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NONLINEAR CHARACTERISTICS OF A SUDDEN EXPANSION FOLLOWED BY SUDDEN CONTRACTION CHANNEL

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ABSTRACT

The flow and heat transfer in a sudden expansion followed by sudden contraction channel are widely applied in industry, and it is also a classic problem for theoretical study. In this article, the SIMPLE algorithm with QUICK scheme were used to study the flow and heat transfer in a sudden expansion followed by sudden contraction channel. The nonlinear characteristics and temperature field were investigated for various Reynolds number and geometrical dimension. The results show that the temperature field evolves from symmetric to asymmetric state with the increasing Re. When Re $\geq$Re$_c$, the flow loses stability and from symmetric to asymmetric via a symmetry-breaking bifurcation; when the Reynolds number continues to increase, the fluid flow and heat transfer oscillation. The nonlinear characteristics of flow and heat transfer within the channel is further analyzed.

KEYWORD: Sudden expansion, Sudden contraction, Numerical simulation, Nonlinear characteristics, Phase space.

INTRODUCTION

In petroleum production, such as natural gas transportation project, the use of the pipeline with abrupt change of section is very common; the research of flow and heat transfer in sudden change section channel has always been a hot topic. For the problem of flow and heat transfer in sudden expansion channel, when the geometry and boundary conditions are symmetric, it is generally believed that the solution is also symmetric. During theoretical studies usually take half of the area as the study area [1].

In fact, the Newtonian fluid flow through a plane symmetric sudden expansion channel becomes asymmetric, as the Reynolds number, expansion ratio, aspect ratio and the Nusselt number are changed. In addition to the presence of symmetric solution, there may be asymmetric solution. Durst et al. [2, 3] experimentally studied the low Reynolds number flow over the sudden expansion channels in two- and three-dimensional cases. It was found that at the lower Reynolds numbers the flow was very stable; at higher Reynolds numbers the flow became less stable and periodicity became increasingly important in the main stream. Badekas and Knight [4] studied the flow in the sudden expansion pipe with Reynolds number from 50 to 200 and the expansion ratio from 1.5 to 6.

The flow and heat transfer problem of sudden expansion followed by sudden contraction channel is a classic problem. Thiruvengadam [5] adopted SIMPLEC algorithm and second order upwind difference scheme to study the effects of sudden expansion of the three-dimensional vertical flow path upward buoyancy and the influence of channel scale bifurcation of mixed convection flow. Macagno and Hung [6] used numerical simulation and experimental methods to analysis the flow in the sudden expansion pipe with the expansion ratio of 2, and the Reynolds number less than 200; the equations of stable and unstable formats were calculated by finite difference method. Iribarne [7] experimentally observed the flow field in the sudden expansion channel with Reynolds number from 90 to 1350 when the expansion ratio was 2. Acrivos et al. [8-10] established the corresponding mathematical model at the sudden expansion flow with numerical simulation, and found that below the critical Reynolds number, N-S equation had stable solution; when the Reynolds number was greater than the critical Reynolds number, the flow was unsteady. Oliveira [11] used finite volume method to solve the laminar flow in axisymmetric sudden expansion pipe. Shen et al. [12] employed the SIMPLE algorithm and unsteady-state model to analyze the flow and heat transfer characteristics and nonlinear phenomena at different Reynolds numbers and channel geometries. Liu et al. [13] used SIMPLE algorithm with QUICK scheme and unsteady-state model to simulate flow and heat transfer in the entire channel, and analyzed the flow and heat transfer characteristics and nonlinear phenomena with the change of Reynolds number and geometry.

In this paper, SIMPLE algorithm with Quick scheme are used to simulate the flow and heat transfer problems in a sudden expansion and sudden contraction channel. The nonlinear characteristics and its influence factors have been studied for various Reynolds number and geometrical dimension. The nonlinear characteristics within the channel is further studied.

2 PHYSICAL AND MATHEMATICAL MODEL

2.1 PHYSICAL MODEL

Figure 1 shows the physical model of the problem that the fluid flows through a two-dimensional symmetric plane sudden expansion followed by sudden contraction channels. The length of AB is L_1, BC is L_2 and CD is L_3, the width of inlet and outlet are h, and the space of upper and lower plate is H. U_0 is the inlet velocity. The expansion ratio is defined as E_R = H/h, the aspect ratio is defined as A_R =L_2/h, and the channel completely symmetrical. The inlet temperature of fluid is T_0 and the wall temperature isT_W.

