Perspectives on Inference for Restricted Stochastic Dominance

David M. Kaplan
University of Missouri

BU Econometrics Seminar
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Prologue

Defining “Better”

“Better”?

Two distributions (of earnings, productivity, . . .)

Which is “better”?

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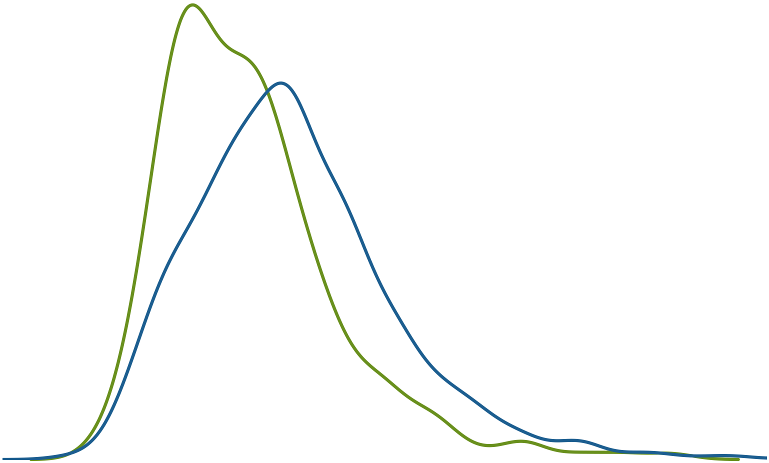
Which is “better”?

- Would you prefer to (live there, buy this, use that, . . .)

"Better"?

2 PDFs



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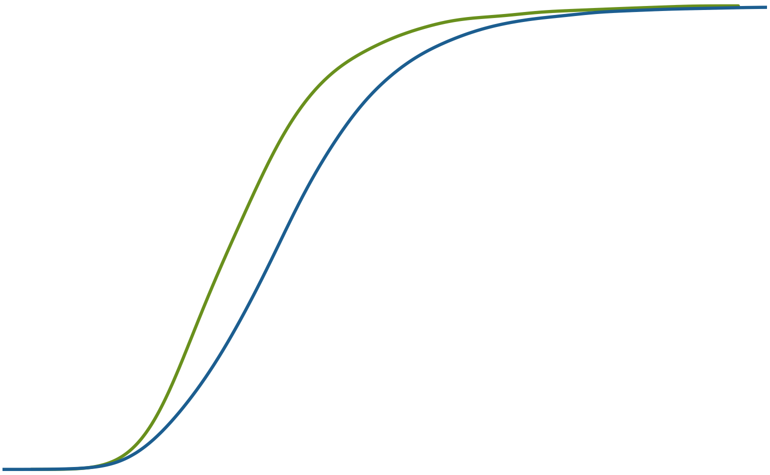

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Most: utility restricted stochastic dominance ($\mathrm{SD}_{\mathcal{U}}$)

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includes second-order SD (etc.)

CDFs (Atkinson, 1987, §1)

Poverty line: v

Headcount poverty: $F_Y(v)$ and $F_Z(v)$

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Condition I of Atkinson (1987, p. 751)

Brief Tangent: Economic Inequality

Literature on measuring inequality, comparing distributions

Similar issue (me/you/all/most), like

- ϵ of Atkinson (1970, p. 257)
- α of Cowell and Flachaire (2017, §4.3)

Inference

Learning from Data

Literature: Testing

Two features in common:

- Single H_0: all-or-nothing
- CDF-based

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$H_0: Y \text{ SD}_1 Z$ (or $\text{SD}_2, \ldots$)

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$H_0\colon Y\ non\text{SD}_1\ Z$ ($H_1\colon Y \text{ SD}_1\ Z$)

- Davidson and Duclos (2013)
- Want stronger evidence for SD_1 (analog: $H_0\colon \beta = 0$)
- Actually $\text{SD}_{[v_1,v_2]}$

New Perspectives

~~Single H_0~~ Multiple testing

- Goldman and Kaplan (2018)
- $H_{0v}\colon F_Y(v) \geq F_Z(v)$ for each $v \in \mathbb{R}$
- Learn about $\mathcal{V} \equiv \{v : F_Y(v) < F_Z(v)\}$ (Y SD$_{\mathcal{V}}$ Z)

