ANALYSIS OF FREEZING IN AN ECCENTRIC ANNULUS

Yuwen Zhang and Amir Faghri
Department of Mechanical Engineering
University of Connecticut
Storrs, CT 06269-3139

Abstract

Freezing in an eccentric annulus is investigated numerically by using a temperature transforming model. Since the effect of the heat conduction along the circular direction on the growth of the freezing layer is very small, an analytical solution by employing integral approximate method is proposed. The freezing rate obtained by the analytical solution agreed very well with that of the numerical solution, although the analytical solution is much simpler than the numerical solution. The effects of the eccentric annulus geometric structure on the freezing process is also investigated.

INTRODUCTION

Thermal energy storage systems are very important in the harnessing of periodical energy sources, such as solar energy. The phase change thermal energy storage system is the most promising since it can store and release a large amount of heat energy during the melting and freezing process. The advantage of the phase change thermal energy storage system, compared with other thermal storage systems, is that it can absorb and release heat at a constant temperature. Heat transfer in the melting and solidification have been intensively studied by many researchers and numerous papers have been published (Viskanta, 1983; Yao and Prusa, 1989; Fukusako and Yamada, 1994).

In a shell- and- tube phase change thermal energy storage system where the Phase Change Material (PCM) filled the tube side, melting or freezing occurred in the tube from the tube wall toward the center. For the melting process in a horizontal tube with natural convection in the liquid phase, there are two different models of melting: fixed melting (Ho and Viskanta, 1984) and unfixed melting (Bareiss and Beer, 1984). The only difference between these two models is that the un-melted solid PCM either remains at the center of the tube or falls downward due to the effect of gravity. For the freezing process in a tube without superheat in the liquid phase, heat conduction is the only mechanism of heat transfer in the latent heat thermal energy storage system.

In order to accelerate the phase change process in the latent heat thermal energy storage system, Betzel and Beer (1988) proposed a concentric annulus, where the phase change process simultaneously occurs from the inner and outer tubes. The experimental results for unfixed melting and solidification showed that the addition of an inner tube can significantly accelerate the phase change process. Cao et al. (1991) studied the two- dimensional change of phase in a PCM cylinder and a PCM concentric annulus conjugated with the forced convection of the transfer fluid. The temperature of the transfer fluid varied along the axial direction instead remaining a constant. Their results also showed that adding an inner tube is an efficient way to accelerate the phase change process.

The latent heat thermal energy storage system in Betzel and Beer (1988) and Cao et al. (1991) are modeled as two concentric annular tubes in which the inner tube and the outer tube have the same center. However, eccentricity can exist in some latent heat energy storage systems due to manufacturing error and for special application requirements. Huang et al. (1989) developed an enthalpy model with a body fitted coordinate system to simulate the freezing in the cylinders of arbitrary cross- section. As an application of this model, they calculated the freezing in an eccentric annulus. The freezing process is a three- dimensional problem since cooling occurs from the inner tube, outer tube and the two ends of the eccentric annulus. The calculation process is very complex and the effect that eccentricity had on the freezing process is not discussed.

In this paper, freezing in a two dimensional eccentric annulus will be investigated by employing numerical and analytical methods. The effect of the eccentricity and the diameter ratios on the freezing will also be discussed.

PHYSICAL MODEL

Fig. 1 shows the physical model of melting in an eccentric annulus. The PCM fills the eccentric annulus with outer radius, r_o, inner radius, r_i, and eccentricity, e. The initial temperature of the PCM is assumed to be at the freezing temperature, T_m^0. It is also assumed that the freezing occurs at a single temperature and that the solid- liquid interface is smooth. At the beginning, $t = 0$, the temperature of the outer and inner walls of the eccentric annulus are suddenly reduced to a temperature, T_c, below the freezing temperature. The freezing process will simultaneously occur at the outer and inner walls of the eccentric annulus.

