Errata

Ordinary Differential Equations with Applications

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- Page 3: In Theorem 1.4 the inequalities $\infty \leq \alpha < \beta < \infty$ should be $-\infty \leq \alpha < \beta < \infty$.
- Page 3: In the italics in the first full paragraph please read: "...can be extended in time until it either reaches the boundary of the domain of definition ...".
- Page 3: "extensibility" is not a word! It should be changed to "Extension".
- Page 3: In Theorem 1.4 remove the words "a point on".
- Page 10, Exercise 1.9: Some readers may worry that $1/F$ might not have an antiderivative? Then, T can be infinity. In the book all functions are smooth! At any rate, add the hypothesis that F is a smooth function.
- Page 12: The paragraph starting with the words "No matter..." should be part of the previous paragraph.
- Page 25: All occurrences of the symbol ϕ should be replaced by φ.
- Page 28: The statements in 3rd paragraph of Section 1.7 are correct. But, there might be a confusion. For example, an arc may be embedded or immersed in some space with dimension larger than one. Thus, although an arc is a manifold, it may not be a submanifold of its ambient space. For example, an arc may accumulate on itself.
- Page 33: Last paragraph line 3: remove the words "the dot product of"
- Page 38: In the line above the first display the inequality $(x-1)^2+y^2 < \sqrt{2}$ should be $(x-1)^2 + y^2 < 2$.
- Page 42: Second line: $Af_i = e_i$ should be $A\{f_1, \ldots, f_n\} \mapsto \{e_1, \ldots, e_n\}$, or the "that is" phrase should just be left out.
- Page 43: Statement of implicit function theorem: $F_p(a,b) \neq 0$ must of course be $\det F_p(a,b) \neq 0$ or "the linear transformation $F_p(a,b) : \mathbb{R}^m \to \mathbb{R}^m$ is invertible".
- Page 48: The third display has unbalanced parentheses. It should read as follows:

$$F_*(p,v) = \big(F(p), DG_2(G_2^{-1}(F(p)))\zeta\big).$$

- Page 52, Ex. 1.74 Add the hypothesis $g(0) = h(0)$.

- Page 57: In the definition of polar coordinates the formula $V \subset \mathbb{R}^2 \setminus \{0, 0\}$ should be $V \subset \mathbb{R}^2 \setminus \{(0, 0)\}$.
- Page 58, third paragraph: The change of variables formula should read: $\dot{y} = Dg(g^{-1}(y))f(g^{-1}(y))$.
- Page 73: Second full paragraph. "...if $P^k(p) = p$ and $DP^k(p)$ has all its eigenvalues..."
- Page 74: Second display should be

$$\tau(0, \xi) = 0, \quad y(0, \xi) = \xi$$

- Page 74: After 3rd display. $t \mapsto (\tau(t, \xi), y(t, \xi))$
- Page 80: Proof of 1.110. $T > 0$ should be $T \in \mathbb{R}$.
- Page 85: The last sentence of the second paragraph should read: (See [109] for the existence of limit cycles for the case $|\mu| \leq 1$.)
- Page 93: The last line of Definition 1.150 should read "then the function $\ldots$ "
- Page 102: Last display:

$$\int_a^b (cf + dg)(t)\,dt = \lim_{n \to \infty} \int_a^b (cf_n + dg_n)(t)\,dt.$$

- Page 103: Last display:

$$\| \sum \mu(I_i) v_i \| \leq \sum \mu(I_i) \sup |v_i| \leq (b - a) \sup |v_i|.$$

- Page 107: Last factor in the third display: $d(x, x_0)$ should be $d(T(x), x)$.
- Page 108: Second sentence of proof: "for all $y \in V$."
- Page 109: Second display: the last equality should be $\leq$.
- Page 111: The definition of fiber contraction should read: $y \mapsto \Psi(x, y)$ is a contraction with contraction constant μ for every $x \in X$. In the displays on this page some indices n should be changed to $n + 1$. For example, the first display should read

$$\Gamma^{n+1}(x, y) = (\Lambda^{n+1}(x), \Psi_{\Lambda^n(x)} \circ \Psi_{\Lambda^{n-1}(x)} \circ \cdots \circ \Psi_x(y)),$$

etc.