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~~CDF-based~~ Utility-based

- Draft circulated for this talk
- $H_{0u}\colon \mathrm{E}[u(Y)] \leq \mathrm{E}[u(Z)]$ for each $u \in \mathcal{U}$
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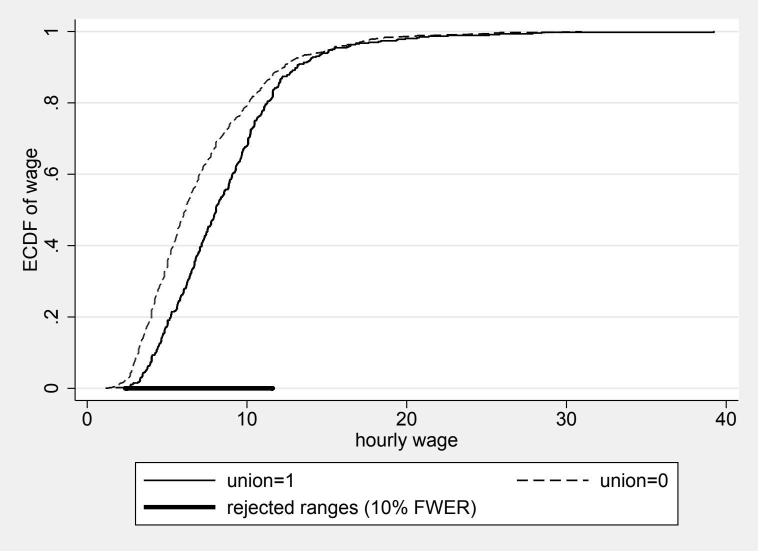
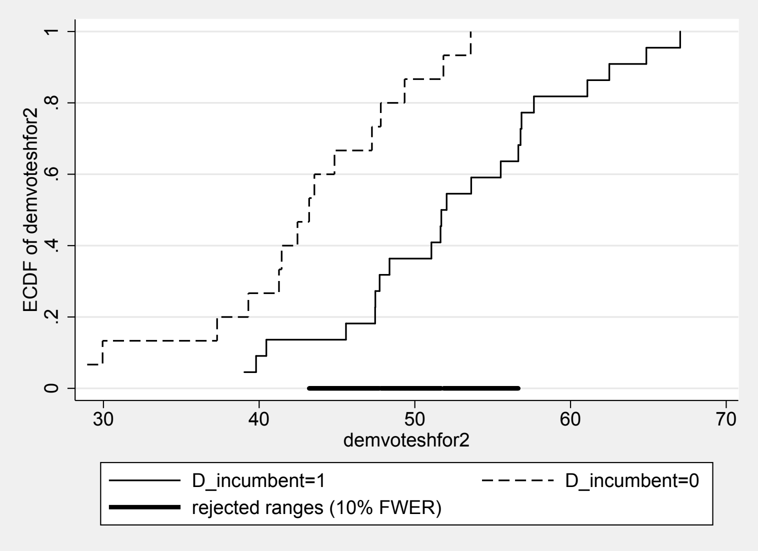
Examples (`distcomp` in Stata)



Examples (`distcomp` RDD)