The freezing problem is described by employing the temperature transforming model using a fixed grid method (Cao and Faghri, 1990). This model assumes that the freezing process occurs over a range of phase change temperatures from $(T_m^0 - \delta T^0)$ to $(T_m^0 + \delta T^0)$, but it can also be successfully used to simulate the freezing process occurring at a single temperature. This model has the advantage of eliminating the time step and grid size limitations that are normally encountered in other fixed grid methods. Since the freezing problem is axisymmetric, only half of the eccentric annulus will be investigated. In the polar coordinate system, where the center is located at the center of the outer wall, the governing equations for the freezing problem are:

$$\frac{\partial (C^0 T^0)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T^0}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T^0}{\partial \phi}\right) - \frac{\partial S^0}{\partial t} \tag{1}$$

$$C^0(T^0) = \begin{cases} C_s^0 & \text{(solid phase)} \\ \dfrac{C_s^0 + C_\ell^0}{2} + \dfrac{\rho H}{2\delta T^0} & \text{(mushy zone)} \\ C_\ell^0 & \text{(liquid phase)} \end{cases} \tag{2}$$

$$k(T^0) = \begin{cases} k_s & \text{(solid phase)} \\ k_s + \frac{(k_\ell - k_s)(T^0 - T_m^0 + \delta T^0)}{2\delta T^0} & \text{(mushy zone)} \\ k_\ell & \text{(liquid phase)} \end{cases} \tag{3}$$

$$S^0(T^0) = \begin{cases} C_s^0 \delta T^0 & \text{(solid phase)} \\ \dfrac{(C_s^0 + C_\ell^0)\delta T^0}{2} + \dfrac{\rho H}{2} & \text{(mushy zone)} \\ C_s^0 \delta T^0 + \rho H & \text{(liquid phase)} \end{cases} \tag{4}$$

where the temperature range for each phase is

$$\begin{array}{ll} T^0 < T_m^0 - \delta T^0 & \text{(solid phase)} \\ T_m^0 - \delta T^0 \le T^0 \le T_m^0 + \delta T^0 & \text{(mushy zone)} \\ T^0 > T_m^0 + \delta T^0 & \text{(liquid phase)} \end{array}$$

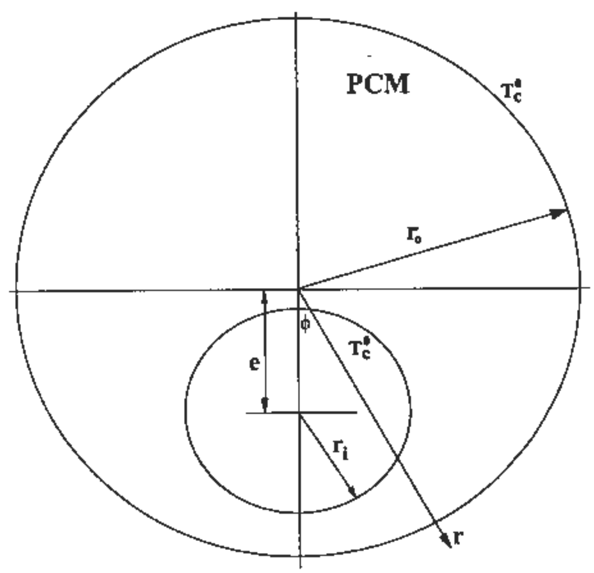

Fig.1 Physical model of freezing problem

The initial condition and boundary conditions can be written as

$$T^0 = T_i^0 \qquad t = 0 \tag{5}$$

$$T^0 = T_c^0 \qquad r = r_o \tag{6}$$

$$T^0 = T_c^0 \begin{cases} r = r_2,\ 0 < \phi < \pi & e < r_i \\ r = r_1, r_2,\ 0 < \phi < \sin^{-1}(\frac{r_i}{e}) & e \geq r_i \end{cases} \tag{7}$$

$$\frac{\partial T^0}{\partial \phi} = 0 \qquad \phi = 0, \pi \tag{8}$$

Where

$$r_2 = e\cos\phi + \sqrt{r_i^2 - e^2\sin^2\phi} \tag{9}$$

$$r_1 = e\cos\phi - \sqrt{r_i^2 - e^2\sin^2\phi} \tag{10}$$

By defining the following dimensionless variables:

$$\left.\begin{array}{llll} R = \frac{r}{r_o} & R_i = \frac{r_i}{r_o} & E = \frac{e}{r_o} & Fo = \frac{\alpha_s t}{r_o^2} \\ C = \frac{C^0}{C_s^0} & C_\ell = \frac{C_\ell^0}{C_s^0} & K = \frac{k}{k_s} & K_\ell = \frac{k_\ell}{k_s} \\ S = \frac{S^0}{C_s^0(T_m^0 - T_c^0)} & & Ste = \frac{C_s^0(T_m^0 - T_c^0)}{\rho H} & \\ \delta T = \frac{\delta T^0}{T_m^0 - T_c^0} & & T = \frac{T^0 - T_m^0}{T_m^0 - T_c^0} & \end{array}\right\} \tag{11}$$

