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NONLINEAR CHARACTERISTICS OF PERIODICALLY FULLY DEVELOPED FLOW IN CROSS-FLOW TUBE BUNDLE

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ABSTRACT

In this paper, a numerical study of periodically fully developed convective heat transfer is performed in horizontal cross flow across tube bundle. The developed mathematical mode is governed by the couple equations of continuity, momentum and energy and is solved numerically by employing SIMPLE algorithm with QUICK scheme. The effects of Reynolds number on fluid flow and heat transfer performance are investigated. It is found that at small Reynolds numbers does not have much influence on the flow field. While at high Reynolds numbers have considerable effect on the flow pattern and Nussle number, and appear unsteady-state. The unsteady state flow and heat transfer exhibited periodic oscillating or chaotic behaviors due to formation of the vortexes behind the tubes. Within the velocity boundary layer is torn because of the pressure evolving negatively at the tube area, large shear stresses occur. As a result, the first vortex cannot persist and emergence of an additional cell is observed when increasing Reynolds number. As a result, the flow and this disequilibrium is the source of instabilities. Fundamental studies on oscillatory convection are expected to clarify the general mechanism of oscillatory convection. Moreover, the deterministic chaos theory is used to the stability analysis in this paper. The maximum Lyapunov numbers are calculated in order to reflect the chaotic degrees.

KEY WORDS: Periodically, Fully Developed Flow, Cross-Flow Tube Bundle, chaos, maximum Lyapunov number

1. INTRODUCTION

Convection in cross-flow around tube bundle has been widely studied theoretically and experimentally over the past five decades because of the important usage in heat exchangers. The heat transfer characteristics for tube arrangements are of practical interest. Both experimental and computational studies have been conducted for the flow and convection in cross-flow around tube bundle. Most of the earlier studies were experimental in nature to show the flow and heat transfer characteristics with different Reynolds numbers (Re) [1-4]. Numerical methods have also been used to simulate the heat transfer in cross flow of pure gases [5-7] and with particles suspended in gas flow without heat transfer [8, 9]. Moreover, Launder et al.[10] , Fujii et al. [11] , Dhaubhadel et al. [12] , Wung et al. [13] ,Murray et al. [14] presented numerical solutions of local heat transfer for the tube bank problem for a wide range of longitudinal and transverse pitches, Reynolds and Prandtl numbers. Detailed flow fields, temperature fields and the distribution of Nusselt number are obtained by developing mathematical models and simulate methods in different thermal boundary conditions [15-19].

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Most studies aforementioned were mainly focus on flow and heat transfer of different Reynolds numbers or size radios. While, beyond these experimental or numerical investigations, rich transient phenomena were captured, which has attracted theoretical attention of nonlinear characteristics in order to clarify the mechanism of the transitions of the flow and heat transfer patterns. Many scholars concern such nonlinear characteristics on heat transfer field in recent years. Kong et al. [20, 21] proposed the high order finite difference method to calculate the heated side at the bottom of the cavity and studied the bifurcation and chaos problem on convection in porous media. Mizushima et al. [22] have calculated the bifurcation diagram of the two different solutions under the assumption of the two-dimensionality and clarified the origins of the change of the flow patterns due to the saddle-node bifurcation for small aspect ratios. Namely, they found the convection which had a saddle-node point in addition to the crescent-shaped convection. It should be noted that, in general, the crescent-shaped convection can be attained from natural initial conditions, while the convection with the saddle-node point can only be realized under the particular conditions and has less physical reality than the crescent-shaped convection. In fact, Labonia et al. [23] as well as Powe et al. [24] have recently performed an experimental study of the transition from steady to chaotic flow in a horizontal annulus for a small aspect ratio and showed an oscillating flow pattern passing through the vertical center-plane above a critical value.

Problem occurs in heat exchanges such as tubes vibration and instability of heat transfer. Yang et al. [25] conducted a numerical calculation of periodically fully developed convective heat transfer in horizontal sweep bundle. He found that all the calculation condition had nothing to do with the time, but under certain parameters, the result was oscillating in flow field and temperature field. Zhao et al. [26] developed the periodically convective heat transfer of horizontal sweep tube bundle system of numerical calculation, and pointed out that the solution existed bifurcation. There is not special research that focuses on the instability degree.

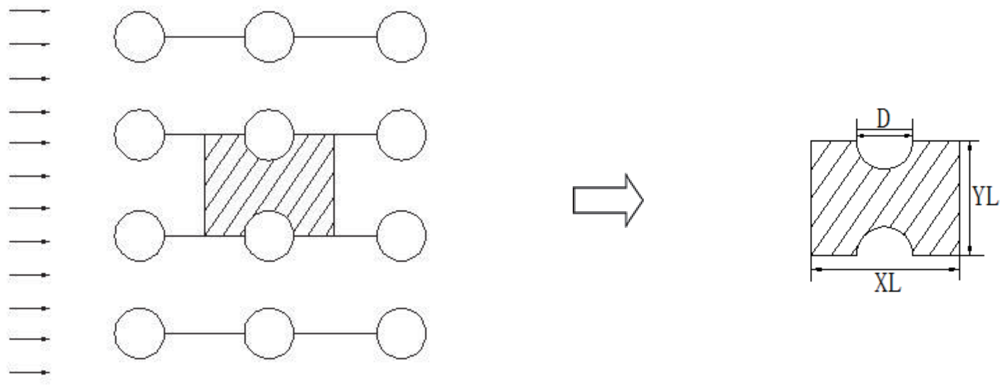
The objective of this article is to numerically simulate convection heat transfer in periodically fully developed flow cross tube bundle. The velocity at the sample point will be studied to investigate the effect of the increasing Reynolds numbers on the routes to chaos. Nonlinear stability theory is applied to explain the phenomenon appeared at the flow pattern by calculating the maximum Lyapunov numbers which reflects instability degree quantitatively. The effects of Reynolds numbers on route to chaos and the study of the periodic, the quasi-periodic, and the chaotic regimes will be investigated in detail by observe change tendency of the maximum Lyapunov numbers. The study will be based on the solution of the unsteady two-dimensional (2D) convection by using a finite volume method under the assumption of fully-developed-periodic flow. The stepped boundary technique is used to approximate the tube surface. Special attention will be paid to the treatment of the tubes, which can be regarded as two isolated islands, and an integrative solution method [27] will be used to solve the problem of isolated islands.