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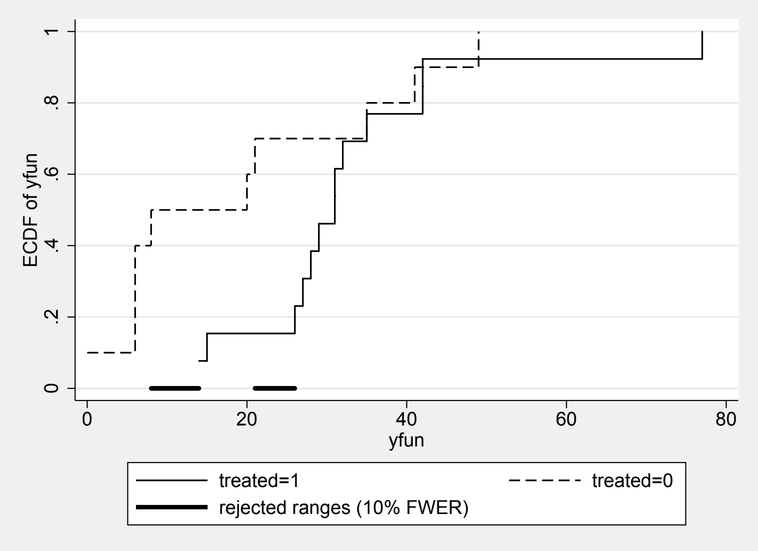
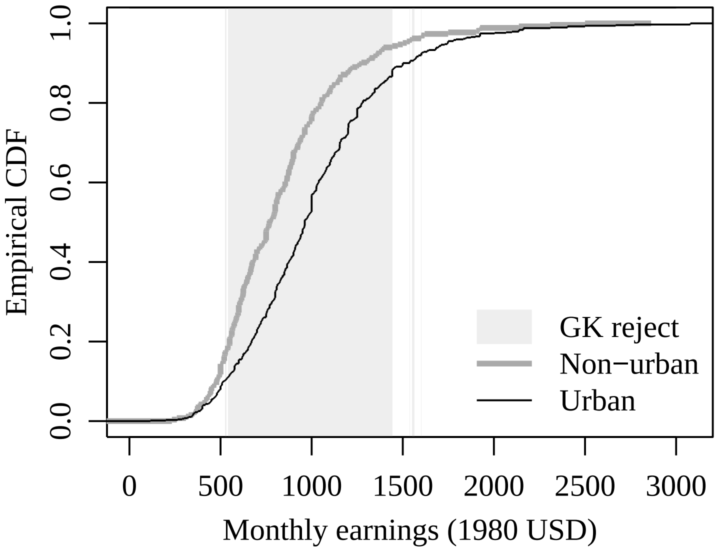
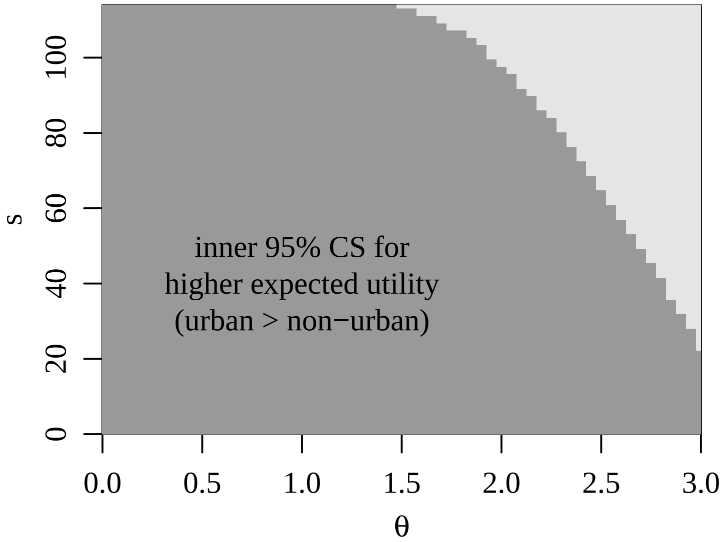


Examples (CDF)



Examples (utility)



Multiple Testing Goal

Multiple testing procedure (MTP)

- Test $H_{0v}\colon F_Y(v) \geq F_Z(v)$ for each $v \in \mathbb{R}$
- $\mathcal{V} \equiv \{v : F_Y(v) < F_Z(v)\}$
- $\mathcal{V}^{\complement} = \{v : H_{0v} \text{ is true}\}$

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Familywise error rate (FWER)

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Expected utility version

- Test $H_{0u}\colon \text{E}[u(Y)] \leq \text{E}[u(Z)]$ for each $u \in \mathcal{U}$
- Strong control of FWER

MTP vs. All-or-Nothing Test

If $H_0\colon Y \ \mathrm{SD}_1 \ Z$ rejected:

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If $H_0\colon Y \text{ SD}_1\ Z$ not rejected:

- MTP shows evidence favoring $Y \text{ SD}_1\ Z$ vs. just uncertainty
- "Reject $H_0\colon Z \text{ SD}_1\ Y$" is a crude version of this idea
- Non-rejection may be type II error if small sample, etc.