the governing equations become:

$$\frac{\partial(CT)}{\partial Fo} = \frac{1}{R}\frac{\partial}{\partial R}\left(KR\frac{\partial T}{\partial R}\right) + \frac{1}{R^2}\frac{\partial}{\partial \phi}\left(K\frac{\partial T}{\partial \phi}\right) - \frac{\partial S}{\partial Fo} \tag{12}$$

$$C(T) = \begin{cases} 1 & T < -\delta T \\ \frac{1}{2}(1 + C_\ell) + \frac{1}{2Ste\delta T} & -\delta T \leq T \leq \delta T \\ C_\ell & T > \delta T \end{cases} \tag{13}$$

$$K(T) = \begin{cases} 1 & T < -\delta T \\ 1 + \frac{(K_\ell - 1)}{2\delta T}(T + \delta T) & -\delta T \leq T \leq \delta T \\ K_\ell & T > \delta T \end{cases} \tag{14}$$

$$S(T) = \begin{cases} \delta T & T < -\delta T \\ \frac{1}{2}(1 + C_s)\delta T + \frac{1}{2Ste} & -\delta T \leq T \leq \delta T \\ \delta T + \frac{1}{Ste} & T > \delta T \end{cases} \tag{15}$$

$$T = T_i, \quad Fo = 0 \tag{16}$$

$$T = -1 \qquad R = 1 \tag{17}$$

$$T = -1 \begin{cases} R = R_2,\ 0 < \phi < \pi\ E < R_i \\ R = R_1, R_2, 0 < \phi < \sin^{-1}(\frac{R_i}{E})\ E \geq R_i \end{cases} \tag{18}$$

$$\frac{\partial T}{\partial \phi} = 0 \qquad \phi = 0, \pi \tag{19}$$

Where

$$R_2 = E\cos\phi + \sqrt{R_i^2 - E^2\sin^2\phi} \tag{20}$$

$$R_1 = E\cos\phi - \sqrt{R_i^2 - E^2\sin^2\phi} \tag{21}$$

NUMERICAL SOLUTION

The freezing problem has been specified by eqs. (12-21). These equations can be solved by a finite difference method described by Patankar (1980). In this methodology, the discretization equations are obtained by applying the conservation laws over a finite size control volume surrounding the grid node and integrating the equation over the control volume. The resulting scheme has the form:

$$a_P T_P = a_E T_E + a_W T_W + a_N T_{N+} a_S T_S + b \tag{22}$$

where the factors in eq. (22) can be found in Patankar (1980).

It is noticed that the PCM filled eccentric annulus has an irregular shape which is difficult to describe by the polar coordinate system in which the center is located at the center of the outer wall. This irregular geometric shape can be transformed to a concentric annulus by employing a Landau coordinate transformation (Ho and Viskanta, 1984), but the energy equation of the problem will become very complex. A simpler method, which will be used in this paper, is to employ the computational region expansion method (Patankar, 1980). In this methodology, the calculation region is the entire cylinder with a radius of the outer wall instead of the eccentric annulus. The dimensionless temperature inside the entire inner cylinder can be set to -1 so that the boundary condition at the inner wall of the eccentric annulus, eq. (18), can be satisfied. In order to set the dimensionless temperature inside the inner wall to -1, a_p and b, for the node located inside the inner wall, in eq. (22) are set to very large values, 10^{30} and -10^{30}, respectively.

Since eq. (12) is a non-linear equation, iteration is needed. During the iteration process, some underrelaxation is necessary. The relaxation factor used in this method is $0.1 \sim 0.2$. In order to simulate the freezing process occurring at a single temperature, a very small dimensionless phase-change temperature range, $\delta T = 0.001$, is used in the calculation and the initial temperature of the system is set to $T_i = -0.001$ instead of $T_i = 0$. The calculations were carried out for a grid of 42 nodes in the circular direction and 82 nodes in the radial direction with a dimensionless time step of $\Delta Fo = 0.01$. Finer grid sizes and smaller time steps were also used in the calculations, but their results did not show a noticeable differencc with the present grid size and time step.