2. PROBLEM STATEMENTS AND GOVERNING EQUATION

2.1 Problem description Fig.1 (a) shows the model of membrane economizer in boiler at power plant. The advantage of this design is to prevent the accumulate ash. It is not normally possible to construct a mesh large enough to describe the gross motion of the fluid within the entire heat exchanger, and fine enough to capture the boundary-layer detail around individual tubes, Spalding [28, 29]. A single typical tube is therefore considered with the flow taken as being fully-developed-periodic deep within the bank [30]. One set of periodic area is taken out to simplify the model as shown in Fig.1 (b). The tube is kept at a constant temperature T_w, while the surrounding fluid interacting with the tubes is labelled as T_b. Tube diameter is D (8cm). The length of periodic model is XL (26cm) and width is YL (15cm).

The following assumptions are made,

- The fluid is assumed to be air (Pr=0.7), incompressible and Newtonian.



(a) (b)

Fig. 1 Physical model

2.2 Problem statement The transient dimensionless conservation equations governing the transport of mass, momentum, and energy in primitive variables are expressed as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\frac{\partial U}{\partial F}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}) \tag{2}$$

$$\frac{\partial V}{\partial F}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}) \tag{3}$$

$$\frac{\partial \Theta}{\partial F}+U\frac{\partial \Theta}{\partial X}+V\frac{\partial \Theta}{\partial Y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}(\frac{\partial^2 \Theta}{\partial X^2}+\frac{\partial^2 \Theta}{\partial Y^2}) \tag{4}$$

Where the dimensionless variables are defined as

$$F=\frac{tu_R}{D},\quad X=\frac{x}{D},\quad Y=\frac{y}{D},\quad U=\frac{u}{u_R},\quad V=\frac{v}{u_R} \tag{5}$$

$$P=\frac{p}{\rho u_R^2},\quad \Theta=\frac{T-T_w}{T_{b,x}-T_w},T_{b,x}=\frac{\int_0^A \rho u c_p T dA}{\int_0^A \rho u c_p dA},\mathrm{Re}=\frac{u_R D}{\nu} \tag{6}$$

Where ν is dynamic viscosity.

2.3 Boundary and initial conditions Faghri et al. [19] predicted fluid flow and heat transfer characteristics in inline tube banks under the assumption of periodic, fully developed flow and used the stepped boundary technique to approximate the tube surface. So in this paper, the left and right boundaries as shown in Fig.1 (b) are set to periodicity conditions as follows.

$$U(X,Y)=U(X+XL,Y) \tag{7}$$

$$V(X,Y)=V(X+XL,Y) \tag{8}$$

$$\Theta(X,Y)=\Theta(X+XL,Y) \tag{9}$$

The top and bottom boundary conditions are set to wall characteristic as follows.

$$Y=0,V=0,U=0,\frac{\partial \Theta}{\partial Y}=0 \tag{10}$$

$$Y=YL,V=0,U=0,\frac{\partial \Theta}{\partial Y}=0 \tag{11}$$

The two tubes are regarded as two isolated islands as shown in Fig.1 (b). The velocities at the grid point in the solid regions should always approximately equal to zero (1×10^{-15}), and the temperatures of the internal tubes part are always 1. The algebraic equations resulting from the control volume approach are written as

$$a_p\phi_p=\sum a_{nb}\phi_{nb}+b \tag{12}$$

To set the desired value of ϕ_p equal to B, one can let $a_p = A$ and $b = A \cdot B$ in Eq. (12), where A is a very large number. The value of ϕ_p obtained from Eq. (12) becomes

$$\phi_p = (\sum a_{nb}\phi_{nb} / A) + A \cdot (B / A) = B \tag{13}$$

This method can also be applied in the intern al region to the momentum equation to obtain zero velocity by setting $B = 1 \times 10^{-15}$. After these treatments aforementioned, the value of U, V and Θ in tube areas are

$$U = 1 \times 10^{-15}, V = 1 \times 10^{-15}, \Theta = 1 \tag{14}$$

Initial conditions are as follows.

$$F = 0, U = 0.33, V = 0, \Theta = 0 \tag{15}$$

3 NUMERICAL METHOD

The transport equations were discretised on structured control volumes using a second-order QUICK-type scheme [31] for the convection terms and by employing the SIMPLE [32] pressure correction algorithm. The fully implicit scheme was used for the discretization. The energy and momentum equations were solved by alternating direction implicit method [33], which leads to a tri-diagonal matrix that was solved with the tridiagonal matrix algorithm. Inner iterations are used to account for the nonlinearity of the equations in every time step.

