}_0 \) under the alternative policies. In the following we show that \( \bar{L}_0 > \tilde{L}_0 \). Suppose the opposite is true; that is, \( \tilde{L}_0 > \bar{L}_0 \). Then,

\[
\Omega(\tilde{L}_0, K_0, H) = \Gamma(\tilde{L}_0, K_0, H, 0) > \Gamma(L_0, K_0, H, 0) = \Omega(L_0, K_0, H, 0).
\]

The first inequality follows from \( \partial \Gamma(L, K, H, G)/\partial L > 0 \), and the second follows from the observation \( \partial \Gamma(L, K, H, x)/\partial x < 0 \). On the other hand, since \( \partial \Omega(L, K, H)/\partial L < 0 \), \( \tilde{L}_0 > \bar{L}_0 \) implies \( \Omega(L_0, K_0, H, 0) > \Omega(\tilde{L}_0, K_0, H, 0) \), which contradicts (A4). Therefore, \( \bar{L}_0 \geq \tilde{L}_0 \) cannot be true. It follows from the assumption on knowledge accumulation (3) that \( H_1 > \bar{H}_1 \). Combining the inequality \( \bar{L}_0 > \tilde{L}_0 \) with Euler equation (9) and the budget constraints yields \( C_{0.1} < \bar{C}_{0.1} \) and \( K_1 > \bar{K}_1 \).

**Proof of proposition 3**

a) The case with \( \eta < 0 \). We prove that, in this case, \( L_1 < \bar{L}_1 \). Suppose not; that is, assume \( \bar{L}_1 \geq L_1 \) is true. Then,

\[
\Omega(\bar{L}_1, \bar{K}_1, \bar{H}_1) = \Gamma(\bar{L}_1, \bar{K}_1, \bar{H}_1, 0) = \Gamma(L_1, K_1, H_1, 0)
\]

\[
\leq \Gamma(L_1, K_1, H_1, 0) = \Omega(L_1, K_1, H_1, 0).
\]

The equalities follow from the definitions.

Meanwhile, with \( \eta < 0 \), we have \( \partial \Omega(L, K, H)/\partial L < 0 \), \( \partial \Omega(L, K, H)/\partial H < 0 \), and \( \partial \Omega(L, K, H)/\partial H < 0 \). Therefore, \( \bar{L}_1 \leq L_1 \) and the inequalities from proposition 2 (i.e., \( K_1 > \bar{K}_1 \) and \( H_1 > \bar{H}_1 \)) imply \( \Omega(\bar{L}_1, \bar{K}_1, \bar{H}_1) > \Omega(L_1, K_1, H_1) \), which contradicts (A5). The contradiction proves \( L_1 > \bar{L}_1 \).

b) The case with \( 0 < \eta < 1 \). When \( 0 < \eta < 1 \), simply reversing the signs of all the inequities in case (a) proves \( L_1 > \bar{L}_1 \).

c) The case with \( \eta = 0 \). It is easy to show that \( L_1 = \bar{L}_1 = (\theta + \beta)/(1 + \beta) \).
Proof of proposition 4

Given the fact that $K_1 > \bar{K}_1$, by iteration one can easily show that for $1 \leq t < T$, $K_t > \bar{K}_t$ and $r_t < \bar{r}_t$ if $\eta < 1$. When the lump-sum tax is collected at period $T$, given $K_{T-1} > \bar{K}_{T-1}$, we still have $K_T > \bar{K}_T$. It is easy to establish then that $K_t < \bar{K}_t$ for $t > T$. Therefore, we have $K_t > \bar{K}_t$ for all $t > 0$. When $t < T$ and $t > T$, the comparison on labour service $L_t$ and $\bar{L}_t$ in cases $\eta \leq 0$ and $\eta < 1$ can be made in the way it can when $t = 1$. Hence, we have $r_t < \bar{r}_t$ for $t < T$ and $t > T$. At period $T$, one can show that if $\eta \leq 0$, then $L_T < \bar{L}_T$; therefore $r_T < \bar{r}_T$. If $0 < \eta < 1$, then there are two scenarios: $L_T \leq \bar{L}_T$ and $L_T > \bar{L}_T$. If $L_T \leq \bar{L}_T$, then obviously $r_T < \bar{r}_T$. If $L_T > \bar{L}_T$, we still have $r_T < \bar{r}_T$. The reason is as follows. From equilibrium conditions $\Omega(L_T, K_T) = \Gamma(L_T, K_T, 0)$ and $\Omega(\bar{L}_T, \bar{K}_T) = \Gamma(\bar{L}_T, \bar{K}_T, \bar{r}_T \bar{B}_T)$ we have

$$L_T/(1 - L_T) = (1 + \beta)((1 - \alpha)\theta/(1 - \theta))^{1/(1 - \eta)}(K_T/L_T)^{\eta/(1 - \eta)} + \beta(1 - \alpha) \tag{A6}$$

$$\bar{L}_T/(1 - \bar{L}_T) = (1 + \beta)((1 - \alpha)\theta/(1 - \theta))^{1/(1 - \eta)}(\bar{K}_T/\bar{L}_T)^{\eta/(1 - \eta)} + \beta(1 - \alpha) + \bar{r}_T \bar{B}_T(\bar{L}_T/\bar{K}_T)^{\alpha}. \tag{A7}$$

If $L_T > \bar{L}_T$, then the left-hand-side of (A6) is larger than that of (A7). It follows from (A6) and (A7) that (note that $0 < \eta < 1$) $K_T/L_T > \bar{K}_T/\bar{L}_T$; therefore, $r_T < \bar{r}_T$.

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