A Monetary Intertemporal Model

Economics 3307 - Intermediate Macroeconomics

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Fall 2013
Introduction

Thus far, agents have exchanged goods directly in all of our models.

What is money, and how does it affect the economy?

Three important functions of money:

▶ **Medium of exchange:** It is accepted in exchange for goods purely because it can be traded for other goods. The advantage of money is its liquidity and transparency.

▶ **Store of value:** Like other assets, money can be used to trade current goods for future goods.

▶ **Unit of account:** Almost all contracts are denominated in terms of money (rather than some other good).
Measuring the Money Supply

- Many forms of money: commodity money, private bank notes, etc.

- The definition of money depends on drawing a line about which assets satisfy the medium of exchange property.

- Various measures of money supply in the United States:
  - $M_0$, referred to as the **monetary base** or **outside money**, represents liabilities to the Fed. $M_0 = \text{currency in circulation} + \text{Fed deposits}$.
  - $M_1$ is a measure of the assets most widely used by the private sector in making transactions. $M_1 = \text{currency in circulation} + \text{travelers' checks} + \text{demand deposits} + \text{transactions deposits}$.
  - $M_2 = M_1 + \text{savings deposits} + \text{retail money market funds}$.
Two approaches to modeling money:

- “Deep models” explicitly model the frictions that lead people to use money as a medium of exchange.
  - Most frictions arise from agents’ **double coincidence of wants**.

- “Applied models” simply assume that money is used in some or all transactions.

For now, we assume that goods must be purchased either with cash on hand or costly transaction services (credit) provided by banks.
The price level— the amount of money required to purchase 1 unit of consumption— is $P_1$ in period 1 and $P_2$ in period 2.

Household portfolios include money, which pays no interest, and nominal bonds, which pay interest $1 + R$ in money in period 2.

The real interest rate on savings is $r$ and is related to $R$ by the Fisher relation,

$$1 + r = \frac{1 + R}{1 + i}, \text{ where } 1 + i = \frac{P_2}{P_1} \Rightarrow r \approx R - i$$

The nominal interest rate on money is $R_m = 0$ and the real rate is $r_m = \frac{1}{1 + i} \approx -i$, so why do agents hold money?
Historical Interest Rate Data

The graph shows historical interest rate data with two lines: one for nominal interest rates and another for real interest rates. The x-axis represents years from 1940 to 2010, and the y-axis represents interest rates in percent. The nominal interest rates peak significantly in the early 1980s, while the real interest rates show a more subdued pattern with fluctuations throughout the period.
In the **money-in-utility** model, utility comes from consumption $C_t$, leisure $l_t$, and period 1 real money balances, $m_1 \equiv \frac{M_1}{P_1}$.

Households receive labor income $P_t w_t (h - l_t)$, dividends $P_t \pi_t$, and pay taxes $P_t T_t$.

Households save in period 1 using money $M_1$ and bonds $S_1$ which pay $M_1 + (1 + R)S_1$ in period 2.

The government sets $G_1$, $G_2$, $T_1$, $T_2$, and $M_1^s$. 
Households

- Nominal budget constraints:

\[ P_1 C_1 + M_1 + S_1 = P_1 (w_1 (h - l_1) + \pi_1 - T_1) \]
\[ P_2 C_2 = P_2 (w_2 (h - l_2) + \pi_2 - T_2) + M_1 + (1 + R) S_1 \]

- Substitute out \( S_1 \) to get the nominal intertemporal constraint:

\[ P_1 C_1 + \frac{R}{1 + R} M_1 + \frac{P_2 C_2}{1 + R} = P_1 (w_1 (h - l_1) + \pi_1 - T_1) \]
\[ + \frac{P_2 (w_2 (h - l_2) + \pi_2 - T_2)}{1 + R} \]

