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STUDY ON THE ROUTE TO CHAOS OF NATURAL CONVECTION IN CYLINDRICAL ENVELOPE WITH AN INTERNAL CONCENTRIC CYLINDER WITH SLOTS

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ABSTRACT

Natural convection heat transfer was investigated numerically in a cylindrical envelope with an internal concentric cylinder with slots. Governing equations are discretized using finite volume method and solved using SIMPLE algorithm with QUICK scheme. Calculations were performed on certain parameters with a Rayleigh number varying from 700 to 20000. The effect of the Rayleigh number on the route to the chaos of the system was analyzed by the phase space of velocity at the sample point. The results show that the system can reach to steady state and symmetric when the Rayleigh number is below 700, and to steady state and asymmetric when the Rayleigh number is equal to 1000. For a Rayleigh number ranged between 1500 and 3000, an asymmetric periodical solution is obtained although the initial field and boundary conditions were symmetric. As the Rayleigh numbers increase further, a quasi-periodic solution of the system is achieved at Ra=2000. There is one more bifurcation and period doubling at successive critical values of Rayleigh numbers from to. It is ascertained that periodicity is lost at Ra=20000. The results show that the oscillatory flow undergoes several bifurcations and ultimately evolves to a chaotic flow.

INTRODUCTION

Natural convective heat transfer in horizontal annuli has attracted many attentions in the past owing to the number of practical applications such as heat transfer and fluid flow in nuclear reactors, thermal storage systems, electrical transmission cables, and and electronic component cooling among others. A voluminous literature exists on experimental and theoretical work, which is thoroughly reviewed in the articles by Morgan et al. (1975) and Churchill et al. (1975). Kuehn et al. (1976) carried out a two-dimensional investigation covering the range of Rayleigh numbers from pure conduction to the laminar boundary layer regime. Rao et al. (1985) investigated the transient oscillatory phenomena and the numerical determination of the critical Rayleigh number at which unsteady flow occurs. Moukalled and Acharya (1996) studied numerically natural convective heat transfer from a heated horizontal cylinder placed concentrically inside a square enclosure. The governing equations in their work are solved in a body fitted coordinate system using a control volume based numerical procedure. Takata et al. (1984) performed an analytical and experimental investigation of natural convection in an inclined cylindrical annulus with a heated inner and cooled outer cylinder. Their results revealed the existence of a co-axial double helical flow pattern inside the cavity. Powe et al. (1969, 1971) visualized the flow patterns with smoke as a tracer and obtained the Rayleigh number at which the flow will change from a steady flow to an unsteady flow. Liu et al. (1994) investigated the stability of natural convection by measuring the total heat transfer coefficient and the distribution of radiation temperature of water, air and silicone. The results indicated that the critical Rayleigh number dictates the transition from steady to unsteady.

Natural convective heat transfer in enclosure has been found to produce a wealth of nonlinear phenomena. Recently there was considerable effort to achieve a clear understanding of how the nonlinear bifurcations are mapped in the space spanned by the two control parameters. It has been shown that the oscillations can follow a period doubling route to chaos (Hunt E. et al, 1993). Chaos can be defined as a system that diverges exponentially from initial positions varying by a small degree (Erramilli A. et al, 1994). Chaos is usually considered as an unfavorable phenomenon. However, it is very useful under certain situations and there is great interest in introducing chaotic behavior in various applications (Ottino J. M. et al, 1992, Yoo J. S., 2003).

One of the routes to chaos is by period doubling. In this case, the period continues to double until there are no more stable states available. When driven at a frequency near the diode's resonant frequency, the circuits can exhibit periodic behavior. As the driving amplitude is increased, the periodic state becomes unstable. The state divides into two frequencies dependent on the resonance. Further increase in the amplitude results in the splitting of the two periods, giving quadrupling, and finally chaos. Yang et al. (2001) examined the effect of the sinusoidal time varying temperature oscillation of the hot and cold walls on natural convection in a tall vertical enclosure. Kazmierczak and Chinoda (1992) analyzed the effect of frequency and amplitude of oscillating hot wall temperature on the flow and heat transfer in an enclosure. Xia er al. examined the same problem configuration for high Rayleigh numbers and showed the impact of the wall temperature oscillation on the flow stability for a fixed value of frequency, Yoo (2003) investigated numerically the bifurcation sequences to the chaos for the natural convection in horizontal concentric annuli in detail. Mizushima et al. (1995) numerically investigated the bifurcation phenomenon and obtained the critical Rayleigh number when flows changing into multi-vortex.

