Assignment 9, Math 8425, Complex analysis I
Spring 2010, Dr. Tanya Christiansen

Turn in on Fri. April 30

Comment: For any problems assigned from Conway that assume a curve is rectifiable, you may substitute the assumption that the curve is piecewise $C^1$.

Please read through this assignment to make sure you understand which problems you need to turn in. If in doubt, ask!

Part 0. Make sure you can do these problems. You do not need to turn these in. HOWEVER, it is important that you can do them-- if you have any doubts at all, please see me or ask another student.

p. 185 # 4

This is a follow up to B below. Read this as part ii of B.

ii) Show that the result of i) may not be true, if the condition $\sum |a_j| < \infty$ is replaced by the weaker condition $\sum a_j$ converges.

Part 1. Selected problems from here will be graded.

From Conway:
pg. 185, # 3, 11(a-d) (for this it is possible you may need to use some facts about the gamma function proved in the book but not in class)

A) (adapted from Greene and Krantz): Prove that

$$\cot \pi z = \frac{1}{\pi} \left[ \frac{1}{z} + \sum_{j=1}^{\infty} \frac{2z}{z^2 - j^2} \right]$$

by following the outline below:

*The righthand side defines a meromorphic function on $\mathbb{C}$, with poles at the integers $n$, and the functions on the left and right hand sides have the same singular parts at the integers $n$.

*The difference between the right and left hand sides is an entire function $h(z)$, and $|h(z)| \leq C(1 + |z|^{1/2})$. Now use a slight strengthening of Liouville’s theorem to conclude $h(z)$ is a constant.

B) (adapted from Greene and Krantz): i) Show that if $\sum_j |a_j| < \infty$ and if $\sigma$ is any permutation of the positive integers, then

$$\prod_{j=1}^{\infty} (1 + a_j) = \prod_{j=1}^{\infty} (1 + a_{\sigma(j)})$$