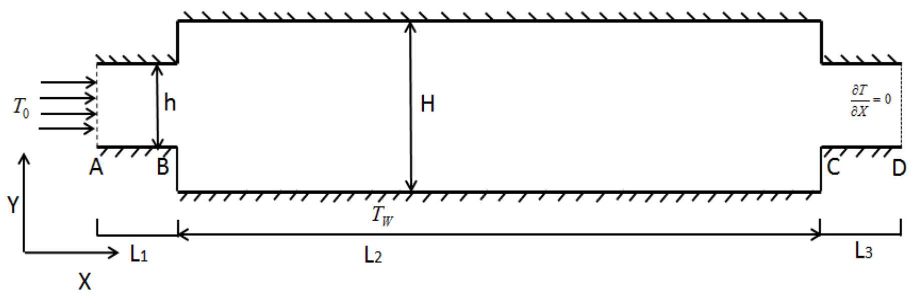


Fig. 1 The physical model

2.2 MATHEMATICAL MODEL AND BOUNDARY CONDITION

In this problem, the fluid is water and considered incompressible. The effects of viscous dissipation and gravity are ignored. The governing equations for laminar flow of the forced convection can be written as following.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}=-\frac{\partial p}{\partial x}+\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}) \tag{2}$$

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho vu)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}=-\frac{\partial p}{\partial y}+\mu(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}) \tag{3}$$

$$\frac{\partial(\rho ut)}{\partial x}+\frac{\partial(\rho vt)}{\partial y}=\frac{\lambda}{c_p}(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}) \tag{4}$$

The boundary conditions are defined as follows:

Inlet: $u=U_0 \;\; v=0 \;\; t=T_0$ (5)

Outlet: $\frac{\partial u}{\partial x}=0 \;\; \frac{\partial v}{\partial x}=0 \;\; \frac{\partial t}{\partial x}=0$ (6)

Wall: $u=v=0 \;\; t=T_w$ (7)

The following dimensionless parameters are introduced:

$$U=u/U_0, \quad V=v/U_0, \quad P=P/(\rho U_0^2), \quad X=x/H,$$

$$Y=y/H, \quad \mathrm{Re}=U_0H/v, \quad \Pr=\frac{v}{a}, \quad F=\frac{\tau U_0}{H}$$

where u and v, are horizontal and vertical velocity components, respectively. U_0 is the inlet velocity. P is the pressure, v, ρ are the kinematic viscosity and density respectively. The governing equations can be rewritten in dimensionless form as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{8}$$

$$\frac{\partial V}{\partial F}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}) \tag{9}$$

$$\frac{\partial V}{\partial F}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}) \tag{10}$$

Reynolds number is defined as:

$$\mathrm{Re}=U_0H/\nu \tag{11}$$

The dimensionless boundary conditions are:

Inlet: $U=1 \; V=0$ (12)

Outlet: $\frac{\partial U}{\partial X}=0 \;\; \frac{\partial V}{\partial X}=0$ (13)

Wall: $U=V=0$ (14)

The simulation results using the average Nusselt number of the channel as the indicators and the width of H as the feature size. The Nusselt number is defined as:

$$Nu=-\frac{\partial\theta}{\partial Y}\bigg|_{Y=0} \tag{15}$$

$$\overline{Nu}=\frac{1}{x}\int_0^x-\frac{\partial\theta}{\partial Y}\bigg|_{Y=0}dx \tag{16}$$

The equations of the study are forced convection equations which can be discretized by the finite volume method [14] and solved numerically. The well-known staggered grid is employed. Due to the nonlinear of the governing equations, iterations are needed. The SIMPLE algorithm with the 2nd order precision QUICK scheme [14] are adopted in this paper.

3 RESULT AND DISCUSSION

The objective of this paper is to study fluid characteristics of flow and heat transfer by changing Reynolds, expansion ratio and aspect ratio in channel as well as the process of flow and heat transition from symmetric to asymmetric**.** In order to validate the code, the Poiseuille flow was simulated, the comparison of velocity distribution between analytical solutions and numerical solutions is shown in Table 1 (Y=0 is the central axis of the channel). The velocity error of numerical simulations and analytical solutions is less than 1.1%, so this numerical simulation method can be considered credible.

