On Understanding the Cyclical Behavior
of the Price Level and Inflation

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I. Introduction
For business cycle researchers, two facts have emerged in the post-World War II data. The two are that the contemporaneous correlation coefficient for the price level and output is negative and the contemporaneous correlation between the inflation rate and output is positive. In other words, the price level is countercyclical and the inflation rate is procyclical.

Kydland and Prescott (1990) initiated this research line when they reported that after detrending, the price level is negatively correlated with output.\(^1\) Other researchers began testing the robustness of this finding. Cooley and Ohanian (1991) and Smith (1992) extend the sample. The two papers broadly agree that before World War II, the evidence suggests the price level was procyclical.\(^2\) After World War II, however, the evidence is consistent with Kydland and Prescott’s finding that the price level has been countercyclical.

Wolf (1991) asks whether the price level is countercyclical over the post-war sample period. He distinguishes between business cycles before and after the 1973 recession. Wolf’s view builds on the question, Are all business cycles alike?\(^3\) In particular, Wolf presents evidence from price indexes constructed from consumer expenditure categories. Based on the temporal break and from the correlation between movements in output and the price indexes, he concludes that he looks at price indexes for consumer expenditure categories. Wolf concludes that the price level has been countercyclical since the 1973 recession, but not before.

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\(^{1}\) Kydland and Prescott reported this result because it contradicted the maintained hypothesis that the price level was procyclical. In their view, theory was needed to account for the negative relationship.

\(^{2}\) In Cooley and Ohanian, the evidence is ambiguous before World War II. During the interwar period, they report that the price level is procyclical. In Smith, the evidence is that the price level is procyclical before the Great Depression.

\(^{3}\) See Blanchard and Watson (1986) for a detailed discussion on this question.
Cooley and Ohanian extend the set of business cycle facts to examine the relationship between the detrended inflation rate and output. They report that inflation rate is procyclical during the post-war sample. Webb (2003) and Kanstantakapoulou, Efthymois and Kollintzas (2009) are more recent contributions. Webb (2003) is especially clear on the stakes involved in the issue of procyclicality of the price level. He states, “The issue is of particular importance to macroeconomists who must choose which model to work with.” Kanstantakapoulou et al. (2009) investigate the robustness of countercyclicality of the price level and procyclicality of the inflation rate for 9 OECD countries using quarterly data, 1960-2004. For example, they state, “We examine the stylized facts …prices are countercyclical; inflation is procyclical.” Given that the inflation rate is the time derivative of the price level, the qualitative difference deserves attention.

Researchers have debated the implications associated with a countercyclical price level. One debate has centered on the competing role of demand shocks and supply shocks as a source or business cycle fluctuations. For example, Kydland and Prescott initially argued that countercyclical prices were a death knell to demand-shock focus that had been built to study monetary policy effects. Judd and Trehan (1995) argued that the cross correlation could come from an aggregate demand-aggregate supply model with sticky prices. Basically, a positive aggregate demand shock resulted in output increasing. Because prices were sticky, output would begin falling (mean reverting) and prices would increase eventually, causing real balances to decline and output to decline further. Of course, the Judd and Trehan explanation begs why prices are sticky at one level. To our knowledge, those researchers participating in the demand-versus-supply literature have not tried to reconcile the countercyclical price level and procyclical inflation.

An atheoretical approach was applied by Haslag and Hsu (2012). Because the inflation rate is the time derivative of the price level, the two correlations are consistent with a phase shift occurring in the cyclical component of the price level relative to the cyclical component of the output. In that paper, Haslag and Hsu document the size of the phase shift that could account for the pair of reported correlation coefficients. The approach does not explain why there is a phase shift, but simply that a phase shift could explain why the price level and its time derivative—the inflation rate—have qualitatively different correlations with output.

There are two broad questions that come to mind based on the literature. First, there are two business cycle facts presented. One goal, therefore, is to develop a methodology that characterizes the joint distribution function. Our aim is represent the probability of any particular data pattern in an empirically disciplined way that also respects “model uncertainty.” We apply this methodology to the particular

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4 See Webb (2003, page 69).
5 See Kanstantakapoulou et al. (2009, page 1).
question of the correlation between the price level and output and the correlation between the inflation rate and output. In our view, the methodology could easily be extended to characterizing the likelihood of other business-cycle correlations. Second, we are interested in providing some theoretical to account for these two facts. We propose rational inattention as a type of friction that could account for this pair of facts. We then provide quantitative results from a model economy to support this view.

In addition, we examine the pair of correlation coefficients from two different detrending approaches. Researchers have applied both trend-stationary and difference-stationary methods to identify the cyclical component of economic time series. We consider both approaches. In both methods, the cyclical component of the price level is negatively correlated with the cyclical component of output. In the trend-stationary approach, which used the H-P filter, the cyclical component of the inflation rate is positively correlated with the cyclical component of output. However, in the difference-stationary case, the cyclical component of the inflation rate is not systematically related to the cyclical component of output.

1.1. On Model Uncertainty

The methodological contribution is related to the stance that was presented in Brock, Durlauf and West (hereafter (BDW)) (2003) and (2007). In BDW, the stance was built on the notion of Bayesian Model Uncertainty where prior probabilities were assigned to each model specification. After estimating each model, the posterior probabilities were assigned based on relative likelihoods. Here, we simply detrend the data using two different methods, i.e., trend stationary and difference stationary in the sense of Nelson and Plosser (1982). We then estimate models with the resulting detrended data and report how the probability of the pattern of interest depends upon the method of detrending. We can compute the probability that the pattern of interest in a data disciplined way by used the estimated standard errors (under appropriate distributional assumptions, e.g. Gaussian) of the parameters of each model fitted to the detrended data. Of course, one could do this same exercise taking into account the uncertainties in the estimated parameters of the trend model also. We ignore this extra source of uncertainty in this paper in the interest of simplicity. We report histograms that show the part of the space where the pattern of interest holds with a set of “skyscrapers” whose height gives the probability of that grid of the space. For example, we are basically just using bootstrapping (Efron (1982), Efron and Tibshirani (1986)) to estimate the probability of the pattern of interest, i.e. the probability that the “stylized fact” under scrutiny in this paper holds under each method of detrending. All this will be explained in greater detail below.

One of the main points we want to get across in this paper is that we think this methodology is a useful way to present data patterns of interest in macroeconomics together with a measure of the

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6 Nelson and Plosser (1982) identified this source of model uncertainty in their study of macroeconomic time series.
uncertainties surrounding each such pattern. To put it another way, we are arguing for a methodology like this to accompany the presentation of any “stylized fact” in macroeconomics.

In this particular application, we apply a univariate autoregressive filter to create time series of the price level and output. We then compute the contemporaneous cross-correlations for the price level and output and the inflation rate and output. With the correlation coefficients, we can compute the likelihood that the filter yields countercyclical price level and procyclical inflation. By adopting the autoregressive filter, the approach stresses the role of persistence and goodness-of-fit to infer the likelihood. In other words, there are three key factors that play roles in determining the likelihood of the observed cross-correlations; namely, (i) the estimated values of the relative persistence in the cyclical components, the standard errors of the parameter estimates and (ii) the unexplained variation in the cyclical components. Our methodology allows us to assess the importance of each factor.