- Divide through by \( P_1 \) and recall that \( 1 + r = \frac{1 + R}{1 + i} = (1 + R) \frac{P_1}{P_2} \) to get

\[ C_1 + \frac{R}{1 + R} m_1 + \frac{C_2}{1 + r} = w_1 (h - l_1) + \pi_1 - T_1 + \frac{w_2 (h - l_2) + \pi_2 - T_2}{1 + r} \]
Households

- Households solve

\[
\max_{C_1, l_1, m_1, C_2, l_2} u(C_1, l_1) + \phi(m_1) + \beta u(C_2, l_2)
\]

subject to

\[
C_1 + \frac{R}{1 + R} m_1 + \frac{C_2}{1 + r} = w_1(h - l_1) + \pi_1 - T_1 + \frac{w_2(h - l_2) + \pi_2 - T_2}{1 + r}
\]

- Optimality conditions (continued on next slide):

\[
\left\{ \frac{u_l(C_1, l_1)}{u_C(C_1, l_1)} = w_1 \right. \\
\left. \frac{u_l(C_2, l_2)}{u_C(C_2, l_2)} = w_2 \right. \\
\frac{\phi'(m_1)}{u_C(C_1, l_1)} = \frac{R}{1 + R}
\]

Within-period decisions:
Optimality conditions continued:

Intertemporal decisions: \[ \frac{u_C(C_1, l_1)}{\beta u_C(C_2, l_2)} = 1 + r \]

\[ C_1 + \frac{R}{1 + R} m_1 + \frac{C_2}{1 + r} = w_1(h - l_1) + \pi_1 - T_1 + \frac{w_2(h - l_2) + \pi_2 - T_2}{1 + r} \]

These conditions tell us that \( m_1 \equiv \frac{M_1}{P_1} \) depends negatively on \( R \) and positively on \( C_1 \) (and therefore positively on aggregate income \( Y_1 \)).

Thus, \( M_1^d = P_1 L(Y_1, R) \), i.e. \( M_1^d = P_1 L(Y_1, r + i) \approx P_1 L(Y_1, r) \).
The firm solves

\[
\max_{N_1, l_1, N_2} P_1 \left( z_1 F(K_1, N_1) - w_1 N_1 - l_1 \right) \\
+ P_2 \left( \frac{z_2 F((1 - d)K_1 + l_1, N_2) - w_2 N_2 + (1 - d)((1 - d)K_1 + l_1)}{1 + R} \right)
\]

Optimality conditions:

Within-period decisions:

\[
\begin{align*}
z_1 F_N(K_1, N_1) &= w_1 \\
z_2 F_N(K_2, N_2) &= w_2
\end{align*}
\]

Investment decision:

\[
z_2 F_K(K_2, N_2) - d = \frac{P_1}{P_2}(1 + R) - 1 = r
\]

The firm’s decisions are independent of nominal variables!
The government’s period budget constraints are

\[ P_1 G_1 = P_1 T_1 + B_1 + M_1^s \]
\[ P_2 G_2 + M_1^s + (1 + R)B_1 = P_2 T_2 \]

Substituting out \( B_1 \) gives the nominal intertemporal constraint

\[ P_1 G_1 + \frac{P_2 G_2}{1 + R} = P_1 T_1 + \frac{R}{1 + R} M_1^s + \frac{P_2 T_2}{1 + R} \]

Dividing by \( P_1 \) gives the real intertemporal constraint

\[ G_1 + \frac{G_2}{1 + r} = T_1 + \frac{R}{1 + R} \frac{M_1^s}{P_1} + \frac{T_2}{1 + r} \]
Monetary Equilibrium

- A competitive equilibrium is prices $P_1, P_2, r, w_1, w_2$; household allocations $C_1, N^s_1, M^d_1, C_2, N^s_2$; firm allocations $K_1, N^d_1, l_1, N^d_2$; and allocations for the government $G_1, G_2, T_1, T_2, M^s_1$ such that:
  
  1. $C_1, N^s_1, M^d_1, C_2, \text{ and } N^s_2$ solve the household’s problem.
  