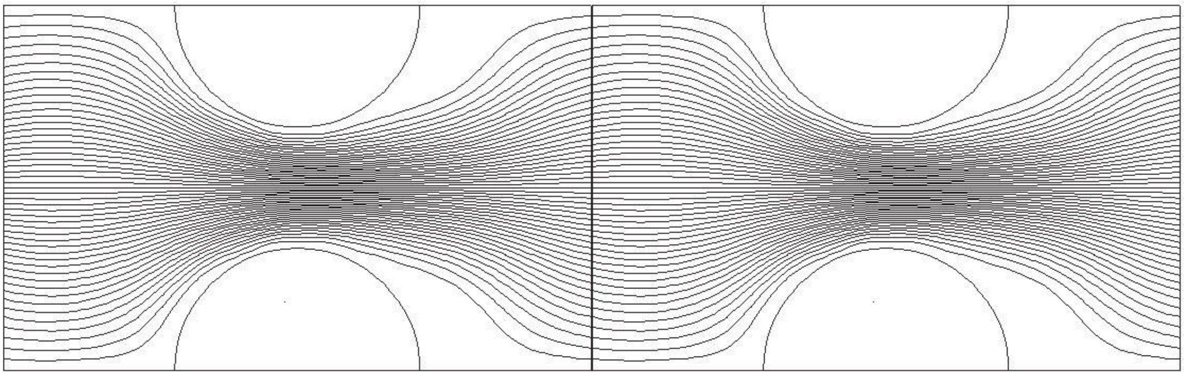
Uniform grid in the X and Y directions was used for all computations. In order to obtain grid independent solution, a grid refinement study is performed. In the present work, six combinations (80×50,120×70,182×122,244×162,260×180,400×250) of control volumes are used to test the effect of grid size on the accuracy of the predicted results. An excellent review of experimental investigations for heat transfer from tube banks in cross flow is given in Zukauskas [2].After compared convergence of average Nusselt number with experimental formula summarized by Zuauskas, grid independence is achieved with combination of (244×162) control volumes where difference is 2.06%.The experimental formula adapt in aligned tube bundles as follows.

$$Nu_f = 0.52 \mathrm{Re}_f^{0.5} \mathrm{Pr}_f^{0.36} (\mathrm{Pr}_f / \mathrm{Pr}_w)^{0.25}, \mathrm{Re} = 10^2 \sim 10^3 \tag{16}$$

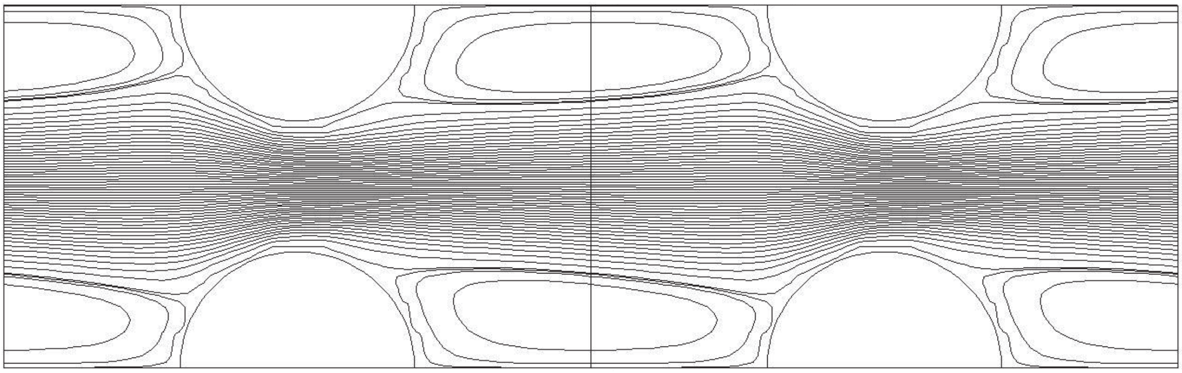
4. RESULTS AND DISCUSSION

4.1 basic features of streamlines and isotherms After plenty numerical simulation at the Re number varied from 1 to 500, typical flow patterns are selected to show in Fig.2 at Re=95,100,195,275,300,400,500. For the case of Re=95, the numerical simulation revealed that the steady symmetrical velocity and temperature field can be reached from the initial uniform velocity field as shown in Fig.2 (a). This shows that although the mathematical equations (Eq. 2, 3) are unsteady ($\frac{\partial U}{\partial F}$), the results have won the steady-state solutions that speed do not change with time. The unsteady (time step equal to 0.001) numerical solutions have the same results compared with which of steady. When the Reynolds numbers are greater than 100, vortexes are appeared at the two sides of the tubes. Attention is especially paid to the appearance of vortexes size. Big vortexes appear at Re=100, and the vortexes become smaller at Re=195. Rise the Re number continually. When Re=275 and 300, big vortexes are observed, and small vortexes appear at Re=400 similar as the patter when Re=175 as depicted by Fig.2 (c) and (f) respectively. Big vortexes are captured at Re=500, which is similar as the pattern when Re=300,275,100 as shown in Fig.2 (g), (e), (d), (b). The oscillation reason is that if the Re is low, cohesive force and inertial force keep balance, so the flow pattern is steady as shown in Fig.2 (a). When raise the Re number, pressure is increased, which tears the viscous boundary layer of tubes. These leads to big vortexes appear as shown in Fig.2 (b). Pressure increases continually, which squeeze the vortexes to small size as shown in Fig.2 (c) when Re number rises at 195. The velocity of mainstream increases when augment Re number continually, which lead the mainstream doesn't expand to the tube area. The stagnation zone is largened so that enlarges vortexes as depicted by Fig.2 (d), (e).

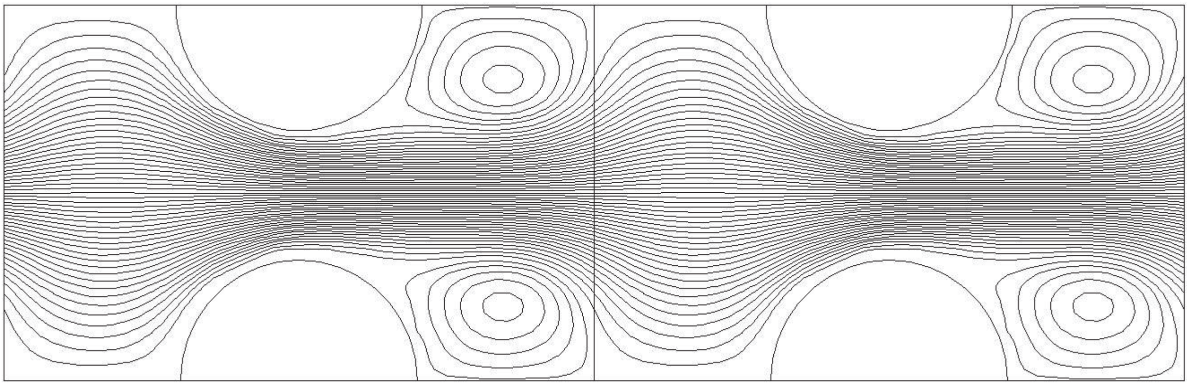
Increasing Re numbers continually, the vortexes shrink again as shown in Fig.2 (f). The process is repeated again and again.