Tab 1 Comparison of velocity value of two parallel plates

Y	Numerical solution	Analytical solution	Error (%)
0	1	1	0
-0.8	0.3563	0.36	1.02
0.8	0.3561	0.36	1.08
-0.4	0.8332	0.84	0.81
0.4	0.8326	0.84	0.88
-0.6	0.6346	0.64	0.84
0.6	0.6352	0.64	0.75

3.1 THE SIMULATION RESULTS OF DIFFERENT REYNOLDS

The sudden expansion ratio and the sudden contraction ratio are both 2, the aspect ratio is 20, and the streamline charts of the fluid under different Reynolds number are shown in Figure. 2(All the flows reach to the steady-state condition). When the sudden expansion ratio and aspect ratio are certain, with the increase of Re, the flow of the fluid flows through the channel from the symmetric to asymmetric, the fluid will deflect to the upward wall, and the nonlinear phenomena of fluid becomes more and more obvious, the degree of deflection of the fluid becomes more and more obvious. The numerical and experimental investigations of the literatures have focused on the attributes initiating instability, the most important of which is critical Reynolds number (Re_c). When fluid flows through the sudden expansion section, there will form a recirculation zone and the length is related to the Reynolds. when the Reynolds is lower than the critical Reynolds, the line of fluid in the channel is symmetrical, when the Reynolds is greater than the critical Reynolds, the passage of fluid in the recirculation zone is greater than the length of the wall, and with the increase of the Reynolds, the length difference between the upper and lower wall of the recirculation zone is become growing, in which the lower length of the fluid of recirculation zone remains stable, the upper length of the fluid recirculation becomes longer as the change of Reynolds.

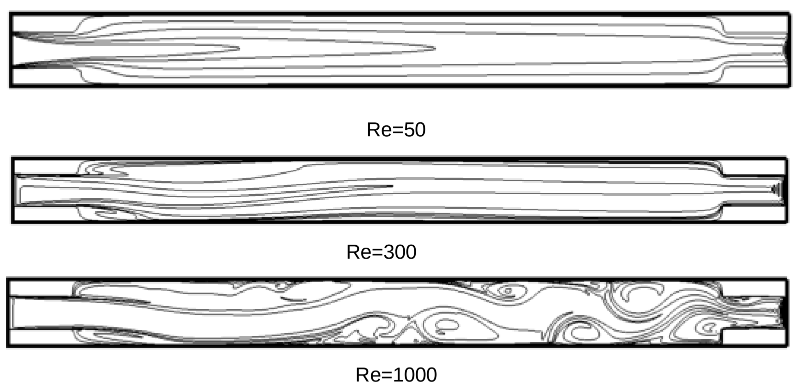
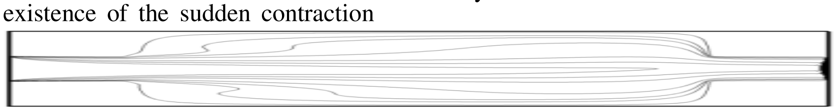


Fig. 2 E_R=2, A_R=20 The isotherm diagram of different Re

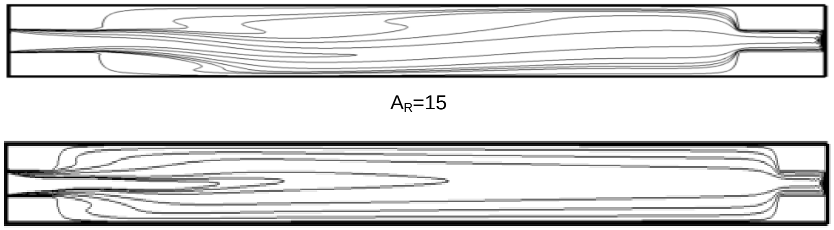
3.2 THE SIMULATION RESULTS OF DIFFERENT ASPECT RATIO

When the Re is 50, the expansion ratio is 3, the streamline charts of the fluid under different aspect ratio are shown in Fig. 3. According to the Fig. 3, it is found that, under the lower Re and certain expansion ratio, the flow of the fluid becomes symmetry to asymmetry with the increase of aspect ratios, when it larger than certain aspect ratio, the flow of fluid will become symmetry, and then the flow of fluid will stay and never change with the increase of the aspect ratio. The sudden contraction channel can also affect the flow characteristics in the channel, in a certain expansion ratio and Reynolds number, the existence of the sudden contraction section can result in the different of the critical Reynolds number.

