

# Comparing latent inequality with ordinal data

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# Outline

- 1 Motivation
- 2 Results
- 3 Bayesian and frequentist inference
- 4 Empirical illustrations
- 5 Simulations
- 6 Conclusion

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# Motivation: health inequality, SRHS

From Deaton and Paxson (1998, pp. 248–9):

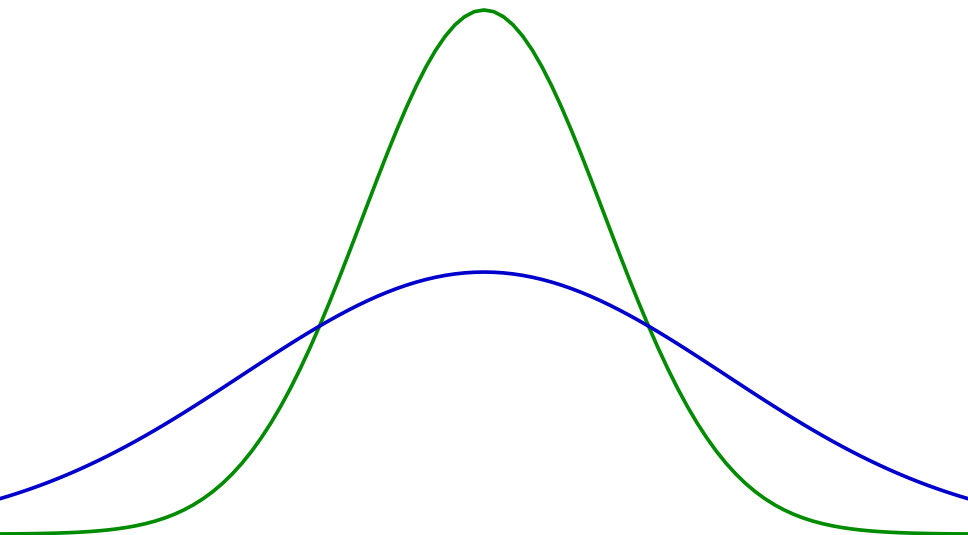
- “Our interest in **health inequality** stems from a more general interest in the **distribution of welfare**.”

# Motivation: health inequality, SRHS

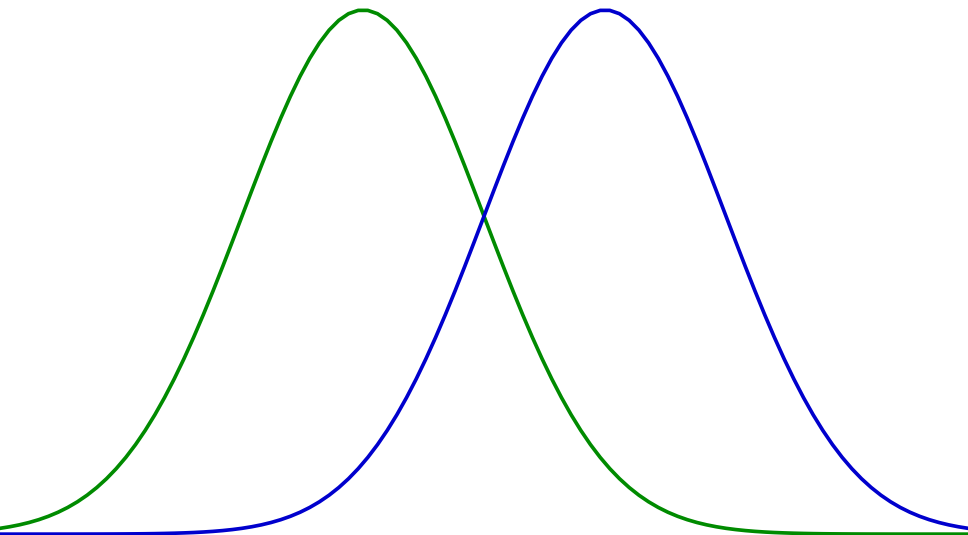
From Deaton and Paxson (1998, pp. 248–9):