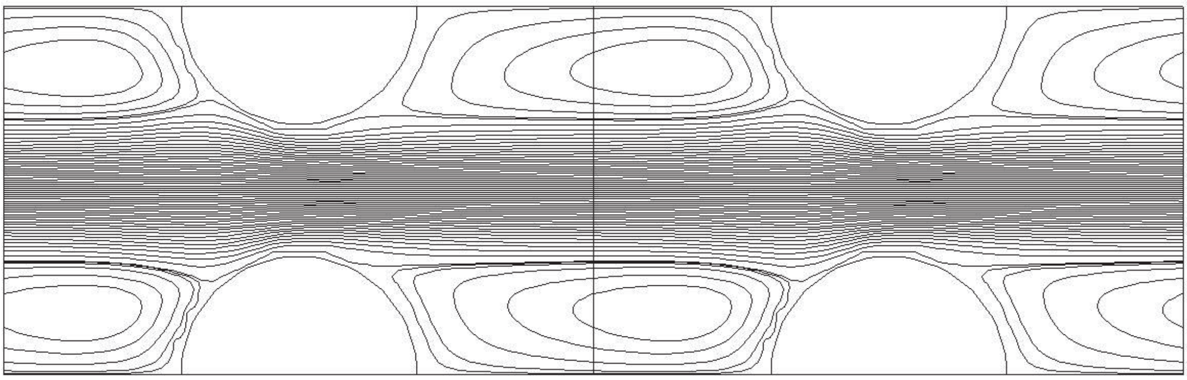
(a) Re=95



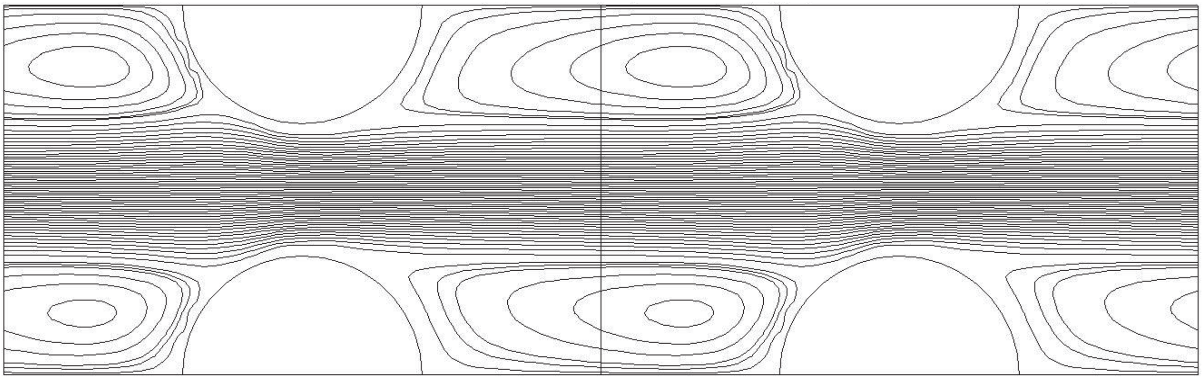
(b) Re=100



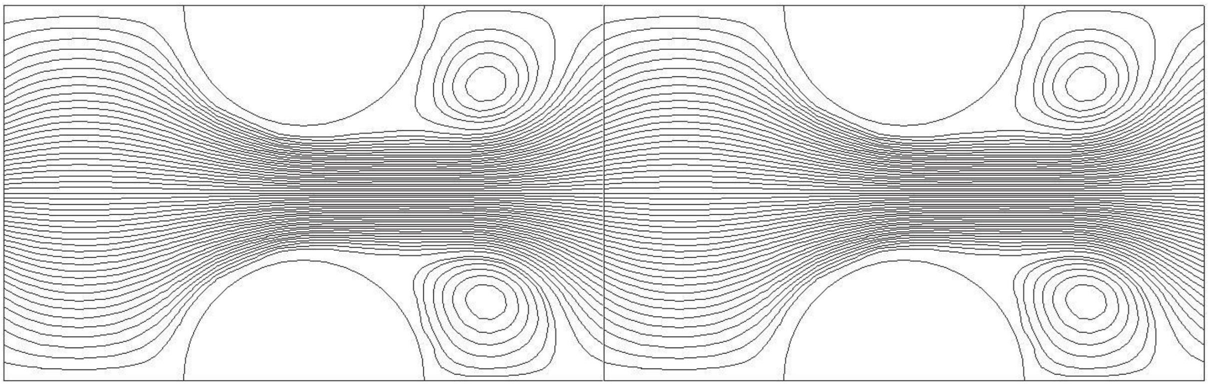
(c) Re=195



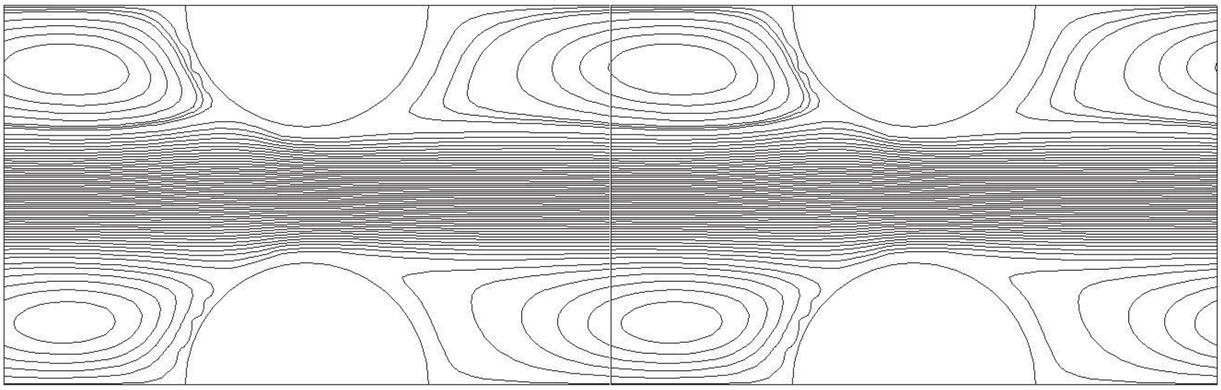
(d) Re=275



(e) Re=300



(f) Re=400



(g) Re=500

Fig. 2 Streamlines for Re=95,100,195,275,300,400,500 respectively

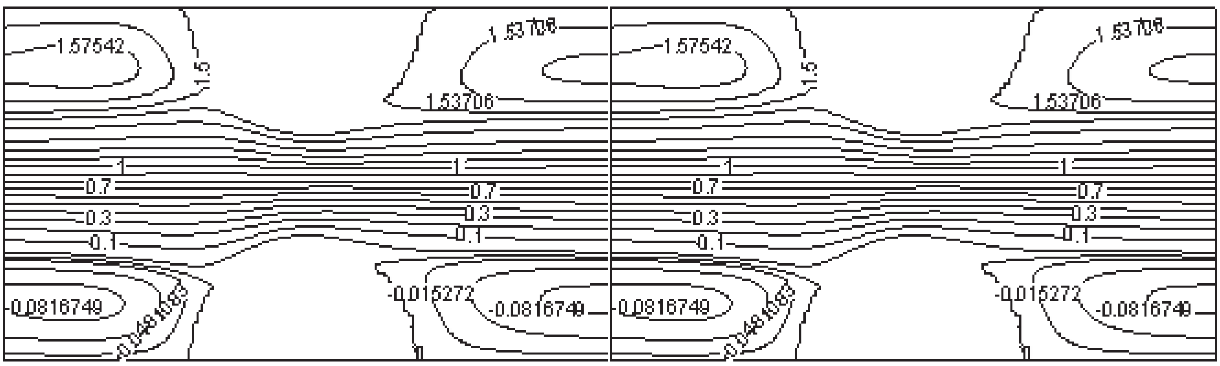


Fig. 3 Streamlines with values for Re=275

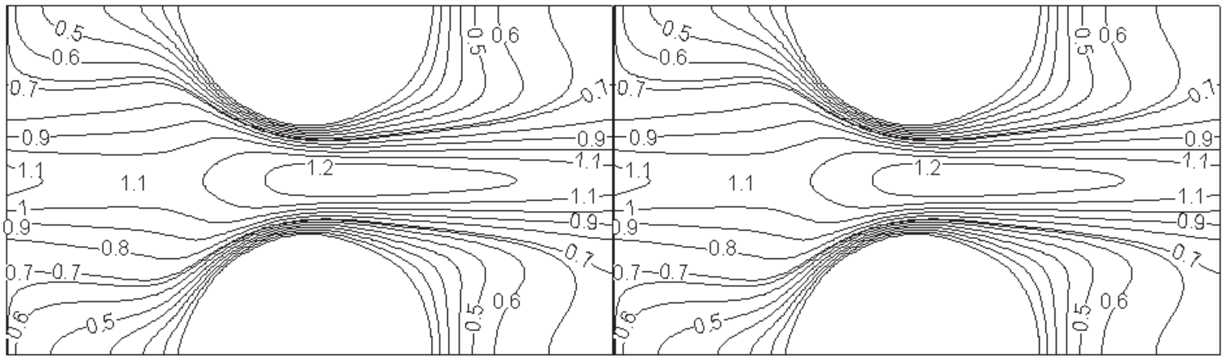


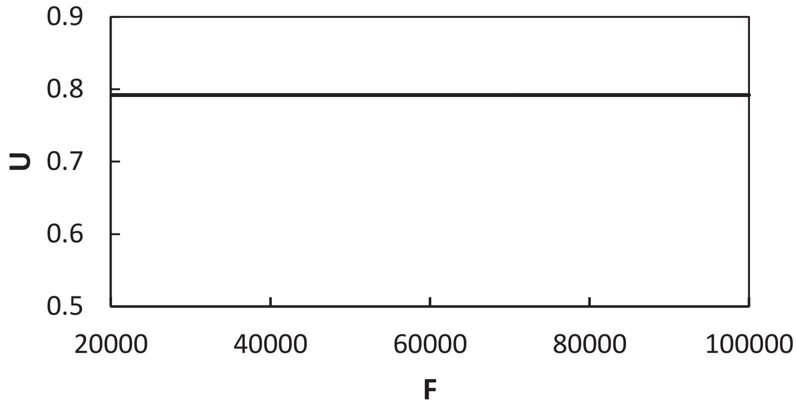
Fig. 4 Contour lines of Θ with values for Re=275

Streamlines and contours lines of Θ with values at Re=275 are described in Fig.3, 4. Clockwise whirlpools are caught at the both sides of the upper tube because the contour value is positive (1.57542). Vortexes contra-rotate at the both sides of the under tube because the contour value is negative (-0.0816749) as shown in Fig.3. It should be noted that the contours lines of Θ plot is closely related to these flow pattern. Since vortexes rotate, the thermal lines have an approximate phase. The thermal contours turn up at the both sides of the upper tube and some other contours at the bottom corners turn down seen as in Fig.4. The fluid vortexes entrainment leads the thermal contours to corners.

