Write out solutions of the following problems. Show all work. The assignment is due on Wednesday 21 September.

I’m not interested in the answer only; to get full credit I want you to explain carefully how you solved the problem. Please write in English sentences. As much as possible, use the methods we studied in class from scratch. If you wish to use a packaged code (in Mathematica or Matlab for example) to check, that is OK; but, to get full credit you should write a code to implement methods we discussed in class.

If you do not use a word processor, please write neatly and please organize your work. Handwritten work is acceptable. Please write some explanatory sentences so I can understand what you are doing.

1. (a) Suppose we have a tank partially filled with water. There is a pipe feeding water to the tank as a variable flow rate and there is also a drain pipe with a computer controlled variable valve hooked to a sensor in the tank. The valve opens exactly enough to let water drain from the tank at a rate proportional to the volume of the tank. The program allows for us to set one number: the constant of proportionality. Write a model for this physical problem. Be sure to define all the variables in your model. (b) Suppose the inflow rate is constant. How should the proportionality constant in the control mechanism be set to keep the tank near a constant desired volume? (c) Suppose the inflow rate is periodic. To be definite let’s say the flow rate is sinusoidal and known exactly, how should the constant of proportionality be set for the controller to best keep the tank at a constant desired volume.