1) (18 points) Suppose that the reliability of a test for disease C is specified as follows: Of people with C, 94% have a positive reaction and 6% have a negative reaction; of people free of C, 90% have a negative reaction and 10% have a positive reaction. From a large population of which 0.7% of the people have disease C, a person is selected at random and given the test.

a) Draw a diagram illustrating the problem above. Make sure to fill in all the probabilities for your diagram.

Find the probabilities that
b) the person has disease C and tests positive.
c) the person tests positive.
d) has disease C given that he/she tested positive.

This problem is almost exactly like Example 2 of §6.6 on page 312.
Please note that as a decimal number, 0.7% = .007.

a) Look at the tree diagram on page 312. Replace “TB” by “C”. From top to bottom the numbers should be .007 and .993, then in the middle, .94, .06, .1, .9, and finally at the ends of the paths (so that you have all the probabilities) .00658, .00042, .0993, .8937.

b) This is the probability that you follow the path C then POS, so \( Pr(C \cap \text{POS}) = .00658 \).

c) Add together the ways to get POS: \( Pr(\text{POS}) = .00658 + .0993 = .10588 \).

d) \( Pr(C|\text{POS}) = \frac{Pr(C \cap \text{POS})}{Pr(\text{POS})} = \frac{.00658}{.10588} \approx .0622 \), or 6.22%.

Note that even with a positive test, you are unlikely to have the disease. This is because there are so many “false positives” coming from the vast majority who don’t have the disease, but still test positive. See page 312 for a discussion of this in more detail.

2) (12 points) You pay $2 to play a game, which consists of placing two yellow balls, and one green ball in an urn and picking one ball at a time at random (without replacement) until you draw the green ball. For each ball that you pick, you are given $1.50. Let the random variable \( X \) be

\( X = \text{net winnings from the game.} \)

Make a probability distribution for \( X \). (a diagram will be helpful)

\[
\begin{array}{c|c}
 k & Pr(X = k) \\
-0.5 & \frac{1}{3} \\
1 & \frac{1}{3} \\
2.5 & \frac{1}{3} \\
\end{array}
\]