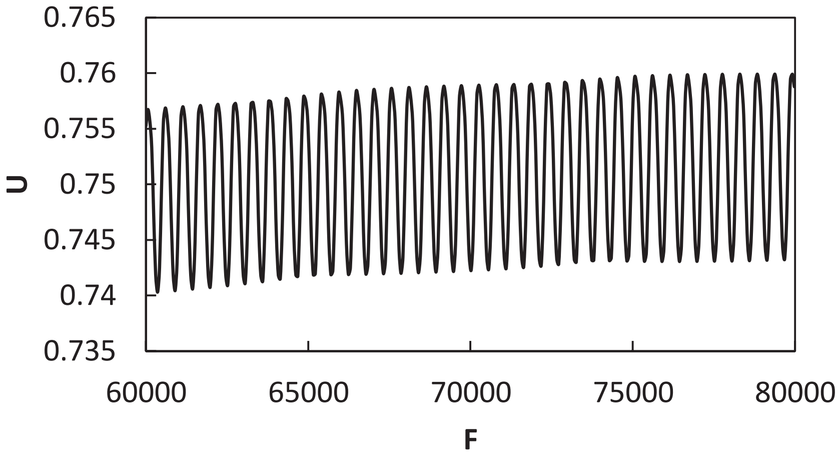
4.2 Nonlinear characteristics The oscillation phenomenon of numerical solution for this scope is now analysed. A simulation of convection from the uniform initial velocity and temperature fields is performed for Re=95,175,500. When time infinities are sufficiently long, steady, periodic and chaos solutions are obtained as shown in Fig.5 (a), (b), (c) respectively. Steady solutions show stable tendency that physical parameters such as U almost do not change with time increasing as shown in Fig. 5(a). The characteristic of periodic solutions appear regular transformation accompany with time marching as shown in Fig. 5(b). Chaos solutions present disordered tendency as shown in Fig. 5(c), where U changes unregularly as time developing.

For the case of Re=175, the left point of (180, 60) as the monitor point were selected to describe the periodic oscillations of numerical solutions. If the solutions were symmetric, the values of U at the monitor point should have only one peak value. However, this is not the case, as seen in Fig.5 (b). With dimensionless time marching, positive peak and negative peak appear alternately as is shown in Fig.5 (b). At Re=175, the periodically oscillated flow occurs as can be seen in Fig.5 (b). At Re=500, intensely oscillated flow occurs as can be seen in Fig.5(c).

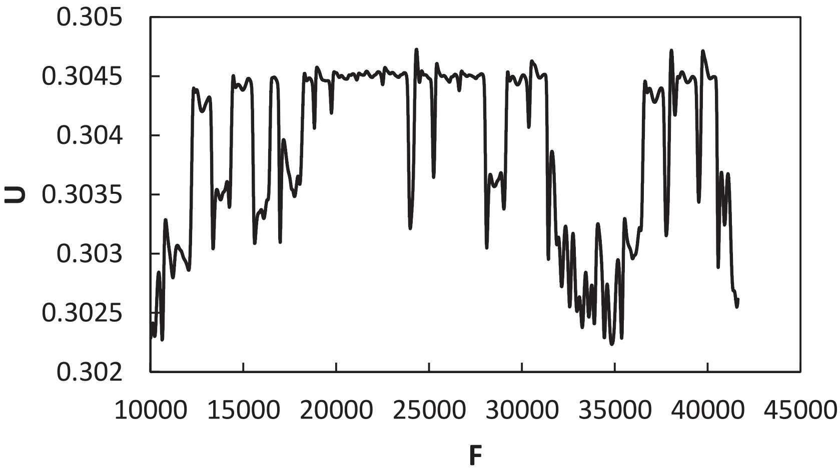
The numerical solutions of velocity at the sample point (180, 60) were obtained for several Reynolds numbers from a low value to 500. The sample point is selected in order to analyse the nonlinear characteristics of the complicated system further. There is no special reason to select a particular point since if the system is periodic it will be so at every point in the area. However, some points near boundary or in the middle may not be suitable due to small amplitude. As the system progresses towards chaos, more peaks occur as shown in Fig.5 (c).The results show that the oscillatory flow undergoes several bifurcations and ultimately evolves to a chaotic flow after the first bifurcation.



(a)Re=95



(b)Re=175



(c)Re=500

Fig. 5 Time variation of U(180,60) at monitor point for Re=95,175,500 respectively

The Lyapunov number is a good digital feature to describe the complexity chaotic characteristic. For a nonlinear dynamic system, if its maximum Lyapunov number is a positive number, then can decide the system is chaotic [34]. If the maximum Lyapunov number is negative real number, then it is a stationary state. So the calculation of maximum Lyapunov number is especially important. Small amount of data method is a simple method to calculate maximum Lyapunov number, the calculate process of which is shown as follows.

For a given time series x_t, t=1,1,…,n, if the time delay between the phase space reconstruction for τ , and the embedding dimension is m, then get phase space reconstruction is $X_i = \{x_i, x_{i+\tau}, ..., x_{i+(m-1)\tau}\}$, $i = 1, 2, ..., m$, and $n = m + (m-1)\tau$. For two points recently, both the spacing can be expressed as

$$d_j(0) = \min \left\| X_j - X_{\hat{j}} \right\|, \left| j - \hat{j} \right| \succ p \tag{17}$$

After a time step i, X_j and $X_{\hat{j}}$ separate at the exponential rate, label as $d(i) \approx C_j e^{\lambda(i\Delta t)}$, λ_{max} as the maximum Lyapunov number, Δt as the Sampling interval, C_j as the initial separation, taking logarithms on both side of Formula3,