- Page 112, Middle display: The statement should be $0 \leq a_k < \frac{1}{2}(1 - \mu)\epsilon$. Also, in the next two displays $n - K - 1$ should be $n - K + 1$.

- Page 113, third paragraph: "The first step of the method is to show that if $\alpha \in \mathcal{C}^1$, then the derivative of $\Lambda\alpha$ has the form

$$(D(\Lambda\alpha))(\xi) = \Psi(\alpha, D\alpha)(\xi)$$

  where...".

- Page 114: Theorem 1.178 is meant to be a transparent warmup for applications of fiber contraction, but perhaps it is too simple—as soon as we show that F has a fixed point, we can take f to be the function with this point as its constant value. A better exercise is to solve the functional equation $F \circ f - f = G$ using the same proof. Also, there is a gap in the proof of Theorem 1.178: no argument is given to show that $\phi \mapsto \Psi(\phi, \Phi)$ is a continuous function. This gap seems to appear often in the applications of the fiber contraction principle, a fact pointed out to me by M. ElBialy. A proof (in the context of Theorem 1.178) to show that Ψ is continuous in its first variable at a given ϕ is easily constructed using the uniform continuity of DF on a ball of radius $\|\phi\| + 1$ centered at the origin.

- Page 115, last display:

$$D\phi_{n+1} = D(\Lambda\phi_n) = DF(\phi_n)D\phi_n = \Psi(\phi_n, D\phi_n) = \Psi(\phi_n, \Phi_n) = \Phi_{n+1};$$

- Page 116, statement of the implicit function theorem: The second to the last sentence should be changed to ... function $\beta : V_0 \to U_0$ such that $\beta(y_0) = x_0$ and $F(\beta(y), y) \equiv 0$.

- Page 121. The space X is not a Banach space. It is a complete metric space.

- Page 136 Prop. 2.17 The nonzero function...
  Also, line -5 $v = u + iw$ not $v = u + iv$.

- Page 137 Second display comma after the ... Also, 5 lines down from this display, Λ should be the imaginary part of the principle fundamental matrix at $t = 0$.

- Page 145. The proof of Theorem 2.24 is not correct! In particular, equation 2.12 is not true in general. This error will be repaired in future editions of the book.

- Page 172 4th line of first proof. Clearly, it follows that $e^{tB}v = e^{\mu t}v$ ...

