Math 426 Homework 3

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1. Exercise 1.34 p. 26 of the book. Consider a Newtonian particle of mass \( m \) moving under the influence of the potential \( U \). If the position coordinate is denoted by

\[
q = (q_1, \ldots, q_n),
\]

then the equation of motion \((F = ma)\) is given by

\[
m\ddot{q} = -\nabla U(q).
\]

If \( q_0 \) is a strict local minimum of the potential, show that the equilibrium \((\dot{q}, q) = (0, q_0)\) is Lyapunov stable. Hint: Consider the total energy of the particle.

2. Determine the stability of the rest points of the following systems. Formulate properties of the unspecified scalar function \( g \) so that the system has a rest point at the origin which is respectively stable, asymptotically stable, and unstable.

1. \[
\begin{align*}
\dot{x} &= 2xy - x^3, \\
\dot{y} &= -x^2 - y^5.
\end{align*}
\]

2. \[
\begin{align*}
\dot{x} &= y + xy^2 - x^3 + 2xz^4, \\
\dot{y} &= -x - y^3 - 3x^2y + 3yz^4, \\
\dot{z} &= -\frac{5}{2}y^2z^3 - 2x^2z^3 - \frac{1}{2}z^7.
\end{align*}
\]

3. Exercise 1.13 p. 13. Determine the flow of the first order system

\[
\begin{align*}
\dot{x} &= y^2 - x^2, \\
\dot{y} &= -2xy.
\end{align*}
\]
Show that (almost) every orbit lies on a circle. Note that the flow gives rational parameterizations for the circular orbits. Hint: Define $z := x + iy$. 