$\ln d(i) \approx \ln C_j + \lambda \ln(i \Delta t)$, j=1, 2, … , m, For each i, we take $\ln d(i)$ the average, note the average value as s(i), such as formula,

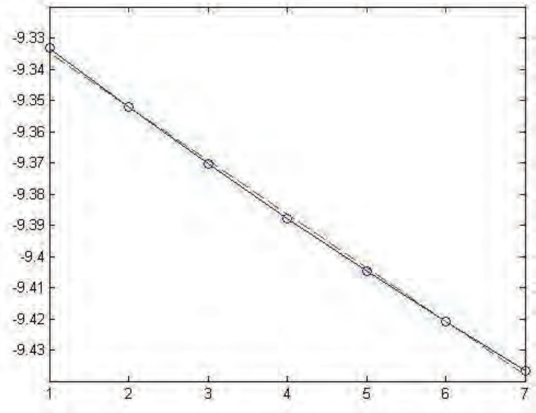
$$S(i) = \frac{1}{k\Delta t} \sum_{j=1}^{k} \ln d_j(i) \tag{18}$$

Among them, the k is the number of $d_j(i)$, fitting curve of s (i) and i, then the slope of the straight line is the Lyapunov number as shown in Fig.6. Maximum Lyapunov numbers is calculated through the periodic oscillation solutions of numerical experiment data by writing the MATLAB program under different Re numbers as shown in Table1. Fig.6 shows Lyapunov number calculating diagram under different Re number. The slop value of the line is just Maximum Lyapunov number. Table1 shows different maximum Lyapunov numbers (label as λ_{max}) and flow states at different Re numbers.

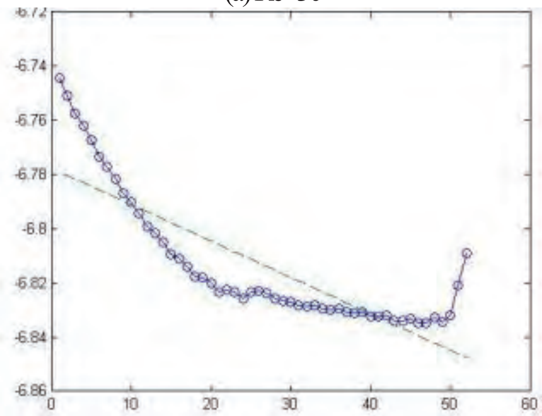
Table 1. Different maximum Lyapunov numbers and flow characteristics at different Re number

Re number	maximum Lyapunov number(λ_{max})	Flow state
50	-0.0172	stationary state
90	-0.0013	quasi-periodic state
100	0.0010	chaos state
120	0.00096	chaos state
140	0.0019	chaos state
160	0.000257	chaos state
180	0.000256	chaos state
200	-0.0039	stationary state
250	-0.0013	stationary state
350	-0.0024	stationary state
450	-0.000688	stationary state

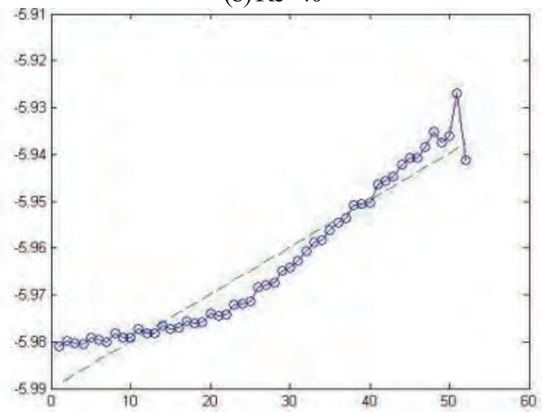
When Re number varies from 50 to 90, the motion type is from stationary state to quasi-periodic state as seen from Table1.When Re number varies from 100 to 180, the flow state enters into chaos, and λ_{max} varies from 0.001 to 0.000256, which means the chaotic degree becomes lighter. Moreover, When Re number varies from 200 to 450, the state changes into stationary state, and the stationary degree becomes lighter. As Wu [35] reports that When the Lyapunov number changed from positive to negative, it means period window starting. On the contrary, when Lyapunov number changed from negative to positive, it does not mean the period window end, and perhaps there is another time of chaos area in the period window. From Table1 we can see, the system exits periodic window in the chaos area, and the starting point of periodic window is exist at Re=200.