The nonlinear dynamics of the two dimensional columns also provides a simple fluid dynamics system which is highly attractive from the point of view of bifurcation theory, because it is large enough to provide a rich spatio-temporal dynamics induced by the symmetry in enclosure, but at the same time not as expensive in calculation time as a three dimensional system. In this context, the equations can be written in the form of a two dimensional with symmetric boundary condition. The governing equations are solved in a body-fitted coordinate system using a control volume based numerical procedure. Natural convection heat transfer was investigated numerically in a cylindrical envelope with an internal concentric cylinder with slots by using SIMPLE algorithm with QUICK scheme.

Although many studies have been conducted on numerical calculations of natural convection of horizontal concentric annuli, very few works regarding problem with a slotted inner cylinder have been published. The nonlinear characteristic of natural convection heat transfer will be investigated in a cylindrical envelope with an internal concentric cylinder with slots. Our main attention for this study was to describe the routes to the chaos and to investigate the periodic, the quasi-periodic and the chaotic regimes.

NOMENCLATURE

a thermal diffusivity, m^2/s
F dimensionless time
g gravitational acceleration, m/s^2
Keq average dimensionless equivalent thermal conductivity of whole cylinder
K relative thermal conductivity
k_s thermal conductivity for solid, W/m k
k_f thermal conductivity for fluid, W/m k
L gap width, m
p pressure, Pa
P dimensionless pressure
Pr Prandtl number
Q heat transfer rate of whole circle, W
ri radius of slotted inner circle, m
ro radius of envelope circle, m
R dimensionless radial coordinate
Ra Rayleigh number
S relative slot width
Ti temperature of slotted inner circle, ^oC
To temperature of envelope circle, ^oC
u tangential velocity, m/s
U dimensionless tangential velocity
UR characteristic velocity, m/s
V dimensionless radial velocity
α thermal diffusivity
β coefficient of thermal expansion, 1/ ^oC
Γ nominal diffusion coefficients in the momentum equations
Γ_T nominal diffusion coefficients in energy equation
δ thickness of slotted inner circle, m
ϕ angle
Θ dimensionless temperature
θ angular coordinate
ν kinematic viscosity, m^2/s
ρ density, kg/m^3
τ time, s

PROBLEM FORMULATION

Consider natural convection heat transfer in an annulus with an internal slotted cylinder. The system is shown schematically in Figure 1. The inner and outer cylinders are maintained at constant and different temperatures, Ti and To , respectively. As a result of the temperature difference between the two circles, density gradients occurs and leads to natural convection. It is assumed that the fluid in the enclosure is of Boussinesq type, and the fluid flow and heat transfer is two-dimensional and laminar.

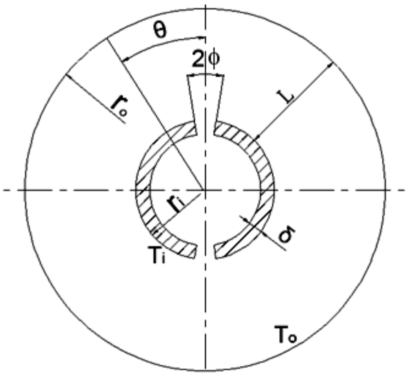


Figure 1 Diagram of computational geometry

The dimensionless governing equations are:

$$\frac{\partial U}{\partial F}+V\frac{\partial U}{\partial R}+\frac{U}{R}\frac{\partial U}{\partial \theta}=-\frac{1}{R}\frac{\partial P}{\partial \theta}+\Gamma_u[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 U}{\partial \theta^2}]+S_\theta \quad (1)$$

$$\frac{\partial V}{\partial F}+V\frac{\partial V}{\partial R}+\frac{U}{R}\frac{\partial V}{\partial \theta}=-\frac{\partial P}{\partial R}+\Gamma_v[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial V}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 V}{\partial \theta^2}]+S_R \quad (2)$$

$$\frac{\partial \Theta}{\partial F}+V\frac{\partial \Theta}{\partial R}+\frac{U}{R}\frac{\partial \Theta}{\partial \theta}=\Gamma[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Theta}{\partial R}\right)+\frac{1}{R^2}\frac{\partial^2 \Theta}{\partial \theta^2}] \quad (3)$$

$$\frac{\partial V}{\partial R}+\frac{V}{R}+\frac{1}{R}\frac{\partial U}{\partial \theta}=0 \quad (4)$$