- “Our interest in **health inequality** stems from a more general interest in the **distribution of welfare**.”
- SRHS: “self-reported health status”
- SRHS scale: excellent, very good, good, fair, poor
- SRHS benefits:
  - 1 “Useful over the complete adult life cycle”
  - 2 Strongly correlated with objective measures
  - 3 Widely available (PSID, NHIS, etc.)
  - 4 Synthesizes all dimensions of health

# Inequality #1: within-group (dispersion)



## Inequality #2: between-group (better/worse)



# SRHS: latent model

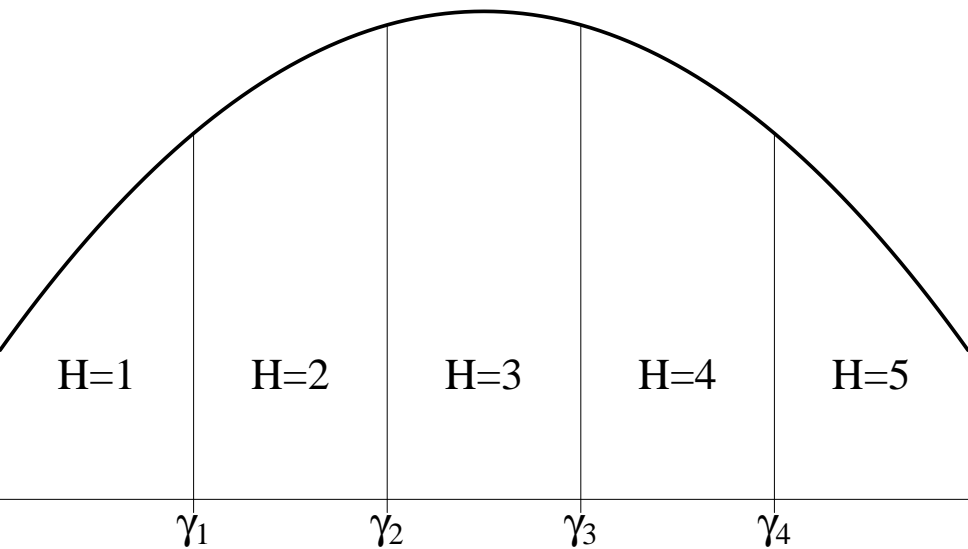
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  - 1 No cardinal meaning (e.g., no mean/variance)
  - 2 Ignores within-category variation (e.g., all "good" identical)



# SRHS: latent model

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  - 1 No cardinal meaning (e.g., no mean/variance)
  - 2 Ignores within-category variation (e.g., all "good" identical)
- Latent health  $H^*$ : cardinal, continuous, of interest, but censored
- SRHS  $H$ , fixed thresholds  $\gamma_j$

## Latent model



# Literature: SRHS inequality methodology

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# Literature: SRHS inequality methodology

- Goal: compare latent health distributions using ordinal data
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- Allison and Foster (2004): “median-preserving spread,” called “the breakthrough in analyzing inequality with [SRHS] data” by Madden (2014, p. 206)
- SRHS-based inequality indexes:
  - Good: “complete ordering” of distributions
  - Bad: many possible indexes/weights

# Our contribution

- Identification: characterize ordinal conditions informative about latent inequality
  - Semi/nonparametric restrictions (not parametric)
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- Inference: frequentist and Bayesian (example code)