In addition, researchers have reported that the combination of the price level-output correlation and inflation rate-output correlation is not robust, and is especially sensitive to the time period being studied. For example, Cooley and Ohanian, Wolf, and Smith each report that the price level-output correlation varies over time going from procyclical to more recently being countercyclical. Haslag and Hsu also report evidence that during the post-war period, the correlation is time varying. The likelihood methodology offers some insight into the time variation.

1.2 On rational expectations and rational inattention

Brock and Hommes (1997) present an interesting framework in which it is costly to acquire the information necessary to form rational expectations. Here, we use an off-the-shelf money-in-the-utility function model, adding a form of rational inattention to determine whether this type of information friction can account for the phase shift in detrended price level. We study a special case of these costs in which agents purchase inexpensive “backward-looking” price expectations. Our quantitative analysis is encouraging in the sense that there is a reasonable set of parameters that result in countercyclical prices and acyclical inflation. The numerical results are consistent with the presence of a phase shift and qualitatively match the results obtained when we assume the cyclical components are derived from a difference-stationary process.

To further illustrate we present the solution to the forward-looking rational expectations equilibrium. In this way, we can contrast the notion of price stickiness that is imparted by rational inattention. In other words, the comparison helps one see why rational inattention works the way it does. We confirm the role that forward-looking rational expectations plays; namely, that prices adjust so quickly to new shocks that the direction of the change in the price level dominates the direction of change in the inflation rate. In other words, forward-looking rational expectations imparts phase-synchronicity between movements in output and movements in the price level. Such synchronous movements explain why models economies
incorporating rational expectations cannot account for the price level being countercyclical and the inflation rate being either procyclical or acyclical.

The paper outline is as follows. In Section 2, we consider an example that analytically illustrates the methodological approach. We construct the numerical analyses in Section 3. Section 4 considers the methodology in a model characterized by difference stationary series. Section 5 develops the model economy and reports the numerical results. In particular, we use the model economy to provide some analytical support and understanding of forces that produce negative correlation of the cyclic component of the price level (positive correlation of inflation) with the cyclic component of real by using the model of Woodford (2003, Chapter 2). Finally, Section 6 is a brief summary, conclusions, and suggestions for future research.

II. An Analytic Example

In this section, we consider a specific autoregressive process to illustrate how the various components affect the likelihood that the correlations will exhibit the pattern in the data.

Suppose both the price level \(p\) and output \(y\) (in log levels) are capable of being decomposed into trend components and cyclical components. Formally, \(x_t = x_t^T + x_t^C\), where \(x = p, y\). The superscript \(T\) denotes the trend component while the superscript \(C\) stands for the cyclical component. We assume that the cyclical component of the price level follows an AR(1) process while the cyclical component of output follows an AR(2) process. Thus,

\[
\begin{align*}
p_t^C &= b_1 p_{t-1}^C + u_t \\
y_t^C &= a_1 y_{t-1}^C + a_2 y_{t-2}^C + e_t
\end{align*}
\]

The cyclical components are computed as deviations from trend. Assume that the cyclical components for both the price level and output are mean zero, stationary processes. We drop the superscripts to write the implication of the stationary process as \(E[p_t y_t] = E[p_{t-1} y_{t-1}] = E[py]\), where \(E[py]\) denotes the covariance of the cyclical components of the price level and output. Under these assumptions, the sign of the covariance determines the sign of the contemporaneous cross-correlation.

We derive the expected value of the product of the cyclical component of the price level and output by substituting equations (1) and (2) into the covariance expression, yielding

\[
a_1 b_1 E[py] + a_2 (b_1)^2 E[py] + E[ue] = E[py]
\]

After rearranging and simplifying, we obtain

\[
E[py] = \frac{E[ue]}{1 - b_1[a_1 + a_2 b_1]} \quad (3)
\]

Based on equation (3), we derive the following lemma.
Lemma 1: With $E[ue] < (>) 0$ and with $1 - b_1[a_1 + a_2b_1] > (>)0$, then the sign of $E[py]$ is negative.

Equation (3) tells us that the sign of the cross-correlation coefficient depends on the sign of the covariance of the unexplained errors from the two AR processes that characterize the cyclical components of the price level and output, respectively. If the residuals are independent, then the correlation coefficient is zero. However, if the unexplained errors are negatively correlated and the denominator is positive, for example, then the correlation coefficient is negative. Note further that the denominator depends on the persistence of the two cyclical components. With $E[ue] < 0$, for example, greater persistence in price level or in output makes it less likely that the correlation between the price level and output will be negative.

Next, we turn to the covariance between inflation and output. Let $E[\pi y] = E[(p_t - p_{t-1})y_t]$ denote the covariance between the inflation rate and output. Substitute for the date-$t$ price level from equation (1) and for output from equation (2), yielding

\[ E[\pi y] = E[ue] \left[ \frac{1-\theta}{1-b_1\theta} \right] \quad (4) \]

where $\theta := a_1 + a_2b_1$. From which, we derive the following Lemma.

Lemma 2: With $E[ue] < (>) 0$, the sign of $E[\pi y] > 0$ if and only if $\frac{1-a_1-a_2b_1}{1-b_1[a_1+a_2b_1]} < (>) 0$.

Equation (4) indicates that the sign of the correlation between inflation and output again depends on the correlation between residuals. If the residuals are negatively correlated, for example, the bracketed term must be negative for the cyclical components of the inflation rate and output to be positive.

Note that the condition in Lemma 2 involves the denominator in Lemma 1. Therefore, the combination of conditions in Lemmas 1 and 2 create a partition for the space of estimated coefficients that must be satisfied for the price level to be countercyclical and the inflation rate to be procyclical. We derive those conditions in the following proposition.

Proposition 1: Based on the conditions derived in Lemmas 1 and 2, the price level is countercyclical and the inflation rate is procyclical for $\frac{1}{b_1} > a_1 + a_2b_1 > 1$ if $E[ue] < 0$ and $a_1 + a_2b_1 > \max \left[ 1, \frac{1}{b_1} \right]$ if $E[ue] > 0$.

By writing down the conditions in Lemma 1 and Lemma 2 and solving for the inverse of the persistence coefficient in price level equation, we obtain the conditions in Proposition 1. Note that the denominator that yields $E[py] < 0$ depends on the sign of the covariance between the unexplained terms. For example, if $E[ue] < 0$, then $E[py] < 0$ if the denominator is positive. It follows from Lemma 2 that $E[\pi y] > 0$ if the numerator is negative.
Combining the conditions in Lemmas 1 and 2, the two correlation coefficients are opposite signs if
\[ \frac{1}{b_1} > a_1 + a_2 b_1 > 1 \text{ if } E[ue] < 0. \] In contrast, if \( E[ue] > 0 \), the condition is \( a_1 + a_2 b_1 > \max \left[ 1, \frac{1}{b_1} \right] \).

The upshot of Proposition 1 is that there exists a range of parameter values that are consistent with the joint observation that the price level is countercyclical and the inflation rate is procyclical. In other words, Proposition 1 derives the values for one specific illustration of a linear autoregressive process that will yield the joint business cycle fact. If the residuals from the two autoregressive processes are negatively correlated, for example, Proposition 1 indicates that the price level cannot be “too persistent” relative to the persistence in the output equation for the joint business-cycle observation to hold. Conversely, if the residuals are positively correlated, the price level must be persistent enough relative to the persistence observed in the output equation for the price level to be procyclical and the inflation rate to be countercyclical.