Fig. 2 shows the locations of the freezing front at different dimensionless times. At the beginning of the freezing process, the solid layer grows from the outside of the inner wall and the inside of the outer wall simultaneously. The freezing front from the inner and outer wall independently grow and the thickness of the solid layer is uniform around the circumference of each wall. The growth of the freezing layer continues in this manner until the two fronts meet at the bottom of the eccentric annulus. After the two freezing fronts meet, the freezing process stops at the locations where the freezing fronts have met, but the freezing front continuously grows in the remaining locations. It is very clear that the thicknesses of the freezing layer at the remaining loeations are still uniform along the circumference of each wall. This result shows that the effect of heat conduction along the circumference of the freezing process is not significant. Based on this fact, an analytical solution will be proposed to predict the freezing rate in the eccentric annulus.

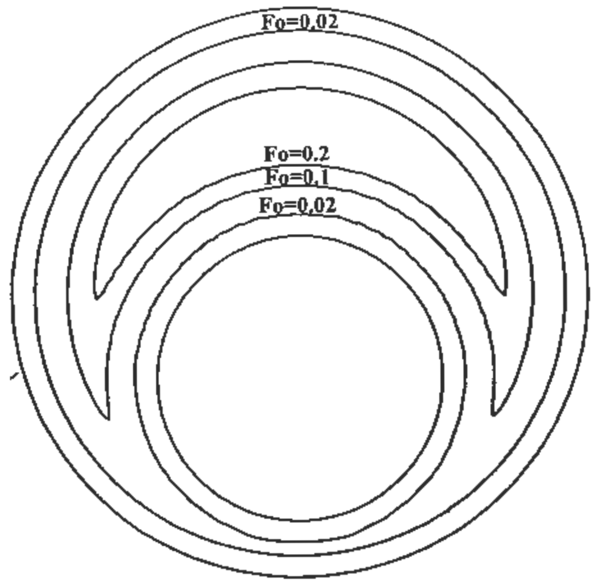

Fig.2 Freezing fronts ($R_i = 0.5, E = 0.3$, $Ste = 0.2$)

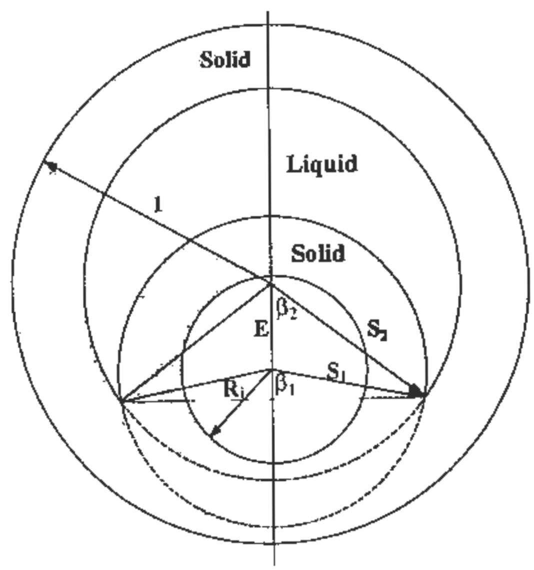

Fig. 3 Physical model for the analytical solution

ANALYTICAL SOLUTION

The point of the analytical solution is that the freezing at the inside of the outer wall and the outside of the inner wall can be solved independently. Before the freezing fronts meet, the freezing volume is the sum of that at inside of the outer wall and the outside of the inner wall. After the freezing the fronts meet, the freezing rate will decrease since the freezing process stopped at the locations where the freezing fronts met. Therefore, the freezing rate needs to be calculated for two different stages: before and after the freezing fronts meet. The radius of the inner and the outer freezing fronts can be determined by an integral approximate solution (Zhang and Faghri 1996).

