A few extra Math 4100 problems: Solve the ODE’s and the IVP’s.

1. \(2\dot{u} + 6u = t, \quad u(0) = 1.\)
2. \(xdy/dx - y = 1, \quad x > 0.\)
3. \(\sin x \sin y dy/dx = \cos x \cos y + 2x.\)
4. \(z^2 d^2y/dz^2 - z dy/dz - 3y = 0, z > 0.\) Hint: Look for a solution of the form \(y = z^r.\)
5. \(x'' + 3x' + x = 0.\)
6. \(y'' + y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 1.\)
7. \((x + 2) \sin y + x \cos y dy/dx = 0.\)
8. Find the general solution of the ODE
   \[t^2 \ddot{u} - t\dot{u} + u = 0.\]
   Hint: \(u = t\) is a solution.
9. Do the functions \(y_1(x) = \sin x\) and \(y_2(x) = 2 \cos(\pi/2 - x)\) form a fundamental set of solutions for the ODE \(y'' + y = 0?\) Explain your answer.
10. Find the monthly payment on a $50,000 loan at 5% annual interest compounded continuously that is to be paid back after 10 years.