Confidence Sets (CDF)

$\mathcal{V} \equiv \{v : F_Y(v) < F_Z(v)\}$

"Inner" CS: $1 - \alpha \leq \mathrm{P}(\hat{\mathcal{V}} \subseteq \mathcal{V})$

- Invert MTP of $H_{0v}\colon F_Y(v) \geq F_Z(v) \quad (H_{0v}\colon v \notin \mathcal{V})$
- $\hat{\mathcal{V}} = \{v : H_{0v} \text{ rejected}\}$
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"2-sided" CS: $1-\alpha \le \mathrm{P}(\hat{\mathcal{V}}_1 \subseteq \mathcal{V} \subseteq \hat{\mathcal{V}}_2)$

- Combine $1-\alpha/2$ inner & outer (Bonferroni)

Confidence Sets (Expected Utility)

Same arguments but with $\mathcal{D}$ instead of $\mathcal{V}$

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CS/MTP from Uniform Confidence Band

Uniform confidence band for $\Delta(\cdot) \implies$ CS/MTP

- $\Delta(v) \equiv F_Z(v) - F_Y(v) \qquad \mathcal{V} = \{v : \Delta(v) > 0\}$
- $\Delta(u) \equiv \mathrm{E}[u(Y)] - \mathrm{E}[u(Z)] \quad \mathcal{D} = \{u : \Delta(u) > 0\}$
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- MTP: equivalent to CS like before

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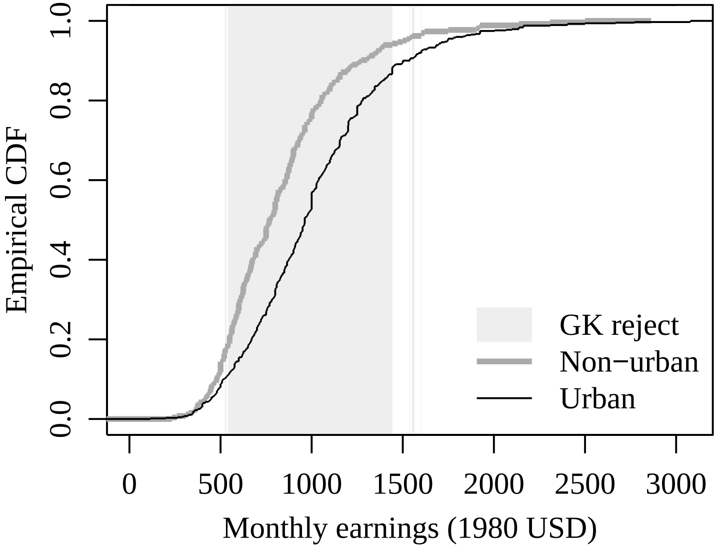
Availability

- CDF diff: asymptotic band, but finite-sample CS/MTP

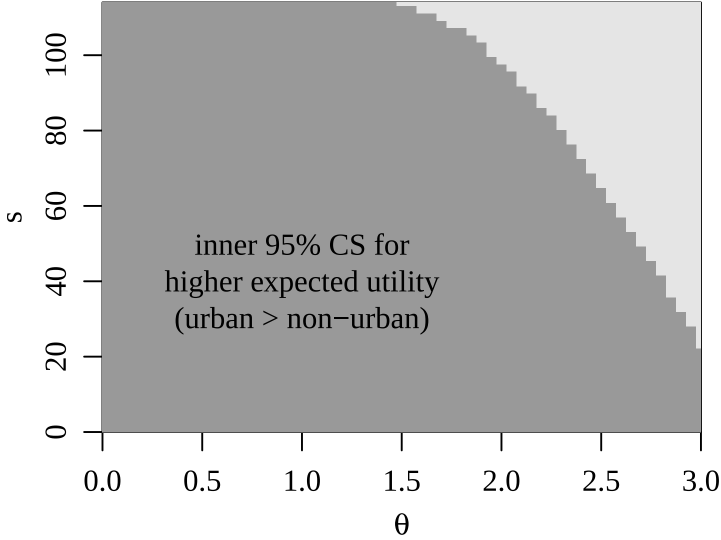
Information vs. comprehension

- EU band more informative, CS/MTP easier to comprehend

CDF-based CS/MTP



Utility-based CS/MTP



Details

Theoretical & Otherwise

CDF: KS vs. Probability Integral Transform

Kolmogorov–Smirnov MTP/CS

- Reject H_{0v} when $\hat{F}_Y(v) - \hat{F}_Z(v)$ exceeds KS critical value
- Prop. 3 of Goldman and Kaplan (2018)

