3-Dimensional Coordinate Systems

3-dim. rectangular system:

Space \( \rightarrow \mathbb{R}^3 \)
point \( P \) \( \leftrightarrow \) \((a,b,c)\)

\( P = (a,b,c) \)
\( O = (0,0,0) \)

Distance formula:

\( P_1 = (x_1,y_1,z_1) \)
\( P_2 = (x_2,y_2,z_2) \)

\[ |P_1P_2| = ? \]

\[ |OH| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \]

\[ |P_1P_2|^2 = |OH|^2 + |HP_2|^2 \]

\[ = (x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2 \]

Ex. 1. Find the distance between the points \((4,1,5)\) & \((1,3,6)\).

Sol.
Distance \(= \sqrt{(4-1)^2 + (3-1)^2 + (6-5)^2} \)

\(= \sqrt{9 + 4 + 1} \)

\(= \sqrt{14} \)

\(\mathbb{R}^3 = \{(a,b,c) \mid a,b,c \text{ real numbers}\} \)
Surfaces:

The graph in \( \mathbb{R}^3 \) of an equation involving \( x, y, \) and \( z \) is called a surface.

Ex. 2: Describe the following surfaces.

(a) \( x = 0 \)
(b) \( y = 0 \)
(c) \( z = 0 \)

Sol. (a) The equation \( x = 0 \) represents all points in the space that are of the form \((0, y, z)\). This is the \( yz\)-plane.

(b) \( y = 0 \) --- the \( xz\)-plane

(c) \( z = 0 \) --- the \( xy\)-plane

Ex. 3: Describe the surface represented by \( y = 2 \).

Sol. The equation represents the set of all pts in \( \mathbb{R}^3 \) whose \( y \)-coord. is \( 2 \). This is the vertical plane that is parallel to the \( xz \)-plane and 2 units to the right of it.

Ex. 5

Def. The sphere centered at the point \( C : (a, b, c) \) and of radius \( r (r > 0) \) is the set of all pts in the space whose distance from \( C \) is \( r \).

Find the equation of the sphere.

Sol. \( P = (x, y, z) \) is on the sphere

\[ \Rightarrow \quad |CP| = r \]

\[ \Rightarrow \quad \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r \]

\[ \Rightarrow \quad (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2. \]

Ex. 6: Describe the surface \( S \) represented by \( x = y \).

Sol. \( S = \{(x, x, z) \mid x, z, \text{ real numbers}\} \)