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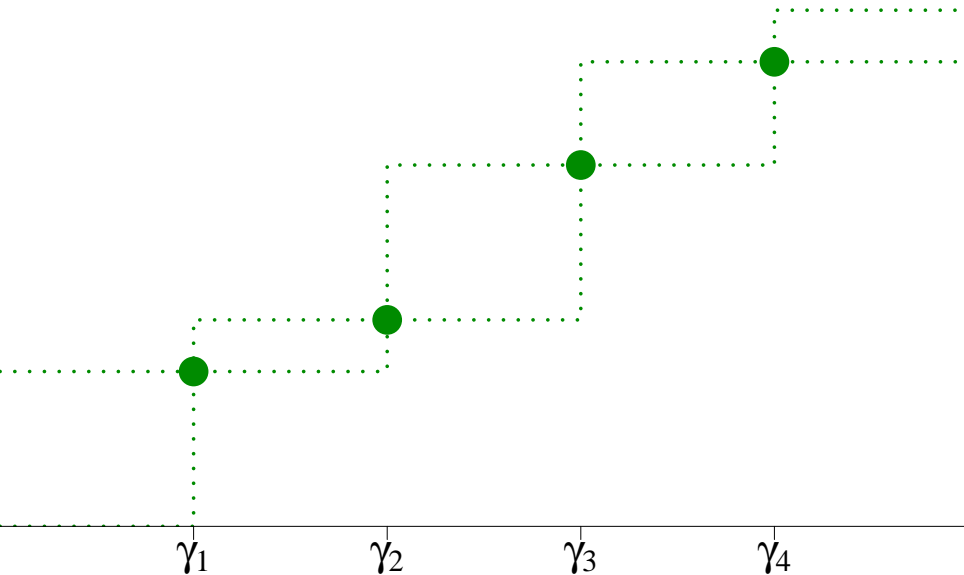
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# Partial Identification

- Given  $H$  distributions, what can we learn about  $H^*$  distributions?
- Latent CDFs partially identified, given  $\gamma_j$
- Related: Stoye (2010)

# Partial Identification



# Thresholds: assumptions

- Do all populations have same  $\gamma_j$  thresholds?
- Evidence of “yes” or “constant shift”: Lindeboom and van Doorslaer (2004), Hernández-Quevedo, Jones, and Rice (2005).

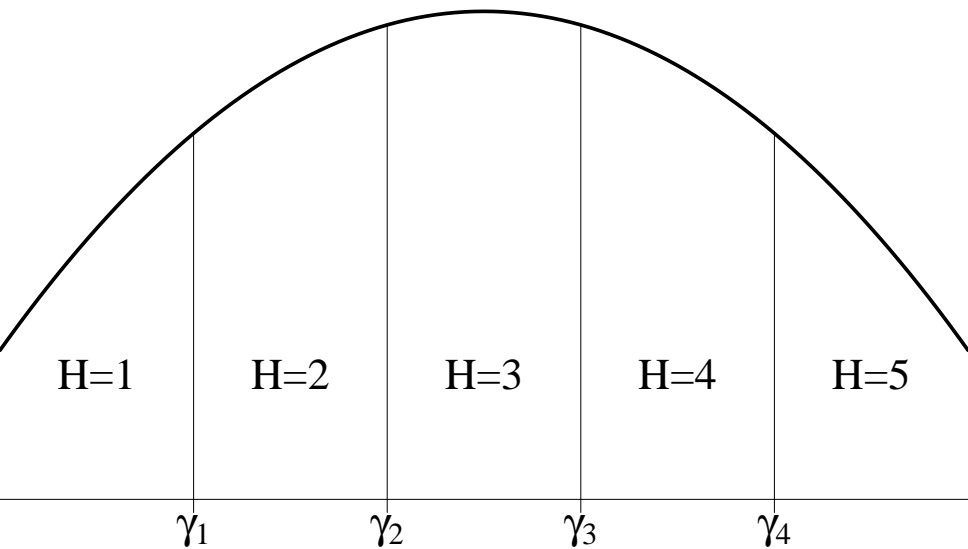
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- “Healthier” if latent first-order stochastic dominance (SD1).
  - $X \text{ SD}_1 Y \iff F_X(\cdot) \leq F_Y(\cdot)$
- Ordinal SD1  $\implies$  latent SD1?
- Latent SD1  $\implies$  ordinal SD1?
- Proposition 2(i–vi) in paper.

## Latent model (again)



## SD1: Proposition 2

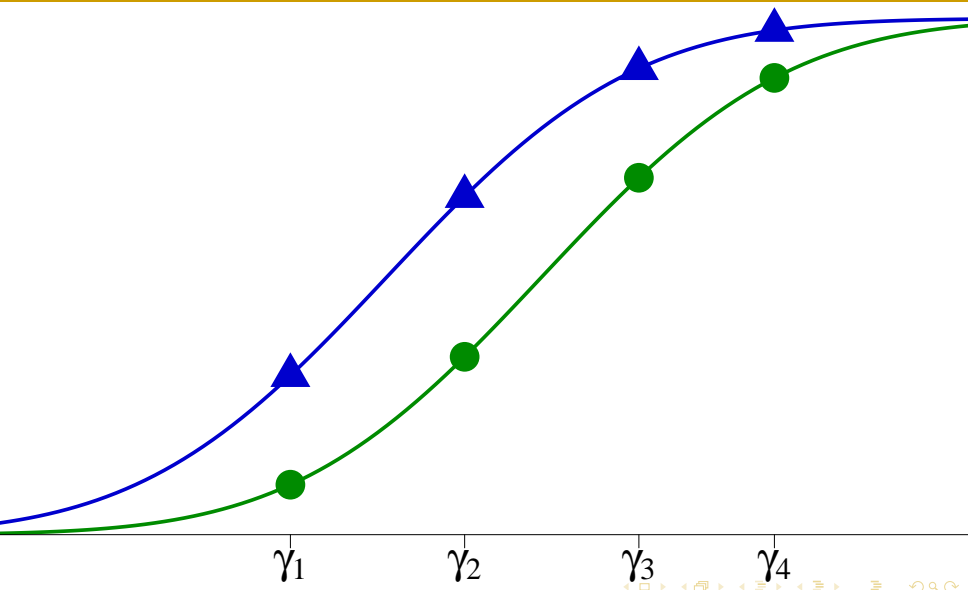
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  - Reject ordinal SD1  $\implies$  reject latent SD1.

## SD1: Proposition 2

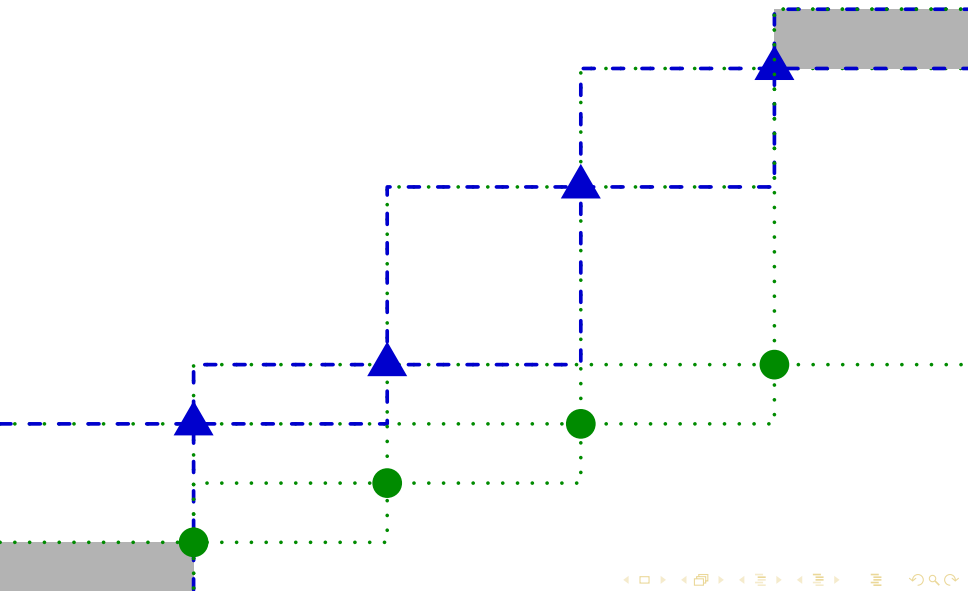
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- Prop 2(v,vi): ordinal SD1  $\implies$  latent “restricted SD1” (Atkinson, 1987).



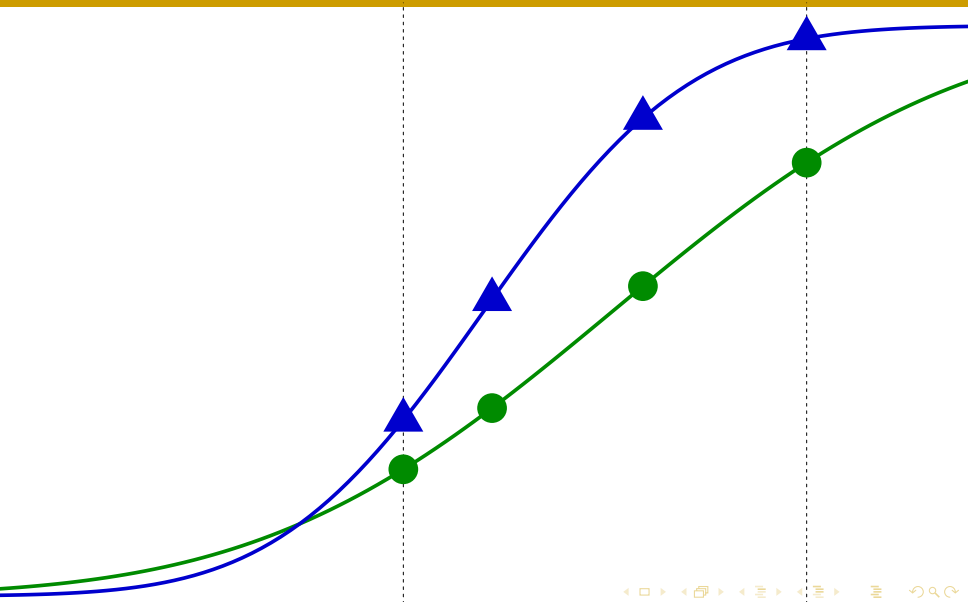
## SD1: Prop 2(i)



## SD1: Prop 2(v)



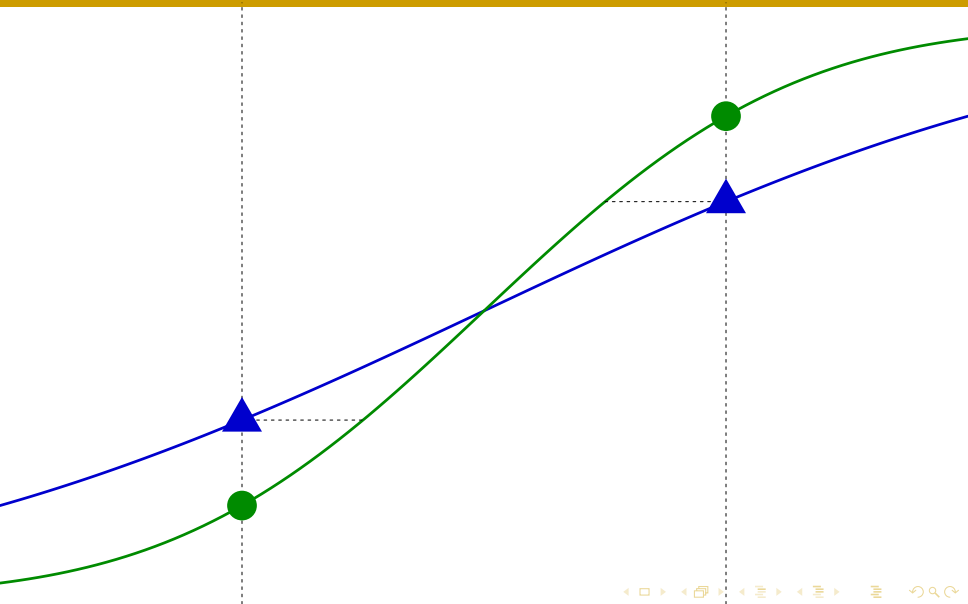
## SD1: Prop 2(vi)



# Dispersion: Proposition 3 (CDF crossing)

- Within-group inequality
- Pure location shift has zero effect on “dispersion.”
- Similarly,  $\gamma_j$  can all shift by a constant (unlike for SD1).
- Prop 3(i): can learn about latent interquantile range (IQR) differences.
- Prop 3(ii): location–scale  $\implies$  extrapolate to all IQR differences.
- Prop 3(iii): even stronger assumptions  $\implies$  latent SD2.

## Dispersion: Prop 3(i,ii)



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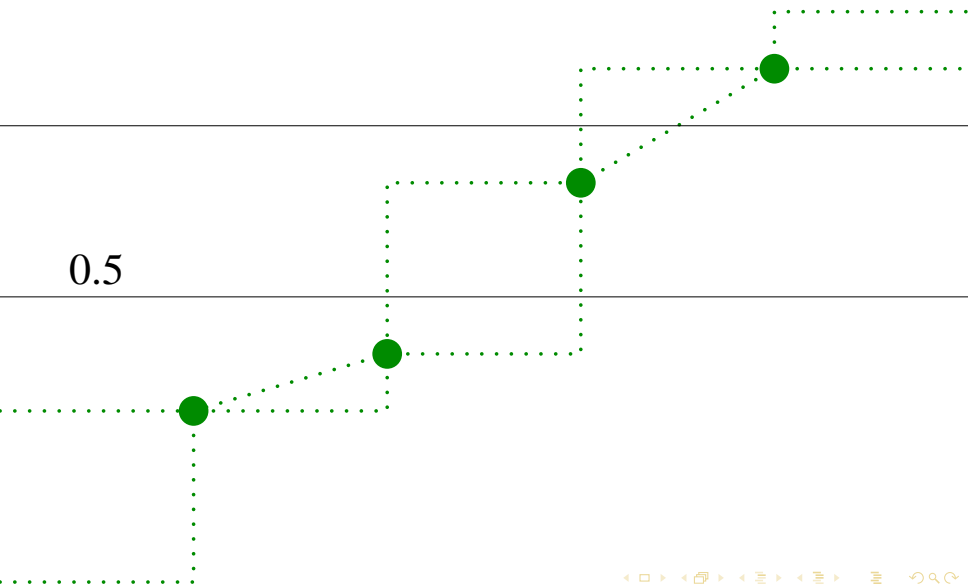
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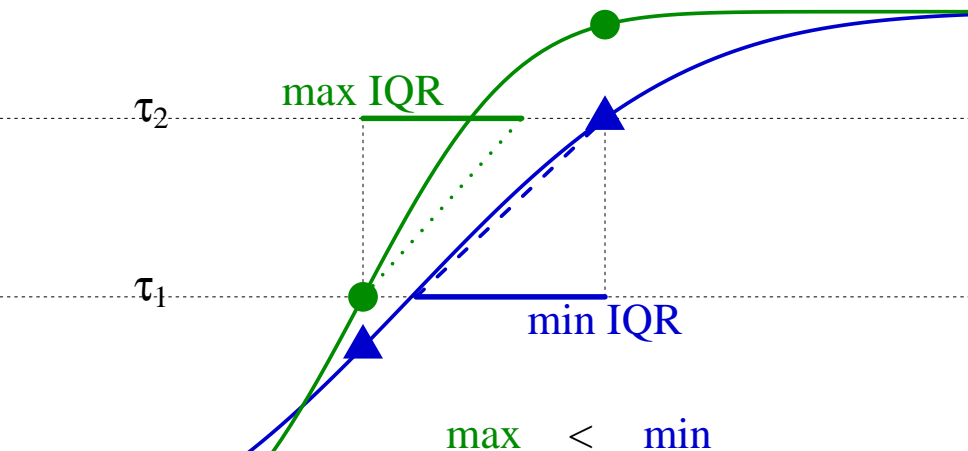
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- Yes, if symmetric, unimodal latent distributions
- Location–scale  $\implies$  extrapolation



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# Ordinal SD1 inference

- Let  $\theta_j \equiv F_2(j) - F_1(j) = \mathbb{E}[\mathbf{1}\{H_2 \leq j\} - \mathbf{1}\{H_1 \leq j\}]$
- Ordinal SD1:  $H_2 \text{ SD}_1 H_1 \iff \theta_j \leq 0, j = 1, 2, 3, 4$

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- Ordinal SD1:  $H_2 \text{ SD}_1 H_1 \iff \theta_j \leq 0, j = 1, 2, 3, 4$
- Frequentist: recent moment inequality tests from Andrews and Barwick (2012), Romano, Shaikh, and Wolf (2014), McCloskey (2015), et al.
- Bayesian: “nonparametric” posterior for category probabilities  $\implies$  posterior probabilities for all relationships

## Other relationships

- Unions and/or intersections of inequalities
- Bayes: just compute posteriors
- Frequentist: intersection–union test (sometimes)

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# Treatment effects

- Jones, Molitor, and Reif (2018): randomized workplace wellness program; measure SRHS
- Can compare treated/untreated latent health distributions: healthier (SD1)? inequality?



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- Can compare treated/untreated latent health distributions: healthier (SD1)? inequality?
- Can examine selection effects, too.
- Stay tuned...

# Comparisons of U.S. states

- Goal: compare latent health across states
- Data: 2011 PSID,  $\sim 50$ –300 observations per state
- Posteriors: Bayesian bootstrap of Dong, Elliott, and Raghunathan (2014) (stratification, clustering, weights)

## PSID 2011 posterior probabilities (%)

$X$	$Y$	$SD_1$		SC		fans out	
		$\succ$	$\prec$	$\succ$	$\prec$	$\succ$	$\prec$
AZ	MO	0	90	4	2	11	34
NY	UT	0	3	0	94	1	70
IL	NY	20	0	66	1	92	3
MN	NY	24	0	57	0	96	3
IA	MO	0	10	2	16	42	98

## PSID 2011 posterior probabilities (%)

X	X SD <sub>1</sub> Y; Y is:					X SC Y; Y is:				
	MO	KS	NE	IA	IL	MO	KS	NE	IA	IL
MO	—	0	10	10	0	—	7	67*	16	30
KS	34*	—	20*	10	3	6	—	24	6	16
NE	3	0	—	6	0	2	0	—	3	0
IA	0	0	6	—	0	2	0	66*	—	6
IL	40*	4	18*	43*	—	4	14	48	15	—

Asterisk (\*): satisfied in-sample

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# Purpose and setup

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- DGP 1:  $P(X = j) = P(Y = j) = 1/5$ ,  $j = 1, \dots, 5$ ; all SD1 inequalities binding.
- DGP 2: change to  $P(X = j) = 1/10$  for  $j = 1, 2, 3$  and  $P(X = 4) = 1/2$ ; only one binding inequality.

# Methods

- KS: Kolmogorov–Smirnov.
- RMS: “refined moment selection” of Andrews and Barwick (2012).
- Bayes: Dirichlet–multinomial, uninformative prior on parameters. Reject if posterior below  $\alpha$ .
- adj: adjust prior to  $P(H_0) = 1/2$ . (Goutis, Casella, and Wells, 1996)



Results:  $\alpha = 0.1$ 

DGP	$n$	$H_0: X \text{ SD}_1 Y$				$H_0: X \text{ SC } Y$		
		KS	RMS	Bayes	adj	RMS	Bayes	adj
1	50	0.038	0.089	0.436	0.204	0.032	0.439	0.175
1	100	0.022	0.084	0.430	0.205	0.029	0.359	0.142
1	500	0.027	0.092	0.447	0.199	0.034	0.428	0.171
1	1000	0.032	0.079	0.454	0.228	0.032	0.408	0.155
2	50	0.004	0.057	0.127	0.032	0.031	0.125	0.034
2	100	0.002	0.068	0.105	0.031	0.085	0.133	0.041
2	500	0.006	0.087	0.098	0.029	0.095	0.114	0.032
2	1000	0.003	0.074	0.084	0.025	0.060	0.084	0.018

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RMS “better” than KS

Even with Bayes (adj), cannot treat posterior as  $p$ -value, or vice-versa

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- Summary: relationships defined by inequalities; frequentist and Bayesian inference (code on website)
- Future work: other shape restrictions? multivariate? regression? health, happiness, ...?
- Thank you!
- (And further questions or comments are welcome)

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