To be more concrete, consider a case in which the cyclical component of the price level is a random walk. If the residuals are negatively correlated, for example, Proposition 1 tells us that the condition which jointly satisfies countercyclical price level and procyclical inflation rate cannot be satisfied. The first inequality in the sequence fails to be greater than one. For the case in which the residuals are positively correlated, then with positive coefficients in the AR process for output, the condition can be satisfied.

III. The Probabilistic Methodology

The data are quarterly observations from the United States for the period 1947:1 through 2007:4. Output is measured by real GDP and the chain-weight index for personal consumption expenditures. We use the H-P filter with \( \lambda = 1600 \) to obtain the detrended, or cyclical, values of output and the price level.

Table 1 reports the summary statistics for the cyclical components of each series. We report the mean and standard deviation for the series. In each case, the mean and standard deviation is close to reported values from other studies using postwar data. The cross correlation is different; in this sample, the cross correlation of price level and output is close to zero. The Bartlett standard error of this estimate is \( \frac{1}{\sqrt{T-1}} = 0.062 \) for our sample. Consequently, the evidence indicates that the price level is countercyclical and the inflation rate is procyclical.

We begin by assessing some standard inference problems. Consider the null hypothesis that the correlation coefficient for the price level and output is equal to zero. With the Bartlett standard error, we reject the null hypothesis with a p-value that is less than one percent. Similarly, consider the null hypothesis that the correlation coefficient for the inflation rate and output is equal to zero. We would also
reject the null hypothesis with a p-value less than one percent. Another question would be to state a joint null hypothesis; namely, that the correlation coefficient for the price level and output is negative and the correlation coefficient for the inflation rate and output is positive.

So, in this paper, we apply standard linear regressions to fit the time series. Armed with these equations and the distributions of the parameter estimates and the residuals, our aim is to generate simulated time series. The simulated series can then be used to estimated correlation coefficients and we can assess the likelihood that a particular observed pattern of the correlation coefficients is present.

Here, we are interested in another probabilistic expression of the joint likelihood that the price level is countercyclical and the inflation rate is procyclical. The approach begins by fitting each cyclical component to an autoregressive process. With the AR process, combined with the noisy of the coefficient estimates and the error terms, we construct a time series for the period 1947:2 through 2007:4. We create 10,000 artificial time series in this way. We compute the inflation rate for each artificial time series and then compute the correlation coefficients for the price level and output and for the inflation rate and output.

Formally, consider the case we use in the analytic illustration in Section 2. The estimated version of the AR(1) process for the price level is represented as

\[ p_t^c = \hat{b}_1 p_{t-1}^c + \hat{\alpha}_t \]

where \( \hat{b}_1 \) is the estimated coefficient for the lagged value of the price level and \( \hat{\alpha}_t \) is the residual. To construct the artificial time series, the date-\( t \) coefficient on the lagged value of the price level is \( \hat{b}_1 + \omega_t \) where \( \omega_t \) is the standard error of the coefficient. Similarly, for output, we estimate

\[ y_t^c = \hat{\alpha}_1 y_{t-1}^c + \hat{\alpha}_2 y_{t-2}^c + \epsilon_t \]

with the date-\( t \) coefficient on the first lagged value of output is \( \hat{\alpha}_1 + v1_t \) and on the second lagged value of output is \( \hat{\alpha}_2 + v2_t \). Note that \( v1_t \) and \( v2_t \) are the standard errors of the coefficients on the first- and second-lagged values of output, respectively. The estimates of the standard errors of coefficients and the standard errors of the estimates are reported in Table 2.

Based on these parameter estimates, the artificial time series is generated as

\[ p_t^* = (\hat{b}_1 + 0.026 \cdot r_t^{PI}) p_{t-1}^* + 0.0042 \cdot r_t^{lI} \] (5)

\[ y_t^* = (\hat{\alpha}_1 + 0.06 \cdot r_t^{PI}) y_{t-1}^* + (\hat{\alpha}_2 + 0.06 \cdot r_t^{PI}) y_{t-2}^* + 0.0084 \cdot r_t^e \] (6)

[8]
where we use “*” superscript to denote the artificial time series. To illustrate the process based on equation (5), we use the estimated coefficient on the lagged price level, which is $\hat{b}_1 = 0.9144$. The price level is generated by using a realization, labeled $r_{t}^{p1}$, from a pseudo-normally distributed random number generator with mean zero and standard deviation one. The error term is a also a draw from a pseudo-normally distributed random number generator denoted $r_t^e$. The same process is applied to equation (6) with $\hat{a}_1 = 1.1629$ and $\hat{a}_2 = -0.372$. Let $r_t^{y1}, r_t^{y2}$ and $r_t^e$ stand for the date-$t$ realization for the coefficient on the first lagged value of output, the second lagged value of output, and the error term, respectively.

Lastly, we consider the contemporaneous correlation between the two residuals, $\hat{u}_t$ and $\hat{e}_t$. We estimate the regression:

$$\hat{e}_t = 0.0 - \frac{0.0298}{(0.0647)} \hat{u}_t + \epsilon_t$$

(7)

So, the evidence suggests that there is no systematic relationship between the two residuals.

From Proposition 1, there are implications for the distribution that occur because the residuals are statistically independent. In the absence of a systematic relationship between the residuals, $E[\pi y] = 0$. Based on equations (3) and (4), the implication is that the correlation between the price level and output and the inflation rate and output are, on average, zero.

The estimated coefficient in equation (7) is negative, but one cannot reject the null hypothesis that the coefficient is different from zero at standard confidence levels. If we consider the sign of the estimated coefficient, we can apply Proposition 1 to say something about the values of the estimated coefficients. Recall that Proposition 1 says that for the case in which the residuals are negatively correlated, then the condition for countercyclical price level and procyclical inflation is $\frac{1}{b_1} > a_1 + a_2 b_1 > 1$. The first inequality is satisfied since $\frac{1}{0.9144} > 1$. However, $1.1629 + (-0.372 \times 0.9144) < 1$, thus violating the condition. The results indicate that for the particular AR processes estimated, the product of the second-order term for output and the first-order term for the price level renders the expression too small. In short, the estimated AR processes do not suggest that, on average, the central tendency for the simulated series do not lies in the probabilistic sweet spot.

To get at the distribution directly, we present the distribution of correlation coefficients from the 10,000 simulations in Figure 1. There are 2316 cases in which the simulated time series generates a negative correlation between the price level and output and a positive correlation between the inflation
rate and output. Indeed, the distribution shows that the mass is centered close to pairs of correlation coefficients. Indeed, there are 6331 pairs of correlation coefficients in which the following joint condition is satisfied: the correlation coefficient between the simulated price level and simulated output is between \(-0.1\) and \(0.1\) and the correlation coefficient between simulated inflation and simulated output is between \(-0.1\) and \(0.1\).

It is worthwhile to offer a brief explanation for why the distribution massed around the combination that the two simulated correlation coefficients are equal to zero. The answer owes chiefly to absence of any correlation between the two residuals. Equations (3) and (4) express the two correlation coefficients as linear functions of the correlation between two residuals, \(e_t\) and \(u_t\). If the two residuals are independent, Equations (3) and (4) indicate that the price level and the inflation rate will be acyclical. Our numerical results reflect the random noise added to the coefficients; by adding a random term to the parameter estimates, our numerical results indicate that the central tendency is consistent with the analytical results derived in Equations (3) and (4).