  2. $N^d_1, l_1, \text{ and } N^d_2$ maximize profits $V = P_1 \pi_1 + \frac{P_2 \pi_2}{1+R}$, given $K_1$.
  
  3. The government’s budget constraint is satisfied:
     
     $$G_1 + \frac{G_2}{1+r} = T_1 + \frac{R}{1+R} \frac{M^s_1}{P_1} + \frac{T_2}{1+r}.$$
  
  4. Labor market clearing: $N^d_1 = N^s_1 \text{ and } N^d_2 = N^s_2$.
  
  5. Credit market clearing: $S^p_1 + S^g_1 = 0 \iff S^p_1 = B_1$.
  
  6. Goods market clearing: $C_1 + l_1 + G_1 = z_1 F(K_1, N^d_1)$ and
     
     $$C_2 + G_2 = z_2 F(K_2, N^d_2) + (1 - d) K_2 \text{ where } K_2 = (1 - d) K_1 + l_1.$$
  
  7. Money market clearing: $M^d_1 = M^s_1$. 

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Rearrange the household constraint and substitute $M_1^d = M_1^s$ to get

$$C_1 + \frac{C_2}{1+r} = w_1(h-l_1) + \pi_1 + \frac{w_2(h-l_2) + \pi_2}{1+r} - \left(T_1 + \frac{R}{1+R} \frac{M_1^s}{P_1} + \frac{T_2}{1+r}\right)$$

Substitute in the government’s budget constraint to get

$$C_1 + \frac{C_2}{1+r} = w_1(h-l_1) + \pi_1 + \frac{w_2(h-l_2) + \pi_2}{1+r} - \left(G_1 + \frac{G_2}{1+r}\right)$$

Thus, the household’s budget constraint in equilibrium looks the same as in the real intertemporal model.
The Classical Dichotomy

We can divide the remaining optimality conditions into two groups:

Real variables:
\[
\begin{align*}
\frac{u_l(C_1, l_1)}{u_C(C_1, l_1)} &= w_1 \quad \text{and} \quad \frac{u_l(C_2, l_2)}{u_C(C_2, l_2)} = w_2 \\
\frac{u_C(C_1, l_1)}{\beta u_C(C_2, l_2)} &= 1 + r \\
z_1 F_N(K_1, N_1) &= w_1 \quad \text{and} \quad z_2 F_N(K_2, N_2) = w_2 \\
z_2 F_K(K_2, N_2) - d &= r
\end{align*}
\]

Nominal variables:
\[
\frac{\phi'(m_1)}{u_C(C_1, l_1)} = \frac{R}{1 + R}
\]

Households/firms make consumption, labor, and investment decisions independent of nominal variables: the classical dichotomy.
Monetary Equilibrium

To find the monetary equilibrium, find $r$ and $Y_1$ from the real equilibrium as before and then solve $M_1^d = M_1^s \iff P_1 L(Y_1, r) = M_1^s$.

Suppose $M_1^s$ jumps to $\tilde{M}_1^s$. Then $\tilde{P}_1 L(Y_1, r) = \tilde{M}_1^s \Rightarrow \frac{\tilde{P}_1}{P_1} = \frac{\tilde{M}_1^s}{M_1^s}$.

Changes in $M_1^s$ cause changes in $P_1$ but not in any real variables: money is neutral.
Effects of Higher Present TFP $z_1$

Output Supply
- Higher $z_1$ increases $MP_N$ this period.
- Labor demand $N^d_1$ increases, causing $Y^s_1$ to shift to the right.

Output Demand
- No changes in $Y^d_1$.

Equilibrium and the Money Market
- Equilibrium $r$ must decrease and $Y_1$ must increase.
- Decreasing $r$ and increasing $Y_1$ cause $M^d_1 = P_1 L(Y_1, r)$ to increase, resulting in lower equilibrium $P_1$. 
Equilibrium $r$ decreases and $Y_1$ increases, causing an increase in money demand $M_1^d = P_1 L(Y_1, r)$.

The price level $P_1$ decreases to clear the money market.