In this approximate analytical solution procedure, the freezing process from inner surface of the outer cylinder and outer surface inner cylinder can be separately treated as two 1-D freezing problems in the cylindrical coordinate system. For the freezing from the inner surface of the outer cylinder of the annulus, the dimensionless governing equation for integral approximate solution will be

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) = \frac{\partial T}{\partial Fo} \quad 1 < R < S(\tau) \quad \tau > 0 \tag{23}$$

$$T = -1 \quad R = 1 \quad Fo > 0 \tag{24}$$

$$T(R,\tau) = 0 \quad R = S_2(\tau) \quad Fo > 0 \tag{25}$$

$$-\frac{\partial T}{\partial R} = \frac{1}{Ste}\frac{dS_2}{d\tau} \quad R = S_2 \quad Fo > 0 \tag{26}$$

It is assumed that the temperature distribution has a second order logarithmic function of the form (Zhang and Faghri, 1996)

$$\theta = -1 + \varphi\left(\frac{\ln R}{\ln S_2}\right) + (1-\varphi)\left(\frac{\ln R}{\ln S_2}\right)^2 \tag{27}$$

where, φ is an unknown variable. It can be seen that eq. (27) can satisfy eqs. (24) and (25) automatically, and φ cau be obtained by differentiating eq. (25):

$$\frac{\partial T}{\partial R}\frac{dS_2}{dFo} + \frac{\partial T}{\partial Fo} = 0 \tag{28}$$

Substituting eqs. (23) and (26) into eq. (28), the following expression is obtaiued:

$$-Ste\left(\frac{\partial T}{\partial R}\right)^2 + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) = 0 \quad R = S_2 \quad \tau > 0 \tag{29}$$

Substituting eq. (27) into eq. (29), an expression for φ is derived:

$$2 + \varphi = \frac{\sqrt{1+2Ste}-1}{Ste} \tag{30}$$

Substituting eqs. (27) and (30) into eq. (26), the location of the freezing front can be expressed as:

$$\frac{dS_2}{dFo} = \frac{\sqrt{1+2Ste}-1}{S_2 \ln S_2} \tag{31}$$

The initial condition of eq. (31) is:

$$S_2(\tau) = 1, \qquad \tau = 0 \tag{32}$$

Integrating eq. (31) in the time interval $(0, Fo)$ gives

$$\left[\frac{1}{2}\ln S_2 - \frac{1}{4}\right] S_2^2 + \frac{1}{4} = \left(\sqrt{1+2Ste}-1\right) Fo \tag{33}$$

For the freezing process at the outer surface of the inner tube, the solving procedure is similar procedure and the result will be

$$\left[\frac{1}{2}\ln\left(\frac{S_1}{R_i}\right) - \frac{1}{4}\right] S_1^2 + \frac{1}{4}R_i^2 = \left(\sqrt{1+2Ste}-1\right) Fo \tag{34}$$

Before the freezing fronts meet ($S_2 > S_1 + E$), the frozen solid volume is

$$V = \pi(S_1^2 - R_i^2) + \pi(1 - S_2^2) \tag{35}$$

and the total volume of the PCM is

$$V_0 = \pi(1 - R_i^2) \tag{36}$$

The freezing rate can be expressed as

$$\overline{V} = \frac{V}{V_0} = 1 - \frac{S_2^2 - S_1^2}{1 - R_i^2} \qquad S_2 > S_1 + E \tag{37}$$

After the freezing fronts meet ($S_2 < S_1 + E$), the freezing process stops at the meeting locations. The illustration of the freezing fronts is shown in Fig. 3. It can be seen that the freezing at the inner wall stops at an angle less than β_1 and the freezing at the outer surface stops at an angle less than β_2. The angles β_1 and β_2 can be determined by the following expressions

$$\beta_1 = \cos^{-1}\left(\frac{S_2^2 - S_1^2 - E^2}{2S_1E}\right) \qquad (38)$$

$$\beta_2 = \cos^{-1}\left(\frac{S_2^2 - S_1^2 + E^2}{2S_2E}\right) \qquad (39)$$

The freezing rate decrease due to the freezing fronts meeting will be

$$d\overline{V} = \frac{(\beta_2 - \sin\beta_2\cos\beta_2)S_2^2 - (\beta_1 - \sin\beta_1\cos\beta_1)S_1^2}{\pi(1-R_i^2)} \qquad (40)$$

The freezing rate after the freezing fronts meet can be expressed as

$$\overline{V} = 1 - \frac{S_2^2 - S_1^2}{1-R_i^2} - d\overline{V} \qquad S_2 < S_1 + E \quad (41)$$