Figure 4 shows the Re = 100, E_R = 3, the Nusselt number of different aspect ratio of the upper and lower wall. It can be seen from Figure 4, When the aspect ratio is small, the Nusselt number of upper and lower wall keep consistent; with increasing of aspect ratio, the Nusselt number of the upper and lower wall appear significantly different, the position of the lower wall under the maximum Nusselt number appearing on later comparing to the upper wall; when the aspect ratio continues to increase, the Nusselt number of the upper and lower wall remained the same again and maintain a symmetrical state.

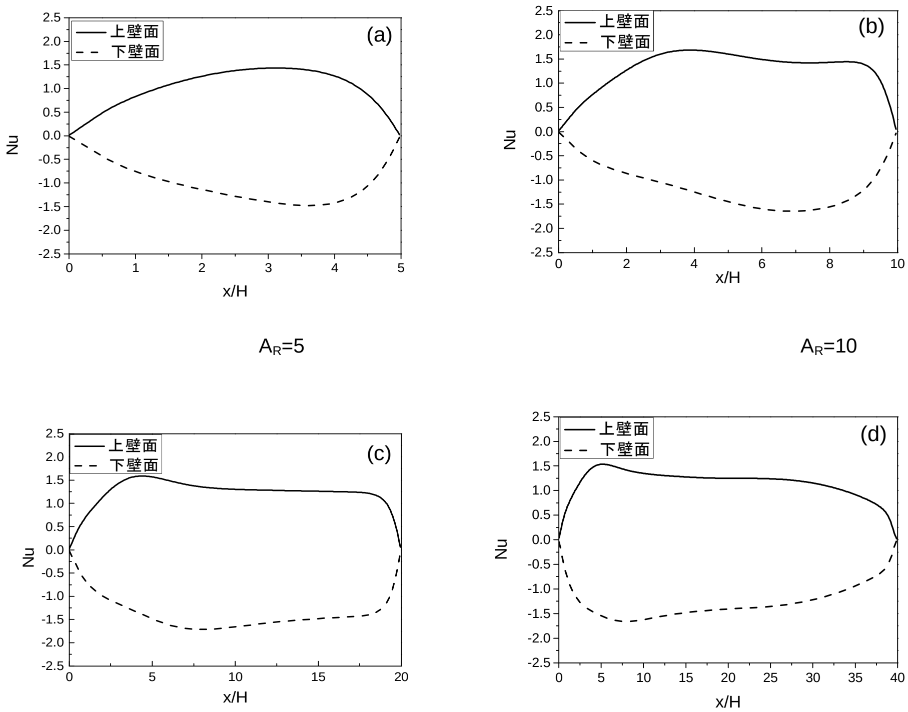


A_R=5



A_R=30

Fig. 3 E_R =3, Re=50, isotherm diagram of different aspect ratio



A_R=20 A_R=40

Fig 4 Re=100, E_R =3, the Nusselt number of different aspect ratio of the upper and lower wall

3.3 THE CHARACTERISTIC SOLUTIONS OF SCALING CHANNEL

As can be seen from the monitoring point of speed in scaling channel, there are three different solutions under different Re, E_R, A_R in the channel, including stable solution, periodic oscillations, aperiodic solutions and oscillation solution.

Figure 5 shows the phase space attractor patterns when the solution is steady and the steady-state solution is obtained by the initial conditions of Re = 250. it can be seen from the figure, the speed performed a fixed and the flow and heat transfer of fluid stabilized finally. In this case, the solution is stable over time, speed U and V become a certain value eventually.

