On the Arrow of Time

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1. Background

• Gas molecules released from half-box into full-box.
• Animals grow older.
• Sugar and water do not spontaneously un-mix.
• Broken glass does not spontaneously fix itself.
• Second law of thermodynamics - entropy.
• Ludwig Boltzmann: The second law (of thermodynamics) can never be proved mathematically by means of the equations of dynamics alone.
• Gas molecules dynamical equations (collisions and free flyings).
• Continuum modeling - Boltamann equations (just a model).
2. Our Theory

We believe the following three ingredients are enough to explain the mystery of the arrow of time:

(1) equations of dynamics of gas molecules,
(2) chaotic instabilities of the equations of dynamics,
(3) unavoidable perturbations to the gas molecules.

Figure 1. The diagram of the arrow of time. The ‘Past’, ‘Now’ and ‘Future’ are coordinate time, and the ‘Entropy’ is the thermodynamic equilibrium entropy.
3. Comments

- Using chaos to explain irreversibility starts from I. Prigogine.
- Many other approaches: quantum perspective by Maccone (wrong).
- A Popular Challenger to Irreversibility — The Poincaré Recurrence Theorem (Physicists making such an argument apply the Poincaré recurrence theorem without its condition: The crucial condition is that one can find a bounded invariant region in the phase space of all the gas molecules coordinated by their locations and momenta).
- Ergodicity: no contradiction with entropy increasing (‘transitional chaotic dynamics’ during the relaxation time - not ergodic; ‘saturated chaotic dynamics’ when the system is in a thermodynamic equilibrium - ergodic)
4. Numerical Simulations

Figure 2. Return probability for particles initially randomly positioned in the half interval \([0, 0.5]\).
Figure 3. The location time series of the right particle in the two particles system.
Figure 4. Time series of the location deviations in Euclidean norm in the 2D case.
Figure 5. The final estimated Liapunov exponent as a function of the size and the number of the disks.
Figure 6. The evolution of the six disks and the evolution of the perturbed six disks. Since the perturbation size is $10^{-6}$, initially the unperturbed and the perturbed disks coincide almost completely. The radius of the disk is 0.25, and the rectangle domain is $8 \times 4$.

(a) $t = 2.59$
(b) $t = 3.286$
(c) $t = 3.42$
(d) $t = 3.954$

Figure 7. The same setup as in Figure 6 except that the perturbation size is $10^{-12}$.

(a) $t = 3.608$
(b) $t = 4.046$
(c) $t = 4.184$
(d) $t = 5.248$