RESULTS AND DISCUSSIONS

Fig. 4 shows the comparison of the freezing rate obtained by the numerical solution and the analytical solution at $R_i = 0.5$ and $E = 0.3$. As can be seen, in the beginning of the freezing process, the freezing rate obtained by the analytical solution is slightly higher than that obtained by the numerical solntion. However, the freezing rate obtained by the two methods becomes very close at the end of the freezing process. The total freezing time obtained by the two methods are almost identical. In order to check the accuracy of analytical solution, it is necessary to compare the results of numerical solution and analytical solntion for a case that approach the worst-case. The worst case for the analytical solution would be small R_i and large E. Fig. 5 shows the comparison of the freezing rate obtained by the numerical solution and the analytical solution at $R_i = 0.2$ and $E = 0.75$. It can be seen that the agreement between the numerical solution and analytical solution is still very good even at the point that approach the worst-case. Since the analytical solution is much simpler than the numerical solution and satisfied result can be obtained by analytical solution, the analytical solution is an useful tool to predict the freezing rate of the freezing in an eccentric annulus. The following discussions about the effect of the eccentricity and the radius ratio on the freezing rate will be based on the results of the analytical solution.

Fig. 6 shows the effect of the eccentricity on the freezing rate at a constant radius ratio, $R_i = 0.5$.

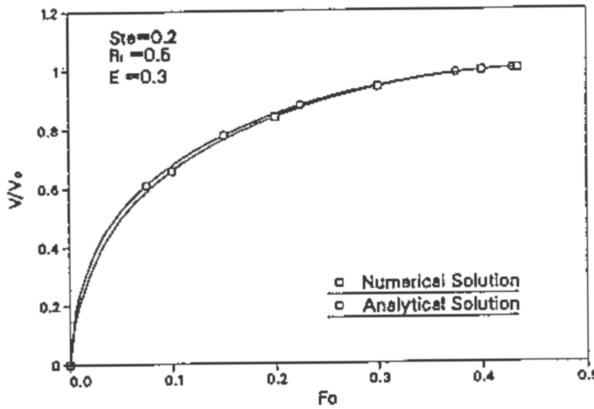

Fig. 4 Comparison of the freezing rate obtained by different methods ($R_i = 0.5, E = 0.3$)

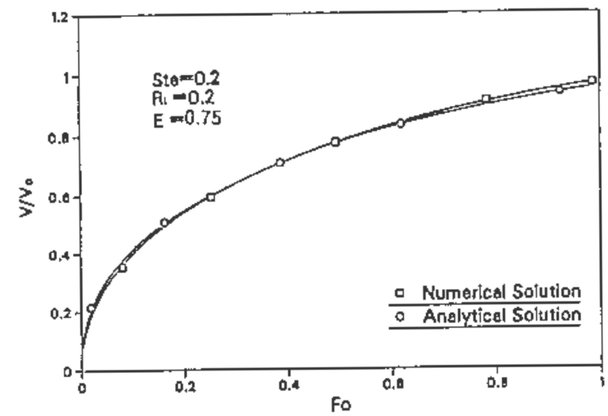

Fig. 5 Comparison of the freezing rate obtained by different methods ($R_i = 0.2, E = 0..75$)

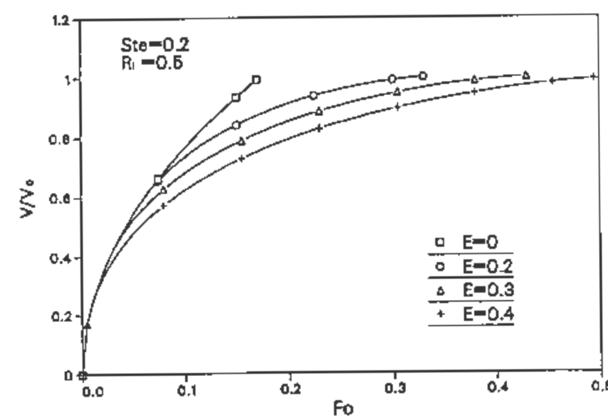

Fig. 6 Effect of eccentricity on the freezing rate

It can be seen that the freezing process is fastest if the eccentricity is zero and the eccentricity will always result in lower freezing rates. For the case of non-zero eccentricity, the freezing curve can be divided into two parts. In the beginning, the freezing rate curves are the same as that of zero eccentricity. It indicates that the freezing fronts from the inner and outer wall did not meet during this stage. After the freezing fronts meet, the freezing rate will be significantly decreased, because the freezing process will stop at the locations where the inner and outer freezing fronts met. As the eccentricity becomes larger, the time in which the freezing fronts meet becomes earlier and the freezing rate decrease is more significant.

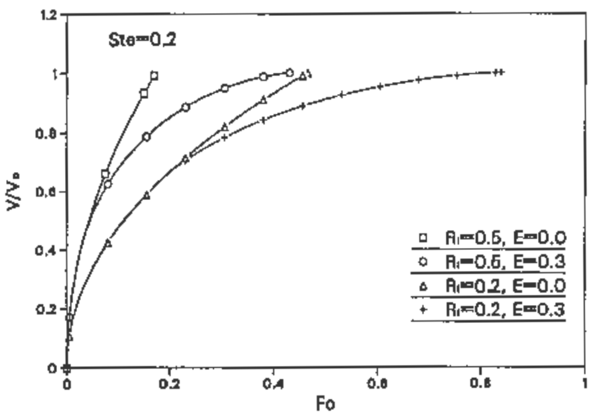

Fig. 7 Effect of eccentricity on the freezing rate at different radius ratios

Fig. 7 shows the effect of the eccentricity on freezing at different radius ratios. It can be seen that smaller radius ratios result in a slower freezing process, since the contact area of the inner wall and the PCM is smaller and the volume of the PCM is larger. The effect of eccentricity on the freezing rate for the smaller radius ratio is similar to that for larger radius ratio. Comparing the freezing curves for different radius ratios in Fig. 7, it can be seen that the freezing rate in which the inner and outer freezing fronts meet becomes larger for the smaller radius ratio. Therefore, the effect of the eccentricity on the freezing rate becomes smaller for a smaller radius ratio.

It is very clear that the existence of the eccentricity results in a lower freezing rate. Therefore, the eccentricity should be as small as possible from the practical view point, since it has a negative effect on the latent heat thermal energy storage system. Although all of the analyses in this paper have been based on the freezing process, it should be pointed out that the above conclusions are also correct for the melting process without natural convection in the liquid phase. The numerical solution and the analytical solution of this paper provided useful tools in predicting the effect of eccentricity on both the freezing and melting process in the eccentric annulus.

CONCLUSION

Freezing in an eccentric annulus has beeu studied numerically and analytically in this paper. The agreement between the numerical solution and the analytical solution was very good. The analytical solution is recommended because it is simpler than the numerical solution. The existence of the eccentricity in the latent heat thermal energy storage system is not recommended since it has a negative effect on the freezing rate. The effect of eccentricity on the freezing and melting process in an eccentric annulus can be predicted by emplnying the numerical solution and the analytical solution presented in this paper.

NOMENCLATURE

a	factor of algebraic equation (22)
b	term in eq. (22)
C^0	heat capacity, $J/(m^3K)$
C	dimensionless heat capacity, C^0/C^0_s
e	eccentricity, (m)
E	dimensionless eccentricity, e/r_o
Fo	Fourier number, $\alpha_s t/r_o^2$
H	latent heat of melting, J/kg
k	thermal conductivity, W/(mK)
K	dimensionless thermal conductivity, k/k_s
r	radial coordinate, m
R	dimensionless radial coordinate, r/r_o
r_i	inner wall radius, m
r_o	outer wall radius, m
R_i	dimensionless radius of the inner wall, r_i/r_o
S^0	term in eq. (1), J/kg
S	$S^0/C^0_s(T^0_m - T^0_c)$
S_1	dimensionless radius of the inner freezing front
S_2	dimensionless radius of the outer freezing front
Ste	Stefan number, $C^0_s(T^0_m - T^0_c)/(\rho H)$
T^0	temperature, K

T dimensionless temperature, $(T_m^0 - T^0)/(T_m^0 - T_c^0)$
V volume of the frozen solid, m^3
V_0 total volume of the PCM, m^3
$\overline{V}$ freezing rate, V/V_0

Greek letters

α thermal diffusivity, m^2/s
β_1 freezing front angle, see Fig. 3
β_2 freezing front angle, see Fig. 3
δT $\delta T^0/(T_m^0 - T_c^0)$
$2\delta T^0$ phase-change temperature range, K
ϕ angular position measured from the bottom of the annulus

subscripts

c cold
E East neighbor of grid P
i initial condition, or radius of the inner tube
ℓ liquid phase
m melting point
N North neighbor of grid P
o outer wall
P grid point
s solid phase
S Sonth neighbor of grid P
W West neighbor of grid P

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