- Page 181 Case 3. $-1 < \phi < 1$

- Page 189 second display, $\|SQAv\|^2$

- Page 193 second display, missing parenthesis,

$$\text{dist}(u(t, u(s, \mathcal{P}^n(\xi))), \Gamma)$$

- Page 230 2nd equation of the 5th display: $\frac{\partial \mathcal{H}}{\partial \mathcal{Q}} = -\frac{\partial \mathcal{L}}{\partial \mathcal{Q}}$.
- Page 241 The scaled equations in this section are not dimensionless. The correct scaling is given by $t = s/\Omega$. With this scaling, the abstract linearized equation for the inverted pendulum remains the same, the derivation is more natural, and the physics is correct!
- Page 244 Second display: $\Phi_\alpha(0) = 0$. Beginning last paragraph: the curve Γ should be defined to be $\Gamma := \{(\alpha, \beta) : \alpha = \gamma(\beta)\}$.
- Page 274 Third display: The change of variables should be $\tau = at$, $\xi = \sqrt{a}/(kx)$.
- Page 284 after the first display: ...negative, positive, and zero real parts, respectively. Also, in the second display η must be replaced by ζ.
- Page 287 third display: $\|f_n\|$ should be $\|f_n\|_{\mathcal{E}}$.
- Page 288, 5th display and the following sentence: The expression $\mathcal{E}$ must be $\mathcal{E}^0$. Also, in the first sentence of the last paragraph $\mathbb{R}^M$ must be $\mathbb{R}^k$.
- Page 293-294, formulas 4.12-4.13 $W(t, \xi, \beta)$ not $W(x(t, \xi, \beta))$. Also, in 4.13 add term e^{tS} before the integral.
- Page 297, line -8 of the last full paragraph, and page 490, line -1 : replace the phrase "cross product" with the word "product".
- Page 299 first display $h(z, y, z)$ must be $h(x, y, z)$.
- Page 311: In the proof of the Hartman–Grobman Theorem the statement that H is a "homeomorphism onto its image is correct". However, to prove that the image is open requires something like Brouwer's theorem on invariance of domain. This fact is not stated explicitly!
- Page 318, first line below the third display: Thus, the external forcing and the nonlinear damping cause energy fluctuations. Energy due to the damping leaves the system while $|x| > 1$ and is absorbed while $|x| < 1$.
- Page 333, In the display in the statement of the preparation theorem, the first chain of equalities should be set equal to zero.
- Page 338, formula for $\mathcal{M}(\xi)$, the differential ds is missing in the integral in the exponent.
- Page 352, line 10 of the first full paragraph: "the resultant of polynomial" should be "the resultant of the polynomial"
- Page 424, the formula for the Laplacian in the third display should read $\Delta f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$.
- Page 450: "...where J_ν is the Bessel..."

- Page 485, line 1 of Proposition 8.2: the phrase "suppose n = 1" is redundant.
- Page 495 third display: $f_{eu}(u,e) \neq 0$ should be $f_{uu}(u,e) \neq 0$
- Page 509 after display 8.21 the second equation in the text should read $\dot{\theta}(t,0,0) = \beta(0) \neq 0$
- Page 511 fifth line up from the bottom: "Theorem 8.2" should be "Proposition 8.2" and the same change on page 512 line 6. Also, in the third display, the first equation should be $\dfrac{dr_\xi}{d\theta} = S_r(0,\theta,\lambda)r_\xi$
- Page 516 Exercise 8.29: for $(x,y) \in U \setminus (0,0)$.
- Page 528 exercise 8.42: This exercise must be rewritten as follows: Prove the following statements. The set $B := \{x^{n-i}y^i \mid i = 0,\ldots,n\}$ is a basis for $\mathcal{H}_n$ and $\mathcal{L}_R$ has the following $(n+1)\times(n+1)$ matrix representation with respect to the given (ordered) basis:

$$\mathcal{L}_R = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ -n & 0 & 2 & 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 1-n & 0 & 3 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 2-n & 0 & 4 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}.$$

The kernel of $\mathcal{L}_R$ on $\mathcal{H}_{2k}$, for $k \geq 2$, is generated by the vector

$$K = (B_{k,0}, 0, B_{k,1}, 0, B_{k,2}, \ldots, 0, B_{k,k})$$

where the numbers

$$B_{k,j} = \frac{k!}{j!(k-j)!}$$

are the binomial coefficients, and

$$\{(a_1,\ldots,a_{2k},0) : (a_1,\ldots,a_{2k}) \in \mathbb{R}^{2k}\}$$

is a vector space complement of the kernel. The operator $\mathcal{L}_R$ on $\mathcal{H}_{2k}$ is represented by the matrix $(\ell_1,\ldots,\ell_{2k+1})$ partitioned by the indicated columns. The matrix representation for $\mathcal{L}_R$, restricted to this complement of the kernel, is given by $(\ell_1,\ldots,\ell_{2k},0)$. Consider $V, H \in \mathcal{H}_{2k}$ and the associated matrix equation

$$(\ell_1,\ldots,\ell_n,K)V = H,$$

where the matrix is partitioned by columns and H is represented in the basis B. The matrix is invertible. If the solution V is given by the vector $(a_1,\ldots,a_{2k},L)$, then H is given by $H = \sum_{j=1}^{2k} a_j\ell_j + LK$ where L is the corresponding Lyapunov quantity. The projection Π_{2k} is given by $\Pi_{2k}(H) := LK$.