Another question that arises is whether the numerical results are sensitive to the order of the autoregressive processes. In addition to the specification in which output follows an AR(2) process and the price level an AR(1) process, we consider four other specifications. In addition to the sensitivity analysis, looking at the AR(1) processes provides a very simple interpretation of the role that persistence plays in characterizing the parameter conditions that satisfy the joint condition that the correlation between the price level and output is negative and the correlation between the inflation rate and output is positive. Table 3 reports the findings for each of the additional specifications.

**Proposition 2:** If \(E[u_e] < 0\), then the price level is procyclical and the inflation rate is countercyclical, in expected value, if \(a_1 b_1 < 1 < a_1\). (See Appendix)

Proposition 2 reduces the expected sign of the correlation coefficients to persistence in the cyclical components. The key is that the cyclical component of output lies outside the unit root and the product of the two persistence coefficients is less than one. It follows that the persistence in the cyclical component of the price level is “low enough” to satisfy the condition.

Panel A of Table 3 reports the results from estimating the output equation as an AR(1) process. The estimated coefficient is 0.8499. So, there is not enough persistence in the cyclical component of output to jointly satisfy the conditions that would, in expectations, yield a countercyclical price level and a procyclical inflation rate.

In both Panel A and Panel B, the evidence indicates that the residuals are negatively, but not significantly correlated. In Panel C, we see that the correlation is negative and significant. Therefore, we
conduct the analysis by setting simulated error in the output equation as follows: \( \hat{\epsilon}_t = 0.0 - 0.3 \hat{u}_t + \epsilon_t \), where \( \epsilon_t \) is drawn from a pseudo-random number generator with mean zero standard deviation equal to 0.008. In this version, we find that 2116 of the 10,000 simulations result in a negative correlation between the price level and output and positive correlation between the inflation rate and output. The frequency distribution for the negatively correlated errors is presented in Figure 2. Therefore, this a slight change in the mass of the frequency distribution resulting in fewer pairs of correlation coefficients that match the observed post-war pair.

We use the regression coefficients to generate simulated time series. Our results are remarkably consistent in the sense that the likelihood that the simulated price level is countercyclical and the simulated inflation rate is procyclical is between 21 and 23 percent.

What does the invariance of the likelihood result tell us about the methodology? For one thing, we know that the order of the autoregressive processes do not have measurable effects on the likelihood that the price level is countercyclical and the inflation rate is procyclical. The analytical example suggests that it is the independence of the residuals that matters. If we incorporate the positive relationship between the residual from the output regression and the residual from the price level regression, we obtain the frequency distribution depicted in Figure 2. Here, the mass is located completely in the non-positive orthant, indicating that this simulated time series generate countercyclical prices and countercyclical inflation in all 100 percent of the simulations.\(^7\)

IV. Difference stationary results

In this section, we consider cases in which the cyclical component is constructed by first-differencing the data. Formally, \( p_t^c = p_t - p_{t-1} \) and \( y_t^c = y_t - y_{t-1} \). For completeness, note that the cyclical component of the inflation rate is defined as follows: \( \pi_t^c = p_t^c - p_{t-1}^c \).

We begin by estimating the correlation coefficients for the difference-stationary definition of the cyclical components. The contemporaneous correlation coefficient for the price level and output is -0.13 and the contemporaneous correlation coefficient for the inflation rate and output is 0.03. Thus, an important difference emerges based on the approach used to construct the cyclical components; we cannot reject the null hypothesis that the price level is countercyclical at conventional confidence levels. However, the inflation rate is acyclical based on the difference stationary definition.

\(^7\) If, however, the residuals were positively correlated, we can show that the simulated time series would yield procyclical price level and procyclical inflation rate in every case.
Consider the cyclical component of the price level as an AR(1) process and the cyclical component of output as an AR(2) process. The results are

\[ p_t^c = 0.002 + 0.8 p_{t-1}^c \]  \hspace{1cm} (8)

and

\[ y_t^c = 0.005 + 0.306 y_{t-1}^c + 0.087 y_{t-2}^c \]  \hspace{1cm} (9)

Here, the root mean square error is 0.004 for both the price level and for output. The contemporaneous relationship between the residuals is represented as

\[ e_t = 0.0 - 0.164 u_t \]  \hspace{1cm} (10)

where \( e \) is the residual from the output regression and \( u \) is the residual from the price level regression. The implication is that there is no statistically significant contemporaneous relationship between the output residual and the price level residual.

We generate 10,000 time series for the price level and for output. Figure 3 plots the frequency distribution for the correlation coefficients between the price level and output and the inflation rate. Based on the frequency distribution, we can compute the likelihood that the correlation coefficient between the price level and output is negative joint with the probability that the correlation between the inflation rate and output is positive. In the first difference setting, the likelihood is 20.8 percent. This value is a slightly less than the probability observed in the trend stationary case.

It seems important to consider another, more narrow question based on the frequency distribution. Specifically, what is the likelihood that the correlation coefficient lies between 0 and -0.2 and the correlation coefficient between the inflation rate and output lies between -0.1 and 0.1. Based on the frequency distribution, the likelihood of this joint event is 42.8 percent. After specifying a tighter joint event, we observe that the likelihood increases in the first-difference stationary setting.

Does the order of the autoregressive process matter for the results? We estimate regressions in which the price level follows an AR(2) and AR(3) process. Similarly, we consider cases in which output is expressed as an AR(1) and AR(3) process. The results are reported in Panels A, B and C of Table 4. In general, the likelihood values are invariant with respect to the changes in the autoregressive processes. The probability is roughly 43 percent of the cases satisfy the joint condition that the correlation between
the price level and output lies between zero and -0.2 and the correlation between the inflation rate and output lies between 0.1 and -0.1.

V. Theory to account for the pair of cross-correlations

We consider a dynamic, stochastic general equilibrium in which money enters directly into the utility function. The MIUF approach does not take a stand on the friction that accounts for why fiat money is valued in the economy. Our aim here is to take an off-the-shelf model economy and derive the conditions in which we could account for the pair of observations presented in this paper.

There are infinite number of discrete time period with $t = 0, 1, 2, \ldots$ There is a single, perishable consumption good. The economy is populated by a measure-one continuum of representative agents. At each $t \geq 0$, each agent is endowed with income represented by $P_t y_t$ where $P$ denotes the price level and $y$ is income measured in units of the consumption good.

Formally, let the representative agent solve the following infinite-horizon, discounted problem:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \xi_{ct} U(c_t) + \xi_{mt} V \left( \frac{M_t}{P_t} \right) \right] \right\}$$

(11)

where $\xi_{jt}$ for $j = c, m$ is a taste shock, following Woodford (2003, Chapter 2), for consumption and real balances, respectively. Further, let increases (decreases) in the money supply over time be distributed as a lump-sum payment (tax) represented by $\tau$. Finally, $M$ is the quantity of money balances held by each person. The functions, $U(.)$ and $V(.)$, are twice continuously differentiable, strictly concave. The marginal utility of each good is nonnegative. Woodford (2003, Chapter 2) considers both the “cashless economy” case, $V(.) = 0$ and the “frictions” case where $V(.)$ is non zero. He discusses the role of the taste shocks as well as various money supply and interest rate rules of the Central Bank in this framework for both cashless economies and monetary economies using this framework. We just take his model “off the shelf” here and use it to study the set of real output, money supply, taste shocks and the set of preferences that produce negative (positive) correlations between the cyclic components of the price level (inflation) with the cyclic component of real output.

In this economy, agents are price takers. The sequence of shocks to tastes and income are drawn from a distribution with positive supports. It is straightforward to derive the first-order necessary conditions for utility maximization. Formally, the Euler equation is
The money supply rule is: \( M_t^S = M_{t-1}^S + T_t \), where \( T_t = P_t \tau_t = F(y_{t-1}, P_{t-1}; s_t) \), where \( s_t \in [\xi_{ct}, \xi_{mt}] \).

In other words, the general setting is one in which the change in money supply depends on lagged output, the lagged price level and the exogenous tastes shocks. The function is written broadly enough to encompass money supply rules such as the Friedman k-percent rule or McCallum’s base rule as well as more exotic versions. The numerical analysis will focus on a k-percent style rule, but these could be modified to be cyclically dependent or an elastic supply rule.

The goods market clears when the quantity of goods consumed equals the quantity of goods available. Thus, for equilibrium concepts that require market clearing, the goods market condition is represented as \( c_t = y_t \).

To obtain some analytical results, we specify specific functional forms. For example, let \( U(c_t) = \ln(c_t), V(M_t / P_t) = \ln(M_t / P_t) \). Further, let real income grow at rate \( g_t \) so that after taking logs, we get \( \tilde{y}_t = \tilde{y}_{t-1} + g_t \) where \( \{g_t\}_{t=1}^\infty \) is a stationary process and “tildes” are used to denote log transforms of the variables. Let \( g_t = \rho g_{t-1} + n_t \) where \( g_0 \) is given and \( n_t \sim IID(0, \sigma_n^2) \). Thus, real income is difference-stationary in this setup. We consider two polar cases of price expectations. The first case assumes that the future price expectations are backward looking so that \( E_t P_{t+1} = P_{t-1} \). The second case is structural rational expectations. We treat the backwards looking case, first.

5.1 The Backwards Looking Case

In this case, we assume that the future price expectations are backward looking so that \( E_t P_{t+1} = P_{t-1} \). With log utility and after substituting the goods market clearing condition, we can rewrite the Euler equation as

\[
\xi_{ct} U'(c_t) = \xi_{mt} V'(M_t / P_t) + \beta E_t [\xi_{ct+1} U'(c_{t+1}) (P_t / P_{t+1})] \quad \forall t \geq 0
\]  

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\]

We define the net inflation rate, \( \pi_t \), as \( (1 + \pi_t)P_{t-1} \). Then, we can use this expression for the date-\( t \) price level, writing the Euler equation as

\[
P_t = \xi_{ct} M_t P_{t-1} / [\alpha \xi_{mt} P_{t-1} y_t + \beta M_t E_t \{e^{-g_{t+1}} \xi_{ct+1}\}]
\]

We define the “price of money”, \( \Lambda_t \equiv 1 / P_t \). Then we may write (13) as follows,
\[
\Lambda_t = (\xi_{mt} / \xi_{ct}) \alpha Y_t / M_t) + \beta E_t \{((\xi_{ct+1} / \xi_{ct}) Y_t^2 / Y_{t+1})\} \Lambda_{t-1} \tag{14}
\]

The main advantage of (14) is that we see immediately that once we specify the real output process, the money supply process, and the taste shock processes, we have a linear stochastic dynamical system in the price of money, \( \Lambda_t \). At the risk of repeating, we follow Nelson and Plosser, 1982, equation (1) for the definition of trend component and cyclical component in the Difference Stationary (DS) case.

We use equation (13) to generate a simulated time series for the price level. We initially consider an economy in which there are no tastes shocks. So, \( \xi_{mt} = \xi_{ct} = 1 \). We initial the economy by setting \( P_{-1} = 1 \) and \( y_{-1} = 100 \). The initial growth rate for output is set at 1.019. For the baseline calibration, we let \( n_t \sim N(0,0.009) \) and the growth rate follows the equation: \( g_t = 0.1019 + 0.9 \cdot g_{t-1} \). We assume the money growth rate is fixed at 4.5 percent so that \( M_t = 1.045 M_{t-1} \). We start with taste parameters set equal to one. We simulated a time series for all the variables for 400 periods. After allowing for initial conditions, we use a time series of 248 observations. We take logs and first difference the price level and output, then take a first difference of the inflation rate. We do this simulation 1000 times.

For the sample of 1000 simulated model economies, we compute the sample means for the standard deviations and contemporaneous correlations. The results are reported in Table 5. The results are qualitatively similar to the actual values reported. We included the standard deviations to indicate that there is some variability in the inflation rate.\(^8\)

The results from the simulated economy highlight the role that “backward” looking price expectations play. In other words, the role that rational inattention could play in terms of accounting for the two observed correlations. Sims (2003) characterized this approach by specifying a signal extraction problem based on communication frictions. In this paper, we adopt the backward-looking price expectations, we are borrowing from the work by Brock and Hommes (1997). In that paper, the authors consider price expectations as a tradeoff between rational expectations, which are costly to form, and a simple rule that next period’s expected price is what the price was last period.\(^9\) What Brock and Hommes showed is that the marginal gain from rational expectations must be enough to offset the marginal cost of resources needed to form rational expectations. Otherwise, agents will opt for expectations that are costless to form. The low-cost expectations are consistent with a kind of rational inattention. Brock and

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\(^8\) One could imagine a case in the simulated economy in which prices and output are negatively correlated and the inflation rate is virtually constant. In such a case, the correlation between inflation and output would be zero. The results indicate there is enough variation in the inflation rate relative to the variation in the price level that correlation, or lack thereof, is caused by the absence of variation in the inflation rate.

\(^9\) Burdett and Judd (1983) and Head, Liu, Menzio and Wright (2012) specify models in which prices exhibit a stickiness owing to the search friction.
Hommes demonstrate how modifying a model along these lines can quantitatively affect the dynamics in a model economy.

So, in equation (13), the simulated time series can quantitatively generate price dynamics that are consistent with the phase shift that explains why the price level is counter cyclical and the inflation rate is acyclical. The backward-looking expectation mechanism induces a stickiness to the price level that moves it out of phase with respect to the movements in output. The stickiness owes to the weight given to last period’s price level in computing the current price level. In contrast, what we observe in the rational-expectations equilibrium is the absence of a phase shift; in other words, the price level adjust too quickly, resulting the price level and the inflation rate both being countercyclical in the model economy.

It is important to note that our reference to price stickiness differs in tow important aspects from what people typically mean by sticky prices. First, prices are sticky in this model economy relative to what they would be in the rational expectations, forward looking agent version of the economy. By setting future price expectations as equal to the last observed price level, the expectations process imparts a backward-looking component that results in the price level not adjusting quantitatively by the same amount compared with the rational expectations equilibrium. The price level, however, is fully flexible. There is no Calvo clock nor menu costs that are imparting a stickiness to the price level. Second, we analyze the equilibrium price level, ignoring commodity differentiation. In order to introduce sticky prices, most models specify economies with multiple goods with some of them subject to timing frictions or menu costs. Indeed, the search friction in Head, et al, examine a model economy in which some of the prices of individual commodities exhibit stickiness in equilibrium.

Put more simply, the model economy generates the date-t price level to clear the money market. Consequently, there are supply factors, like the money supply, and demand factors, like output, that enter into the equilibrium expression. As we developed in Propositions 1 and 2, the results depend on the part of the parameter space. Even with the backward looking expectations, the discount factor determines how much weight is given to expectations of future output growth and tastes shocks. Consider, for example, the case in which $\beta = 0.8$. For this model economy, the average correlation between the price level and output increases to 0.104. The average correlation between inflation and output is 0.007. So, why is the price level procyclical and the inflation rate acyclical? The intuition owes to the fact that the agent is discounts the future less. In this model economy, an increase in the discount factor effectively increases the demand for assets. Since money is the only asset, the demand for real balances is larger. Unexpected increases in output have a smaller impact on money demand. Since the money supply is increasing at a constant rate, the numerical results convey the weight given to the competing factors; we
are simultaneously considering a money supply channel through the growth rate process and a money demand channel through the output shocks. Basically, when money demand is smaller, a given increase in output has a proportionately larger impact on money demand and the price level declines. With a higher discount factor, the demand for money is larger and output shocks have a smaller effect on the demand for money, leaving the money supply growth channel as being the dominant force over time, driving the correlation between prices and output. However, the correlation between the current price level and output is different qualitatively compared with what we observe in the data.

We introduced parameters pertaining to two different preference shocks. It is the relative size of the two shocks that really matter. We assume the distribution for the two shocks is uniform over the unit interval. For purposes of the experiment, we consider $\xi_{ct} \sim U(0.5, 0.1 * \frac{1}{12})$ and $\xi_{mt} \sim U(0.5, 0.025 * \frac{1}{12})$. For these settings, we see the mean correlation between the price level and output is 0.005 and the means correlation between the inflation rate and output is 0.7e-04. In other words, both the price level and the inflation rate are acyclical. In this experiment, the consumption taste shock is more volatile than the taste shock for real balances. The results indicate that with tastes shocks, the standard deviation for the price level is 0.08, roughly ten times greater than the volatility observed in the data. The implication is that taste shocks create much larger swings in the price level. Further, we see that the increase in volatility swamps any correlation between output and the price level.

Overall, the numerical analysis tells us that it is possible to construct a model economy that can account for the countercyclical price level and the acyclical inflation rate. The results are obtained with the contribution of one key assumption; namely that the price level expectations are determined by consumers exhibiting rational inattention. In our particular version, the expected price level next period is set equal to last period’s price level. With these expectations, price stickiness is incorporated into the model economy. We show that there exists a set of parameter values such that the price stickiness is sufficient to generate countercyclical prices and acyclical inflation. The results are not terribly robust to changes in parameter settings, but constitute a valuable first step toward a more complete understanding of the relationship between the price level and output over the business cycle. We turn now to a discussion of forward looking rational expectations.

5.2 Forward Looking Rational Expectations

In this case Equation (14) becomes,

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10 Though not reported here, if the money shock is more volatile than the consumption shock, the results are essentially the same.
\[ \Lambda_t = \frac{\xi_m}{\xi_t} (aY_t / M_t) + \beta E_t \left\{ \left( \frac{\xi_{t+1}}{\xi_t} \right) (Y_t / Y_{t+1}) \Lambda_{t+1} \right\} \]  

(15)

We set the taste shocks all equal to one for each date, \( Y_t = Y_{t-1}e^{\rho_t}, M_t = M_{t-1}e^{m_t} \), treat \{aY_t / M_t\} as a dividend process, solve (15) recursively forward as in asset pricing theory and obtain,

\[ \Lambda_t = aY_t \left\{ 1 / M_t + \beta / M_{t+1} + \beta^2 / M_{t+2} + \ldots \right\} \]  

(16)

assuming the “no bubble” condition that the tail term goes to zero. If

\[ M_{s+1} = e^{m_{s-t}} M_s, \quad s = 1, 2, ... \]  

(17)

where the process, \( \{m_s\} \) is IID with finite mean and finite variance, we may write the solution (16) in the form,

\[ \Lambda_t = (aY_t / M_t) \{ 1 / (1 - \beta e^{-m_t}) \} \]  

(18)

Note that no restriction has been made on the real output process in getting this solution. Here, the cyclical component is treated as difference-stationary log levels. Since this analysis applies to the difference-stationary representation, we use the DS-cyclic. As such, we may now use (18) to compute the covariances of the DS-cyclic components of the price level and inflation with the DS-cyclic component of real output. We have,

\[ \text{cov}(p^C_t, y^C_t) = \text{cov}(m_t - g_t, g_t) \]  

(19)

\[ \text{cov}(p^C_t - p^C_{t-1}, y^C_t) = \text{cov}(m_t - g_t - (m_{t-1} - g_{t-1}), g_t) \]  

(20)

The simplest case that yields potentially interesting results on the two correlations above is the case of a constant money supply growth rule, \( m_t = \bar{m}, \quad t = 1, 2, \ldots \) and an AR(1) for the DS-cyclic component of real output,

\[ g_t = \mu_g + \rho g_{t-1} + n_{gt}, \quad t = 1, 2, \ldots \]  

(21)

where \( \{n_{gt}\} \) is IID with zero mean and finite variance. For this simplest case, we obtain a negative correlation for (19) and,
\[
\begin{align*}
\text{cov}(p^C_t - p^C_{t-1}, y^C_t) &= \text{cov}(m_t - g_t - (m_{t-1} - g_{t-1}), g_t) \\
\text{cov}(-\rho g_{t-1} + g_{t-1} - n_{gr}, \rho g_{t-1} + n_{gr}) &= (1 - \rho)\rho \text{cov}(g_{t-1}, g_{t-1}) - \text{cov}(n_{gr}, n_{gr}) = -\text{cov}(n_{gr}, n_{gr}) / (1 + \rho)
\end{align*}
\]

which is also negative. So, this analytic example still does not get us the pattern of signs of covariances of cyclic components that we seek. It can be shown that even if \(\text{corr}(m_t, n_{gr}) = 1\), because

\[
\text{cov}(m_t, n_{gr}) = \text{corr}(m_t, n_{gr})[\text{var}(m_t) \text{var}(n_{gr})]^{1/2}
\]

the variance of the growth rate of money has to be quite large relative to the variance of the DS-cyclic component of real output, i.e. the condition

\[
\text{var}(m_t) > \text{var}(n_{gr}) / (1 - \rho^2)^2
\]

is needed.

Woodford (2003, Chapter 2, pages 102 and 103) discusses the role of the taste shocks in the utility function in the MIUF model. Hence if the Central Bank is trying to implement a specific policy, e.g. inflation targeting or price level targeting, then the real disturbances modeled by taste shocks may play an important role. Hence we bring the taste shocks back into the picture to locate potentially useful sufficient conditions for the correlation between the DS-component of inflation (price level) with the DS-component of real output to be weakly positive (negative). If we assume,

\[
\xi_{m,t+1} = \xi_{m,t} e^{\rho_{m,t+1}} , \ t = 1, 2, ...
\]

where \(\{\varphi_{m,t}\}\) is IID with finite mean and variance, we may use the same type of solution procedure to obtain the solution,

\[
1 / P_t = \Lambda_t = \{a_{m,t} / [u(Y_t, \xi_{m,t})M_t]\} [1 / (1 - \beta\bar{e}^{\bar{\phi} - \bar{h}})]
\]

Note that (26) permits just about any specification of the utility function \(u(Y_t, \xi_{m,t})\) as well as just about any specification of the \(\{\xi_{m,t}, \{Y_t\}\) processes and (26) still holds. This opens up several channels of influence that might lead to a positive correlation of the DS-cyclic component of inflation with the DS-cyclic component of real output, which seems to be the most difficult correlation to be positive once one restricts influences so that the correlation of the DS-cyclic component of the price level with the DS-component of real output to be negative. The main observation here seems to be that one needs some
type of persistence in the \( \{ \xi_t \} \), \( \{ Y_t \} \) processes to obtain a positive correlation of the DS-cyclic component of inflation with real output, when the price level correlation is negative. We start exploration by looking at the log utility, multiplicative influence of taste shock case,

\[
u(Y_t, \xi_t) = \xi_t \ln(Y_t)
\] (27)

For the log utility case, we have, from (26)

\[
P_t = (1 - \beta E e^{\hat{\varrho}-\hat{\varphi}}) M_t \xi_t / \{ a \xi_m / Y_t \}
\] (28)

\[
\text{cov}(p^C_t, y^C_t) = \text{cov}(\ln \xi_t - \ln \xi_{t-1} + m_t - \varphi_t - g_t, g_t)
\]

\[
\text{cov}(p^C_t - p^C_{t-1}, y^C_t) = \text{cov}(\ln \xi_t - \ln \xi_{t-1} + m_t - m_{t-1} - (\varphi_t - \varphi_{t-1}) - (g_t - g_{t-1}), g_t)
\] (29)

We may now use (29) to take logs and compute the covariance of the DS-cyclic component of the price level and inflation with the DS-cyclic component of real output. These covariances are given by,

\[
\text{cov}(p^C_t, y^C_t) = \text{cov}(\ln \xi_t - \ln \xi_{t-1} + m_t - \varphi_t - g_t, g_t)
\]

\[
\text{cov}(p^C_t - p^C_{t-1}, y^C_t) = \text{cov}(p^C_t, y^C_t) - \text{cov}(p^C_{t-1}, y^C_t)
\]

\[
= \text{cov}(p^C_t, g_t) - \text{cov}(p^C_{t-1}, g_t) = \text{cov}(p^C_t, g_t) - \text{cov}(p^C_{t-1}, \rho g_{t-1} + n_{gt})
\] (30)

\[
= \text{cov}(p^C_t, g_t) - \rho \text{cov}(p^C_{t-1}, g_{t-1})
\]

\[
= (1 - \rho) \text{cov}(p^C_t, g_t).
\]

Here, we use the IID property of \( \{ n_{gt} \} \) which implies, \( \text{cov}(p^C_{t-1}, n_{gt}) = 0 \). The last line follows by stationarity.

What do we learn from (30) and the AR(1) specification of the DS-cyclic component of real output, equation (21)? If the processes in the expression for \( \text{cov}(p^C_t, y^C_t) \) are such that this covariance is negative then the covariance, \( \text{cov}(p^C_t - p^C_{t-1}, y^C_t) \), is still negative, but is smaller when \( 0 < \rho < 1 \). We also expect it to be close to zero when \( \rho \) is close to one. This finding is fairly close to what the data show for DS-components in the sense that the correlation of the DS-component of inflation with the DS-component of real output is very weakly positive while, at the same time the correlation of the DS-component of the price level with the DS-component of real output is substantially negative.
VI. Summary, Conclusions, and Suggestions for Future Research.

In this paper, we focus on the relationship between the price level and output at business cycle frequencies. In particular, we are interested in characterizing two contemporaneous correlations: one is the relationship between the price level and output and the other is the relationship between the inflation rate and output. Because the inflation rate is the rate of change in the price level, the two facts convey some underlying feature of the economy. In our case, the evidence consistently points to the price level being countercyclical while the inflation rate is either procyclical or acyclical, depending on detrending method employed.

One thing we want to do is to develop a methodology to assess how much confidence we should put in a “stylized fact” in a data disciplined way. In particular, our methodology is proposed as a data disciplined way to assess the amount of uncertainty surrounding a “stylized fact” that takes into account, not only estimation uncertainty but also model uncertainty. We illustrated our methodology with assessing the uncertainty surrounding the stylized fact that the price level is countercyclical and the rate of inflation is procyclical using U.S. data under two methods of detrending, e.g. difference stationary and trend stationary (Nelson and Plosser (1982). We found a rather high level of uncertainty in our illustrative example. This should give pause to any theorist who wants to take a strong stand on which model or theory to work with based on this particular “stylized fact.”

We see a promising and useful strand of future research that extends our methodology to combined evidence from more than one country’s data. For example, the recent paper by Konstantakopoulou et al. (2009), reports results on the correlation of detrended real output and detrended price level and the correlation of real output and rate of inflation for many countries and several methods of detrending. The countercyclicality of the price level and the procyclicality of the rate of inflation appears to be quite robust. However, one must be wary of thinking that that each country’s evidence represents independent support of this particular “stylized fact”. This is so because the business cycles of these 9 OECD countries are quite likely to be cross correlated and this cross correlation must be taken into account when applying the bootstrap (Efron (1982), Efron and Tibshirani (1986)) to compute the probability of the cyclicality pattern of price level and inflation under scrutiny here.

The other contribution of this paper is to specify a model economy in which rational inattention is present. Our goal is to quantitatively assess whether such a model can account for the observed pair of correlations. The idea is pretty straightforward. The quantitative results indicate how important price flexibility is to accounting for the negative correlation between the price level and output. In many model economies, consumers are forward-looking, and when combined with persistent output shocks and central
bank operating rules, the quantitative analysis indicate that both the price level and the inflation rate are countercyclical. Following the price expectations approach associated with Sims (2003) and formulated in work by Brock and Hommes (1997), we propose a model economy that can account for the pair of correlations observed; namely, that in a difference-=stationary setting, the price level is countercyclical and the inflation rate is acyclicld. The key is that price expectations are not rational, thereby imparting a price stickiness. Our numerical results demonstrate that by putting enough weight on backward-looking price expectations, we can induce the phase shift in the price level that is consistent with the observed pattern of correlations.

Other directions of future research that we see as potentially fruitful are (i) developing extensions of the simple methods of our paper towards something like a “sturdy” reporting of stylized facts rather in the style of the “sturdy econometrics” advocated by Leamer (1994) and extension and adaptations of the simple methods illustrated in our paper towards Bayesian Model Uncertainty methods of reporting better measures of uncertainty when reporting a “stylized fact” than the more usual reporting of parameter estimates with their corresponding standard errors.
Appendix

Proof Proposition 2:

Assume that the expected value of the product of the price level and output is constant over time. We derive the following condition: \( E[p_t y_t] = [b_1 p_{t-1} + e_t][a_1 y_{t-1} + u_t] \). After collecting terms and simplifying, we can write this as \([py] = \frac{E[eu]}{1-\alpha_1\beta_1}\). With \(E[eu] < 0\), it follows that \(E[py] < 0\) iff \(\alpha_1\beta_1 < 1\).

Similarly, \(E[\pi y] = ([b_1 p_{t-1} + e_t - p_{t-1}][a_1 y_{t-1} + u_t])\). After collecting terms and simplifying, we get \([\pi y] = \frac{E[eu](1-\alpha_1)}{1-\alpha_1\beta_1}\). With \(E[eu] < 0\), and with \(\alpha_1\beta_1 < 1\), it follows that \(\alpha_1 > 1\) for \(E[\pi y] > 0\). ■

References


Leamer, Edward, 1994, Sturdy Econometrics, Edward Elgar, Cheltenham, UK.


### Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Cross-correlation with y</th>
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<tr>
<td>$y$</td>
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<td>0.017</td>
<td>1.0</td>
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<tr>
<td>$p$</td>
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<td>0.01</td>
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<tr>
<td>$\pi$</td>
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<td>0.004</td>
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### Table 2

<table>
<thead>
<tr>
<th>Estimated Parameter</th>
<th>Standard Error</th>
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<tr>
<td>$\omega_t$</td>
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</tr>
<tr>
<td>$\nu_1_t$</td>
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</tr>
<tr>
<td>$\nu_2_t$</td>
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<td>$\hat{u}_t$</td>
<td>0.0042</td>
</tr>
<tr>
<td>$\hat{\epsilon}_t$</td>
<td>0.0084</td>
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Table 3

Panel A: output is AR(1) and price level is AR(1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient Estimate</th>
<th># of cases in $\rho(p, y) &lt; 0$ and $\rho(\pi, y) &gt; 0$</th>
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</thead>
<tbody>
<tr>
<td>output</td>
<td>$y_t^c = 0.0 + 0.85 y_{t-1}^c$ (0.035)</td>
<td>2345</td>
</tr>
<tr>
<td>price level</td>
<td>$p_t^c = 0.0 + 0.91 p_{t-1}^c$ (0.026)</td>
<td>std errors in parentheses</td>
</tr>
<tr>
<td>error terms</td>
<td>$e_t = 0.0 - 0.067u_t$ (0.065)</td>
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</tr>
</tbody>
</table>

Panel B: output is AR(2) and price level is AR(2)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient Estimate</th>
<th># of cases in $\rho(p, y) &lt; 0$ and $\rho(\pi, y) &gt; 0$</th>
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</thead>
<tbody>
<tr>
<td>output</td>
<td>$y_t^c = 0.0 + 1.16 y_{t-1}^c - 0.37 y_{t-2}^c$ (0.06) (0.06)</td>
<td>2241</td>
</tr>
<tr>
<td>price level</td>
<td>$p_t^c = 0.0 + 1.44 p_{t-1}^c - 0.58 p_{t-2}^c$ (0.05) (0.05)</td>
<td>std errors in parentheses</td>
</tr>
<tr>
<td>error term</td>
<td>$e_t = 0.0 - 0.23u_t$ (0.16)</td>
<td></td>
</tr>
</tbody>
</table>
Panel C: output is AR(3) and price level is AR(3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient Estimate</th>
<th># of cases in $\rho(p, y) &lt; 0$ and $\rho(\pi, y) &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>$y_t^C = 0.0 + 1.09y_{t-1}^C - 0.12y_{t-2}^C - 0.22y_{t-3}^C$</td>
<td>2338</td>
</tr>
<tr>
<td></td>
<td>(0.06) (0.09) (0.06)</td>
<td></td>
</tr>
<tr>
<td>price level</td>
<td>$p_t^C = 0.0 + 1.29p_{t-1}^C - 0.22 p_{t-2}^C - 0.22 p_{t-3}^C$</td>
<td>std errors in parentheses</td>
</tr>
<tr>
<td></td>
<td>(0.06) (0.10) (0.06)</td>
<td></td>
</tr>
<tr>
<td>error term</td>
<td>$e_t = 0.0 - 0.3u_t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4

Results from estimating various AR process:

The First-difference Case

Panel A: output is AR(1) and price level is AR(1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient Estimate</th>
<th># of cases in 0.0 &gt; ( \rho(p, y) &gt; -0.2 ) and 0.1 &gt; ( \rho(\pi, y) &gt; -0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>( y_t^C = 0.005 + 0.335 y_{t-1}^C )</td>
<td>4248</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td></td>
</tr>
<tr>
<td>price level</td>
<td>( p_t^C = 0.002 + 0.80 p_{t-1}^C )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td></td>
</tr>
<tr>
<td>error terms</td>
<td>( e_t = 0.0 - 0.155 u_t )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.151)</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: output is AR(2) and price level is AR(2)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient Estimate</th>
<th># of cases in 0.0 &gt; ( \rho(p, y) &gt; -0.2 ) and 0.1 &gt; ( \rho(\pi, y) &gt; -0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>( y_t^C = 0.005 + 0.306 y_{t-1}^C + 0.087 y_{t-2}^C )</td>
<td>4280</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td></td>
</tr>
<tr>
<td>price level</td>
<td>( p_t^C = 0.001 + 0.644 p_{t-1}^C + 0.194 p_{t-2}^C )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td></td>
</tr>
<tr>
<td>error terms</td>
<td>( e_t = 0.0 - 0.119 u_t )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.154)</td>
<td></td>
</tr>
</tbody>
</table>
Panel C: output is AR(3) and price level is AR(3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient Estimate</th>
<th># of cases in 0.0 &gt; $\rho(p, y) &gt; -0.2$ and 0.1 &gt; $\rho(\pi, y) &gt; -0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>$y_t^c = 0.005 + 0.32y_{t-1}^c + 0.12y_{t-2}^c - 0.12y_{t-3}^c$</td>
<td>4337</td>
</tr>
<tr>
<td></td>
<td>(0.064) (0.067) (0.063)</td>
<td></td>
</tr>
<tr>
<td>price level</td>
<td>$p_t^c = 0.001 + 0.62p_{t-1}^c + 0.12p_{t-2}^c - 0.12p_{t-3}^c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.065) (0.067) (0.064)</td>
<td></td>
</tr>
<tr>
<td>error term</td>
<td>$e_t = 0.0 - 0.14u_t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td></td>
</tr>
</tbody>
</table>
Table 5

Simulated and actual values for difference-stationary economies (Baseline parameter settings)

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price level</td>
<td>0.0067</td>
<td>0.0085</td>
</tr>
<tr>
<td>Output</td>
<td>0.0098</td>
<td>0.0194</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.0043</td>
<td>0.0108</td>
</tr>
<tr>
<td><strong>Cross-correlation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(p,y)$</td>
<td>-0.13</td>
<td>-0.145</td>
</tr>
<tr>
<td>$\rho(\pi,y)$</td>
<td>0.03</td>
<td>-0.013</td>
</tr>
</tbody>
</table>
Figure 1

Frequency distribution of

\[ \rho (p^c, y^c) \text{ and } \rho (\pi^c, y^c) \]
Figure 2

Frequency distribution for

\[ \rho(p^c, y^c) \text{ and } \rho(\pi^c, y^c) \] with negatively correlated residuals
Figure 3

Frequency Distribution for 1st Difference Version

With p as AR(1) and y as AR(2)