and such that
\[
\frac{1}{c(s,n)} \leq \frac{G_s(x)}{H_s(x)} \leq c(s,n)
\]
when \(|x| \leq 2\), \hspace{1cm} (1.2.12)
where \(H_s\) is equal to
\[
H_s(x) = \begin{cases} 
|x|^{s-n} + 1 + O(|x|^{s-n+2}) & \text{for } 0 < s < n, \\
\log \frac{2}{|x|} + 1 + O(|x|^2) & \text{for } s = n, \\
1 + O(|x|^{s-n}) & \text{for } s > n,
\end{cases}
\]
and \(O(t)\) is a function with the property \(|O(t)| \leq |t|\) for \(t \geq 0\).

Now let \(z\) be a complex number with Re \(z > 0\). Then there exist finite positive constants \(C'(\text{Re}z,n)\) and \(c'(\text{Re}z,n)\) such that when \(|x| \geq 2\), we have
\[
|G_z(x)| \leq C'(\text{Re}z,n) \left| \Gamma \left( \frac{1}{2} \right) \right| e^{-\frac{|x|}{2}}
\]
and when \(|x| \leq 2\), we have
\[
|G_z(x)| \leq c'(\text{Re}z,n) \left| \frac{\Gamma \left( \frac{1}{2} \right)}{\Gamma \left( \frac{z}{2} \right)} \right| \begin{cases} 
|x|^{\text{Re}z-n} & \text{for } \text{Re}z < n, \\
\log \frac{2}{|x|} + 1 & \text{for } \text{Re}z = n, \\
1 & \text{for } \text{Re}z > n.
\end{cases}
\]

**Proof.** For \(A > 0\) and \(z\) with Re \(z > 0\) we have the gamma function identity
\[
A^{-\frac{1}{2}} = \frac{1}{\Gamma \left( \frac{1}{2} \right)} \int_0^\infty e^{-tA^{\frac{1}{2}}} \frac{dt}{t},
\]
which we use to obtain
\[
(1 + 4\pi^2 |\xi|^2)^{-\frac{1}{2}} = \frac{1}{\Gamma \left( \frac{1}{2} \right)} \int_0^\infty e^{-t} e^{-\pi^2 |2\sqrt{\pi} \xi|^2} \frac{dt}{t}.
\]
Note that the preceding integral converges at both ends. Now take the inverse Fourier transform in \(\xi\) and use the fact that the function \(e^{-\pi |\xi|^2}\) is equal to its Fourier transform (Example 2.2.9 in [156]) to obtain
\[
G_z(x) = \frac{(2\sqrt{\pi})^{-n}}{\Gamma \left( \frac{1}{2} \right)} \int_0^\infty e^{-t} e^{-\pi^2 |t| \frac{x^2}{2}} \frac{dt}{t}.
\]
This identity shows that \(G_z\) is smooth on \(\mathbb{R}^n \setminus \{0\}\). Moreover, taking \(z = s > 0\) proves that \(G_s(x) > 0\) for all \(x \in \mathbb{R}^n\). Consequently, \(\|G_s\|_{L^1} = \int_{\mathbb{R}^n} G_s(x) dx = \hat{G}_s(0) = 1\).