Turn in on Wed. Feb. 24.

Please read through this assignment to make sure you understand which problems you need to turn in. If in doubt, ask!

**Part 0.**

Do not turn in:
p. 83, # 3

Do not turn in, but do think about: how many different (but equivalent) ways can you think of (that we’ve seen) to define a (complex) analytic function on an open set $G$? I’ll get you started: $f : G \to \mathbb{C}$ is analytic if for every $z \in G$, $\lim_{h \to 0} \frac{f(z+h)-f(z)}{h} = f'(z)$ exists and $f'(z)$ is continuous. (As I’m typing this, I can think of two other ways we’ve already seen, and a fourth we should see before you turn in this assignment. Maybe you can think of one I’ve missed.)

**Part 1.** Selected problems from here will be graded.

p. 83 # 4
p. 87 # 7, 8
p. 96 # 5, 11
p. 100, # 4

A) (adapted from Marsden) i) Let $f : \mathbb{C} \to \mathbb{C}$ be continuous, and analytic on $\mathbb{C} \setminus \mathbb{R}$. Is $f$ entire? (prove or give counterexample)
ii) Let $f : \mathbb{C} \to \mathbb{C}$ be analytic on $\mathbb{C} \setminus \mathbb{R}$. Is $f$ entire? (prove or give counterexample)

B) (adapted from Marsden) Let $A$ be a region, and $u : A \to \mathbb{R}$ be harmonic and nonconstant. Show that $u$ is an open mapping.

************************************************************************** Math culture ***************

A function $f : (\alpha, \beta) \to \mathbb{R}$ is **real analytic** if for every $a \in (\alpha, \beta)$ there is an $\epsilon > 0$ so that for some $\{a_n\}$, $f(x) = \sum_0^\infty a_n (x-a)^n$ for all $x \in (a-\epsilon, a+\epsilon)$ (and the series converges for these values of $x$). A real analytic function is $C^\infty$. It is NOT true that a $C^\infty$ function must be real analytic. Here’s a classic, and important, example:

$$f(x) = \begin{cases} 0 & x \leq 0 \\ e^{-1/x} & x > 0 \end{cases}$$

(The problem is at 0.)