Due 12:00 Noon on 18 December 2014

Please be sure to write some words in English to make sure I understand your answers. If you present graphics and numerics, be sure to explain what I am supposed to see in your graphs or tables of numbers. I would rather see some of the problems done well, than sketchy work on all of the problems.

It is possible that I have made an error somewhere on this exam. Please tell me if you believe there is an error.

You may send your solutions by e-mail or hand deliver them to my office. If I am not in my office when you stop by, simply leave your paper with one of the secretaries outside my office. Be sure your name is on the solutions you hand in.

Good luck!

Only the first question for the take-home final is written here. More later.

1. Recall the previous exercise that involved two springs:

Imagine an object with mass \( m \) sliding on a horizontal plane connected by identical springs (each with spring constant \( K \)) to fixed positions on the plane at a distance \( L = 2(\ell + \alpha) \) apart, where \( \ell \) is the natural length of each spring and \( \alpha > 0 \) is the extra distance each spring is stretched to make its attachment to the mass. When the mass is pulled in the direction of the perpendicular bisector of the line connecting the attachments to the plane and let go from rest, it moves along the perpendicular bisector due to the symmetry of the apparatus. Suppose the mass is pulled out a distance \( d \) units from its rest position on the intersection of the line and the perpendicular bisector and released from rest. The idealization just described might be a crude model of a crossbow. At least two phenomena are of interest: The frequency of oscillation and (for the crossbow application) the velocity of the mass at the moment it passes through the equilibrium position of the mass-spring system. Both of these quantities are functions of all the parameters: \( L, \ell, \alpha, M, \) and \( d \). The initial value problem for the displacement of the object from equilibrium (ignoring damping) can be expressed in the form

\[
M\ddot{x} = -2K x \left( 1 - \frac{\ell}{\sqrt{x^2 + (\ell + \alpha)^2}} \right), \quad x(0) = d, \quad \dot{x}(0) = 0.
\]
Consider the following data for the mass-spring system:

\[ K = 200 \text{ kg} / \text{sec}^2, \quad M = 0.25 \text{ kg}, \quad \ell = 0.4 \text{ m}, \quad \alpha = 0.05 \text{ m}. \]

The mass is to be pulled exactly \( d \) meters from its equilibrium position and released from rest. The problem is to determine \( d \) so that the mass crosses the equilibrium position at time \( T = 0.1 \text{ sec} \). What is \( d \)? Explain in words exactly what you did to solve the problem. Write code that automatically returns an approximation of the value of \( d \) so that your code could be used to approximate \( d \) for other choices of \( T \). Is there a critical value (for some parameter or combination of parameters) that must be considered to ensure that a solution exists? Hint: Use the shooting method. Perhaps rescaling the equations to a dimensionless form would be useful in your analysis of the existence of a critical value.