When is a non-self-adjoint Hill operator a spectral operator of scalar type? Part II

V. Tkachenko, Beer-Sheva, Israel

We consider a Hill operator

\[ H = -\frac{d^2}{dx^2} + V(\cdot), \quad x \in \mathbb{R}, \]

in \( L^2([\mathbb{R}; dx]) \) with a complex-valued \( \pi \)-periodic potential \( V(\cdot) \) such that \( V \in L^2_{\text{loc}}([\mathbb{R}; dx]) \) and prove a criterion for it to be a spectral operator of scalar type in the sense of Dunford [1]. This criterion is stated in two versions.

The first version is given in terms of three entire functions which are independent parameters uniquely determining the potential \( V \) (cf. [2]), and the second one is formulated in terms of algebraic and geometric properties of spectra of periodic/antiperiodic and Dirichlet boundary problems generated by \( H \) in the space \( L^2([0, \pi]; dx) \).

The problem of deciding which Hill operators are spectral operators of scalar type appeared to have been open for about 50 years.

This is joint work with F. Gesztesy published in [3].

In this second part we will provide additional details omitted in last week’s Colloquium.

References: