Final Exam, Math 2300, FS13. NAME: ________________________________

No calculators, notes or books are permitted. 20 points/problem.

1. Compute:

\[ \int \int_D \frac{y}{2xe^{x^2}} \, dA, \]

where D is the 2-D region given by \( 0 \leq x \leq 1, \, 0 \leq y/(2xe^{x^2}) \leq 1 \). **Hint:** make the change of variable \( u = x, \, v = y/(2xe^{x^2}) \); solve for \( x \) and \( y \) in terms of \( u \) and \( v \) to compute the Jacobian.
2. Use the method of Lagrange multipliers to write down, but do not solve, four equations with four unknowns which can be used to find the maximum and minimum of \( f(x, y, z) = x^2y + xz \) on the ellipsoid \( x^2 + 2y^2 + 3z^2 = 5 \).

3. Find the net outward flux of the vector field \( \mathbf{F}(x, y, z) = (x^3, y^3, z^3) \) across the boundary of the region enclosed between the sphere \( x^2 + y^2 + z^2 = 4 \), and the cone \( z = \sqrt{x^2 + y^2} \) (i.e., compute the flux integral \( \int_S \mathbf{F} \cdot \mathbf{n} \, d\sigma \), where \( S \) is the boundary of the stated region).
4. Find the critical points of $f(x, y) = x^3 + 3x^2y - 3x^2 - 3y^2$. Use the second
derivative test to find all local extreme values and saddle points.

5. Set up, but do not evaluate, an integral which expresses the surface area
of the parametric surface $\mathbf{r}(\theta, z) = (\cos \theta, \theta, z)$, $0 \leq \theta \leq 2\pi$, $-2 \leq z \leq 2$. 
6. The point (1,0,1) lies on the intersection of the sphere $x^2 + y^2 + z^2 = 2$, and the parametric surface $\mathbf{r}(\theta, z) = (\cos \theta, \theta, z)$. Compute the Cosine of the angle between the the tangent planes to these two surfaces at (1,0,1).

7. Let $S$ denote the boundary surface of the cube bounded by $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$. Let $u(x, y, z) = x^2 + y^2 + z^2$. Calculate the outward flux across $S$ of $\nabla u$ (i.e., compute $\iint_S \nabla u \cdot \mathbf{n} \, d\sigma$, where $\mathbf{n}$ denotes the outer unit normal to $S$).
8. Find the work done along the closed path \( C \) with counter-clockwise orientation, which encloses the region \( R \) bounded above by the line \( y = x \) and below by the parabola \( y = x^2 - 2x \), by the vector field \( \mathbf{F}(x, y) = (3xy, 2x^2) \).

(I.e., compute the “work” integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \)).

9. Find the parametric equation of the line through the point \((1, 2, 3)\) perpendicular to the plane \( x - 2y + 7z = 14 \).
10. Let $F(x, y, z) = (z, x, \cos(y^2))$. Compute $\iint_S \text{Curl} F \cdot n \, d\sigma$, where $S$ is the upper hemisphere $x^2 + y^2 + z^2 = 4$, $z > 0$, and $n$ denotes the upward unit normal to the surface.