KS: well-known low tail power

- ```
  ks.test(c(1:15/21,10^6+1:5),punif)
  D = 0.25, p-value = 0.1376
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  ```

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If cts, $F_Y(Y_i) \sim \text{Unif}(0,1)$

- Retain finite-sample properties
- Power more even than KS across distribution
- Goldman and Kaplan (2018): two-sample MTP, RDD, computation
  ```

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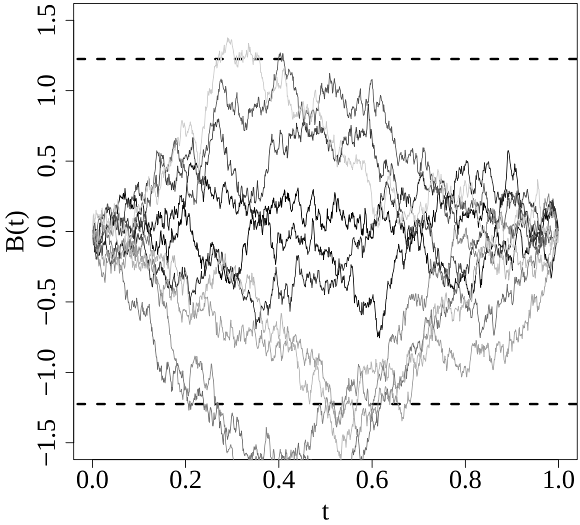

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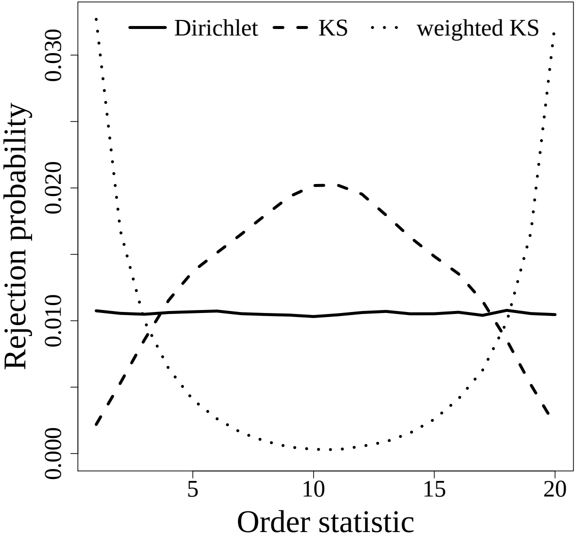

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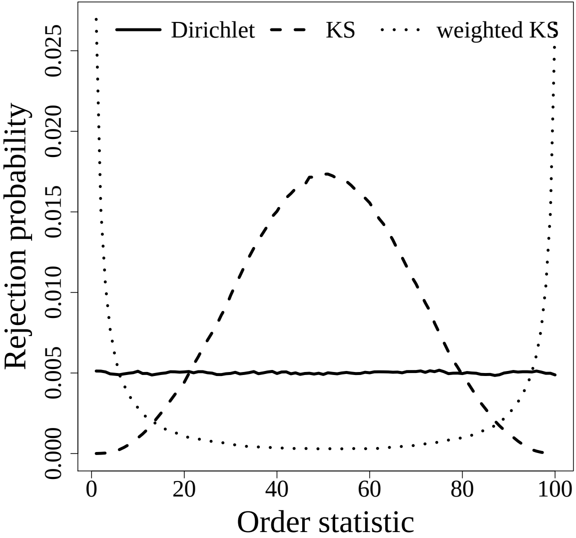

