There was a mistake on the second problem of the last homework, no wonder some people were confused. In the future, please let me know immediately if you believe there is an error in the notes or on a homework problem. I might have made a mistake typing. I am especially eager to correct mistakes in the book.

The following is a correct statement of the problem on the last homework. I am assigning it again with some modifications.

1. Consider the system of differential equations
   \[
   \begin{align*}
   \dot{x}_1 &= -k_1 x_1, \\
   \dot{x}_i &= k_{i-1} x_{i-1} - k_i x_i, & i = 2, 3, 4, \ldots, n-1, \\
   \dot{x}_n &= k_{n-1} x_{n-1}.
   \end{align*}
   \]

   It arises in situations that may schematically be described by a process \(X_1 \to X_2 \to X_3 \to \cdots \to X_n\) where the amount or concentration \(x_i\) of some substance in a region \(X_i\) (perhaps a tank) is determined by the amount of the substance coming into \(X_i\) from \(X_{i-1}\) minus the amount going out. The parameter \(k_i\) is the rate constant for the amount of substance leaving \(X_i\). Suppose initial data \(x_i(0) = \xi_i\) is also given. (a) Show that the system can be solved explicitly. (b) Let \(n = 10\), \(k_i = i/1000\), and \(\xi_i = 1 - 10i/101\). Determine \(x_{10}\) at time \(t = 2000\). Compare the exact solution with approximations using numerical methods for ODEs. Use Euler's method, Improved Euler, and (if you have access) a commercial code such as NDSolve in Mathematica or ode45 in Matlab. Note that you could find the exact solution using Mathematica, for example.

2. Do Exercise 2.12 in the book:
   (a) Consider the system
   \[
   \begin{align*}
   \dot{x} &= 1, \\
   \dot{y} &= axy,
   \end{align*}
   \]
   where \(a\) is a parameter. Solve this system with initial data \(x(0) = y(0) = -1\), and show that the exact value of the solution at \(t = 2\) is
\((x, y) = (1, -1)\) independent of \(a\). (b) Generalize the result of part (a); that is, given \(x(0) < 0\), there is a time \(T > 0\), which is independent of \(a\), such that at \(t = T\) the solution starting at \((x(0), y(0))\) is at the point with coordinates \((-x(0), y(0))\). (c) Use various numerical methods, at least Euler and Improved Euler, to solve the system of ODEs at least for the parameter values \(a = 1, 10^2\), and \(a = 10^4\) with \(x(0) \leq -1\). Do your computer codes produce correct results? Discuss your experiments.