1) (16 points) Solve the system of equations below by putting the equations into matrix form and doing Gaussian elimination on the matrix. Be sure to indicate in some way what row operation(s) you are doing at each step.

\[
\begin{align*}
  z - 2w &= 2 \\
  x + 4y + z + w &= 1 \\
  -x - 4y + z - 5w &= 3
\end{align*}
\]

\[
\begin{bmatrix}
  0 & 0 & 1 & -2 \\
  1 & 4 & 1 & 1 \\
  -1 & -4 & 1 & -5
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  1 & 4 & 1 & 1 \\
  0 & 0 & 1 & -2 \\
  -1 & -4 & 1 & -5
\end{bmatrix}
\]

\[
R_3 = \rightarrow R_3 + R_1
\]

\[
\begin{bmatrix}
  1 & 4 & 1 & 1 \\
  0 & 0 & 1 & -2 \\
  0 & 0 & 0 & 0
\end{bmatrix}
\]

There are infinitely many solutions:
y = anything and w = anything
x = \(-4y - 3w - 1\), and z = \(2w + 2\).

2) For the word problem below, do what is asked for, but do not solve the word problem.

I have $1.50 worth of change in nickels, dimes and quarters. There are a total of 17 coins, and if I removed one nickel, then I would have three times as many nickels as dimes and quarters combined. How many of each type of coin do I have?

a) (3 points) Name variables for the problem and indicate precisely what each stands for.

Let \(x\) = \# of nickels, \(y\) = \# of dimes, and \(z\) = \# of quarters.
(you could have used different letters, or a different order)

b) (9 points) List all the equations that the word problem (implicitly) gives.

The equations are \(.05x + .10y + .25z = 1.50\) (or if you think in terms of cents you could have \(5x + 10y + 25z = 150\), instead).
\(x + y + z = 17\), and \(x - 1 = 3(y + z)\).
The last equation can be written as \(x - 3y - 3z = 1\).

c) (2 points) Write down the first matrix that your equations in part (b) lead to, if you were to do Gaussian elimination (do not do the Gaussian elimination).

\[
\begin{bmatrix}
  .05 & .1 & .25 & 1.50 \\
  1 & 1 & 1 & 17 \\
  1 & -3 & -3 & 1
\end{bmatrix}
\]