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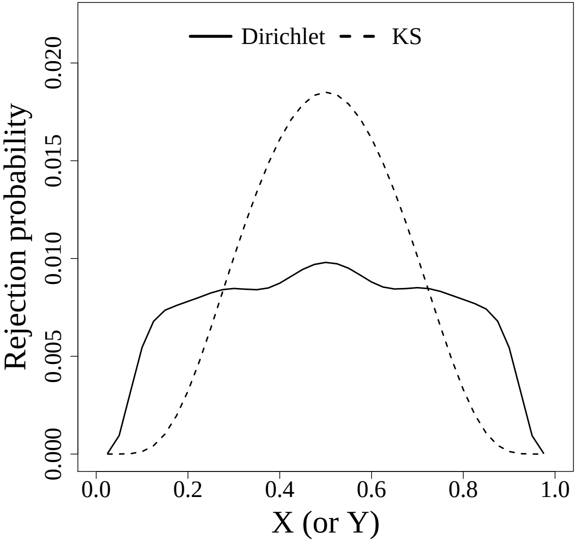

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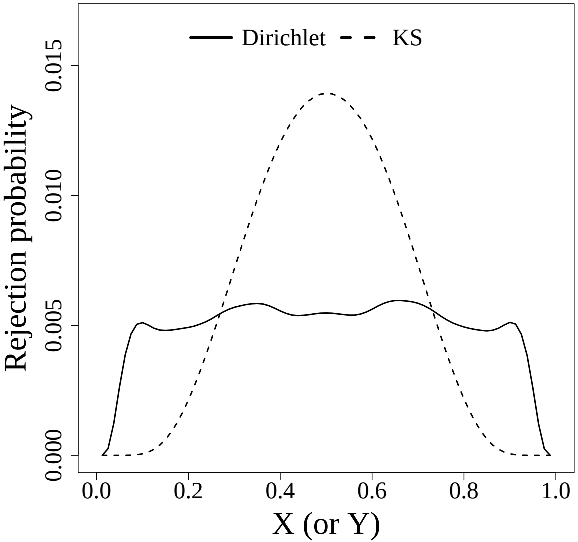

Expected Utility: Asymptotics

$\Delta(v) \equiv F_Z(v) - F_Y(v) = \mathrm{E}[\mathbb{1}\{Z \leq v\}] - \mathrm{E}[\mathbb{1}\{Y \leq v\}]$

- $\{f_v(\cdot) : f_v(t) = \mathbb{1}\{t \leq v\}, v \in \mathbb{R}\}$ is Donsker
- $\hat{\Delta}(\cdot)$: Gaussian limit

$\Delta(u) \equiv \mathrm{E}[u(Y)] - \mathrm{E}[u(Z)]$

- $\hat{\Delta}(\cdot)$: Gaussian limit and bootstrap consistency if Donsker $\mathcal{U}$

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Cor. 3.1 of van der Vaart (1996): $\mathcal{U}$ Donsker if

- non-decreasing
- bounded from below (or above)
- $2 + \delta$ moments of envelope function

Expected Utility: MTP

$H_{0u}\colon \Delta(u) \equiv \mathrm{E}[u(Y)] - \mathrm{E}[u(Z)] \leq 0$, each $u \in \mathcal{U}$

Define pointwise t-statistics:

- $\hat{T}_u = [\hat{\Delta}(u) - \Delta(u)]/\widehat{\mathrm{SE}}_u$
- $\hat{T}_u^0 = \hat{\Delta}(u)/\widehat{\mathrm{SE}}_u$

Bootstrap cv: $1 - \alpha$ quantile of $\sup_{u \in \mathcal{U}} \hat{T}_u$

FWER $=$ P(reject any true) $\leq \mathrm{P}(\sup_u \hat{T}_u > \mathrm{cv}) \to \alpha$

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Stepdown (Holm, 1979)

- $\le$ maybe very conservative if many $\Delta(u) > 0$
- Re-compute bootstrap cv using only non-rejected u
- Iterate: bounded by oracle test using true $\{u : H_{0u}$ true$\}$

Can also pre-test to remove $\hat{\Delta}(u) \ll 0$, etc.