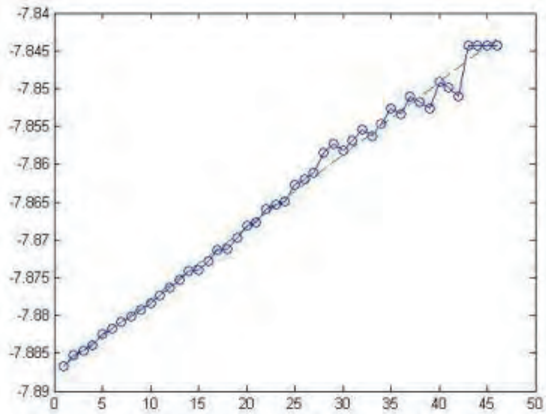
(a) Re=30



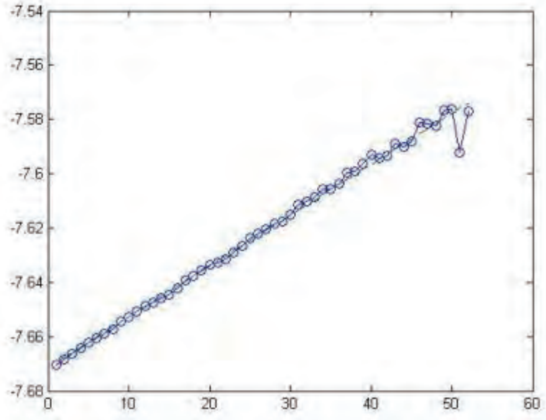
(b) Re=40



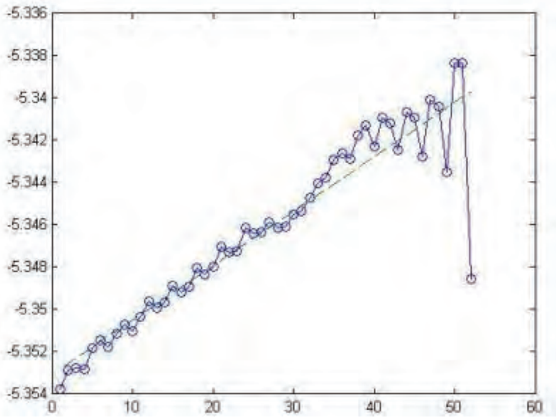
(c) Re=50



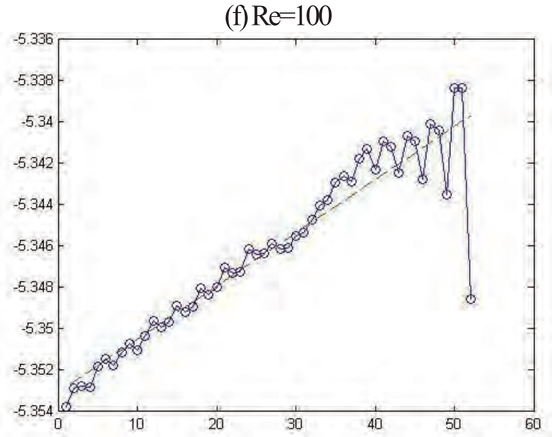
(d) Re=55



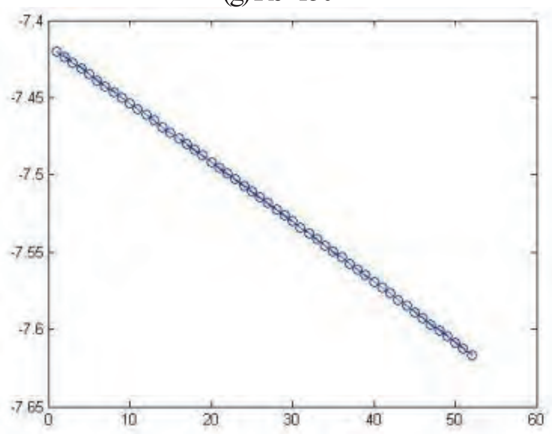
(e) Re=75



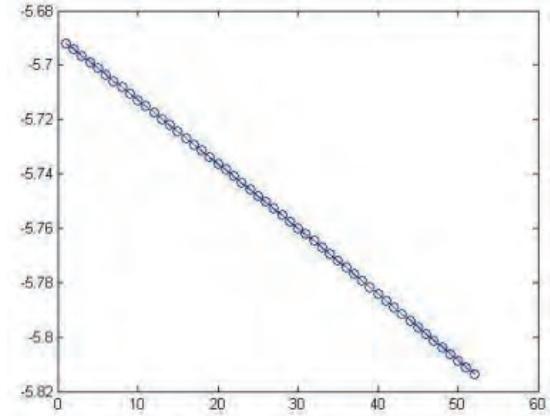
(f) Re=100



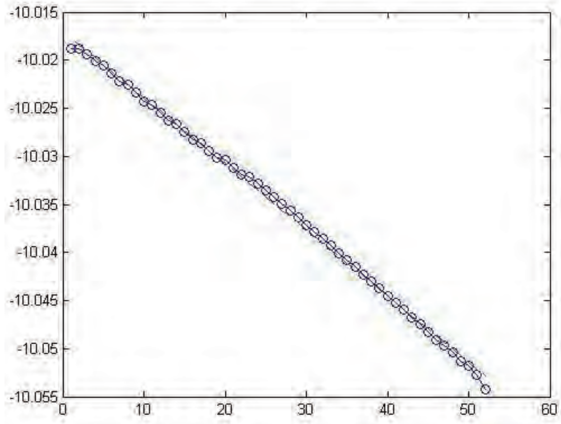
(g) Re=150



(h) Re=200



(i) Re=350



(j) Re=450

Fig.6 Fitting curve of s (i) and i for Re=30, 40,55,75,100,150,200,350,450 respectively

5. CONCLUSIONS

Numerical analyses of periodically fully developed convective heat system in cross-flow tube bundle have been presented. Governing equations are solved using the SIMPLE algorithm with QUICK scheme. Periodic boundary condition is set to the in and out flow, while wall boundary condition is set to up and down boundaries. The tube parts have been regarded as the isolated solid regions, and the integrative solution method is used to solve the problem of isolated islands. We can get the following conclusion:

(1) Flow and heat transfer characteristics are investigated for a Reynolds number range of 5 to 500, while the Prandtl number is taken to be 0.7.
(2) When the Reynolds numbers are below 95, the system is steady. For the cases with Reynolds numbers vary 100 to 500, the convection turns to asymmetric periodical solution at after a short time, and finally developed into non-periodic oscillations. These led to Karman vortex.
(3) The maximum Lyapunov numbers are calculated to show a transition to the determinist chaos degree with increasing Reynolds numbers.
(4) The motion exit periodic window in the chaos area and the starting point of periodic window exists at Re=200.

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NOMENCLATURE

Nu	Nusselt number	x_t	time series

Re	Reynolds number	*P*	dimensionless pressure
Pr	Prandtl number	Θ	dimensionless temperature
Pe	Peclet number	T_b	tube wall temperature
h	Heat transfer coefficient of the fluid [W/(m^2·K)]	T_w	fluid temperature
U,	dimensionless velocity X direct	*F*	dimensionless time
V	dimensionless velocity of Y direct	*f*	resistance coefficient
x, y	Cartesian coordinates	λ	Lyapunov number
X,Y	dimensionless coordinates	$\lambda_{\max}$	The maximum Lyapunov number

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