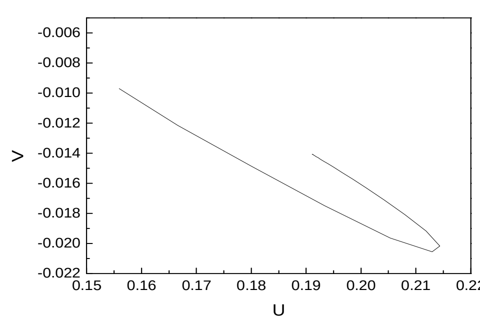


Fig 5 The velocity phase space of Re=250，E_R=4，A_R=30

Figure 6 shows the phase space limit circle attractor in Re=850, E_R=4, A_R=30, from figure 10 can be seen, the attractor is a two-dimensional torus. Numerical results show that with the increase of computing time, the final fluid UV map will form a torus.

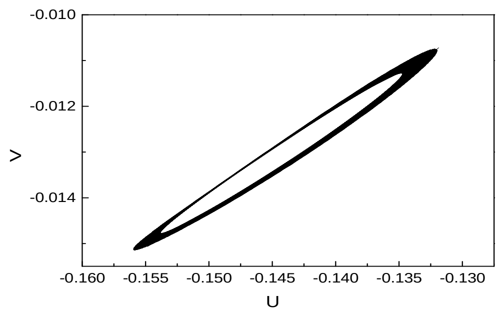


Fig 6 The velocity phase space of Re=850，ER=4，AR=30

Numerical results show that with the increase of Reynolds, the velocity of the fluid is still make a periodic oscillation with the change of time, in addition to the change of the period of oscillation, the velocity of the fluid UV phase diagram also had some changes. Figure 7 shows that the UV phase diagram in Re=900, E_R=4, A_R=30 and can be seen from figure 7, the attractor is a two-dimensional surface domain. In this case, the velocity of the fluid UV forms a surface domain.

Figure 8 is the phase space attractor shape for the periodic oscillation, from the figure can be seen with time, the velocity of fluid turned out non-periodic change and the line of speed become complex, the infinite times overlap, close, but never intersect, and system shows chaotic characteristic eventually.

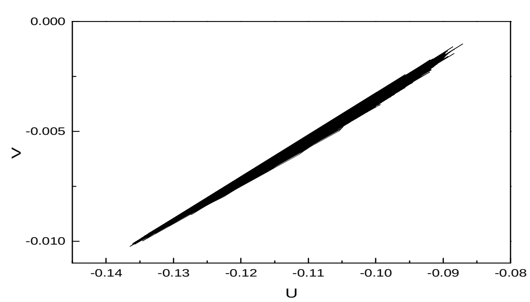


Fig 7 The velocity phase space of Re=900, E_R=4, A_R=30

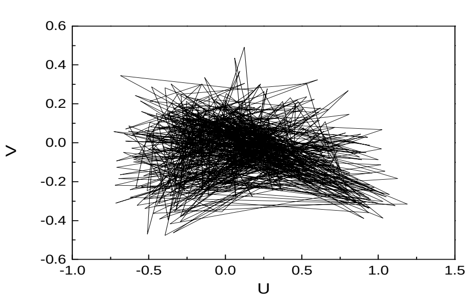


Fig 8 Velocity phase space of Re=1100, E_R=4, A_R=30

4 CONCLUSION

Effects of expansion ratio, Reynolds number and aspect ratio on flow in sudden expansion followed by sudden contraction channel is studied in this paper. According to the numerical results, there is a critical Reynolds number in this nonlinear problem, when the Re lower than this value of Re, there is only a unique stable solution and the flow is symmetric; when the Re reaches the critical Reynolds number, the flow loses stability and changes from symmetric to asymmetric via a symmetry-breaking bifurcation. When the Re above the critical Reynolds number, the problem has three solutions, of which the original solution is now unstable. The sudden contraction channel can also affect the flow characteristics in the channel; in a certain aspect ratio, the existence of the sudden contraction section can result in the different of the critical Reynolds number.

The monitoring points of speed in different Re, E_R and A_R exist steady solution, periodic oscillation solution and non-periodic oscillation solution. Numerical results show that the flow of fluid has different solutions for different parameters. For steady state solutions, which tends to be a fixed phase diagram; for periodic oscillation, its phase diagram is a two-dimensional torus or surface domain; for non-periodic oscillation solution, its phase diagram unlimited overlapping close.

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