Expected Utility: CS

Invert MTP to get CS

Simulation

Performance of New Methods

Setup

$Y_i \overset{iid}{\sim} \log \mathrm{N}(0, 1) + 0.1,\ i = 1, \ldots, n$

$Z_i \overset{iid}{\sim} \log \mathrm{N}(\mu, \sigma) + 0.1,\ i = 1, \ldots, n$

$\mathcal{U}$: CRRA w/ risk aversion $\theta \in [0, 3]$

Band for $\Delta(u) = \mathrm{E}[u(Y)] - \mathrm{E}[u(Z)]$

- Equivalently: $\Delta(\theta)$ on $\theta \in [0, 3]$

CSs for $\mathcal{D} \equiv \{u(\cdot) : \mathrm{E}[u(Y)] > \mathrm{E}[u(Z)]\}$

- Equivalently: $\mathcal{D}$ is subset of $\theta \in [0, 3]$

Results: $n = 40$

			Coverage ($1 - \alpha = 0.9$)			
σ	μ	$\{\theta : u_\theta \in \mathcal{D}\}$	band	2s CS	inner	outer
0.7	-0.3	[0.0, 2.8]	0.873	0.960	0.968	0.992
0.7	0.0	[0.0, 1.1]	0.865	0.972	0.990	0.982
0.7	0.3	[ ]	0.855	0.998	0.998	1.000
1.0	-0.3	[0.0, 3.0]	0.920	0.999	1.000	0.999
1.0	0.0	[ ]	0.938	0.972	0.972	1.000
1.0	0.3	[ ]	0.922	0.995	0.995	1.000
1.3	-0.3	[0.2, 3.0]	0.896	0.965	0.967	0.998
1.3	0.0	[1.2, 3.0]	0.883	0.976	0.988	0.988
1.3	0.3	[2.5, 3.0]	0.861	0.962	0.994	0.968

Results: $n = 100$

			Coverage ($1-\alpha=0.9$)			
σ	μ	$\{\theta : u_\theta \in \mathcal{D}\}$	band	2s CS	inner	outer
0.7	−0.3	[0.0, 2.8]	0.907	0.968	0.975	0.993
0.7	0.0	[0.0, 1.1]	0.897	0.977	0.993	0.984
0.7	0.3	[ ]	0.908	0.999	0.999	1.000
1.0	−0.3	[0.0, 3.0]	0.934	1.000	1.000	1.000
1.0	0.0	[ ]	0.929	0.965	0.965	1.000
1.0	0.3	[ ]	0.922	1.000	1.000	1.000
1.3	−0.3	[0.2, 3.0]	0.901	0.974	0.979	0.995
1.3	0.0	[1.2, 3.0]	0.900	0.983	0.987	0.996
1.3	0.3	[2.5, 3.0]	0.887	0.964	0.992	0.972

Results: $n = 250$

			Coverage ($1-\alpha=0.9$)			
σ	μ	$\{\theta : u_\theta \in \mathcal{D}\}$	band	2s CS	inner	outer
0.7	−0.3	[0.0, 2.8]	0.920	0.978	0.983	0.995
0.7	0.0	[0.0, 1.1]	0.912	0.981	0.995	0.986
0.7	0.3	[ ]	0.893	0.998	0.998	1.000
1.0	−0.3	[0.0, 3.0]	0.920	1.000	1.000	1.000
1.0	0.0	[ ]	0.937	0.968	0.968	1.000
1.0	0.3	[ ]	0.942	1.000	1.000	1.000
1.3	−0.3	[0.2, 3.0]	0.927	0.976	0.978	0.998
1.3	0.0	[1.2, 3.0]	0.902	0.979	0.988	0.991
1.3	0.3	[2.5, 3.0]	0.892	0.974	0.994	0.980

Epilogue

Past & Future

Conclusion

"Better": restricted stochastic dominance based on

- CDF
- Expected utility

Inference on set of:

- values with lower CDF
- utility functions with higher expected utility

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Future:

- non-iid, improve power, implement richer utility family
- economic inequality
- restricted stochastic monotonicity
- other ideas?

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Thank you / further questions & comments welcome

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