This homework is due on Friday 24 October.

(a) For the case of one spatial variable, use the porous medium equation and the boundary conditions given in the posted notes, zero density in the porous medium at the initial time, and the other data given in the section on porous media to approximate the gas flux at the middle cross section of the porous block as a function of time until one minute after the initial time taken to be \( t = 0 \). Report the flux at the middle cross section in kg/s and the time in seconds.

(b) For the case of one spatial variable, use pencil and paper to determine the general solution of the porous medium equation at steady state. Impose the boundary conditions and use a numerical computation to approximate the solution of the steady state boundary value problem. Use your result to determine the gas flux at the downstream boundary when the flow is in steady state.

(c) How long is the transient in hours measured from the instant the flow starts until the root mean square distance of the density profile in the porous block is within 1% of the steady state density profile.

Hints: One good way to debug a numerical code is to use a stable steady state as initial data. Stepping forward in time should leave the steady state unchanged. For part (a) it might be wise to use a second-order accurate numerical method to approximate the velocity at the cross section. This can be achieved by using a centered difference across the section; that is, by approximating the first derivative of an appropriate function \( f \) via \( (f(x + \Delta x) - f(x - \Delta x))/(2\Delta x) \). The root mean square distance between two functions is the square root of the integral over the spatial domain of the square of the difference between the two functions divided by the length of the interval \( (\sqrt{1/(b-a)} \int_a^b |f(x) - g(x)|^2 dx \).