The source terms are

$$S_\theta=-\frac{UV}{R}+\Gamma_u\left(-\frac{U}{R^2}+\frac{2}{R^2}\frac{\partial V}{\partial \theta}\right)-\Theta\sin\theta \quad (5)$$

$$S_R=-\frac{U^2}{R}+\Gamma_v\left(-\frac{V}{R^2}+\frac{2}{R^2}\frac{\partial U}{\partial \theta}\right)+\Theta\cos\theta \quad (6)$$

Where the dimensionless parameters are defined as follows

$$R=\frac{r}{L},\quad F=U_R\frac{\tau}{L},\quad \Theta=\frac{T-T_o}{T_i-T_o},\quad P=\frac{p}{\rho U_R^2},\quad U=\frac{u_\theta}{U_R},\quad V=\frac{v_r}{U_R} \quad (7)$$

The dimensionless parameters Rayleigh number (Ra), Prandtl number (Pr), and referenced velocity U_R are defined as

$$Ra=\frac{\beta g L^3(T_i-T_o)}{a\nu},\quad \Pr=\frac{\nu}{a},\quad U_R=(Ra\Pr)^{1/2}\frac{a}{L}. \quad (8)$$

Where β , α and ν are thermal expansion coefficient, thermal diffusivity and kinematic viscosity of the fluid, respectively is the gap width of annulus, g is the acceleration due to the gravity.

The nominal diffusion coefficients Γ and Γ_T in the momentum and energy equations are respectively defined as

$$\Gamma=\frac{\Pr}{(Ra\Pr)^{1/2}},\quad \Gamma_T=K\frac{1}{(Ra\Pr)^{1/2}} \quad (9)$$

where $K=k/k_f$ is the relative thermal conductivity, which takes the value of one in the fluid region and k_s/k_f in the solid region.

The boundary conditions are as follows:

$$R=R_i,\quad U=V=0,\quad \Theta=1. \quad (10)$$

$$R=R_o,\quad U=V=0,\quad \Theta=0. \quad (11)$$

The periodical boundary conditions are given at $\theta=0$ and $\theta=2\pi$

$$U(\theta=0,R)=U(\theta=2\pi,R) \quad (12)$$

$$V(\theta=0,R)=V(\theta=2\pi,R) \quad (13)$$

$$\Theta(\theta=0,R)=\Theta(\theta=2\pi,R) \quad (14)$$

Apart from above boundary conditions, the following two additional conditions must be satisfied

$$R_i-\delta/L\le R\le R_i,\quad \gamma\le\theta\le\pi-\gamma,\quad U=V=0,\quad \Theta=1. \quad (15)$$

The initial condition is

$$F=0,\quad U=V=\Theta=0. \quad (16)$$

To observe the total heat transfer effect, average dimensionless equivalent thermal conductivity based on the whole outer circle is defined as

$$K_{eq}=\frac{Q}{2\pi k_f(T_i-T_o)}\ln\frac{r_o}{r_i} \quad (17)$$

where Q is the total heat transfer rate.

The slot width is defined as

$$S=2\alpha/\pi \quad (18)$$

NUMERICAL PRODURE

Present analysis is based on upon the control volume method to discretize the governing non-dimensional equations (Patankar, 1980). Staggered grid procedure was used in primitive variables with a Quick differencing scheme for the convection terms. To handle the pressure, temperature and velocity coupling, the SIMLPE algorithm was utilized. The energy and momentum equations were solved by Alternating Direction Implicit (ADI) method. ADI leads to a tri-diagonal matrix which was easily solved with the Tri-diagonal Matrix Algorithm (TDMA). The internal concentric slotted cylinder can be satisfied at the grid points in the isolated solid regions.

A 100×50 grid in $\theta \times R$ coordinates was used for the calculation domain forming the cylindrical envelope. The non-uniform grid was chosen, the grid lines being more closer packed in the slotted place and an internal concentric slotted cylinder. Figure 2 shows an example for the grid system used in this study.

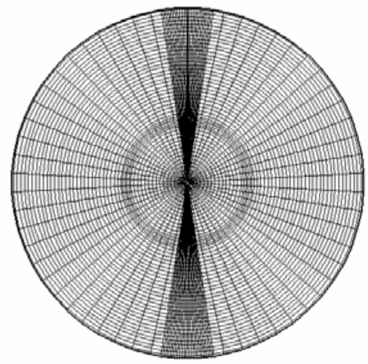


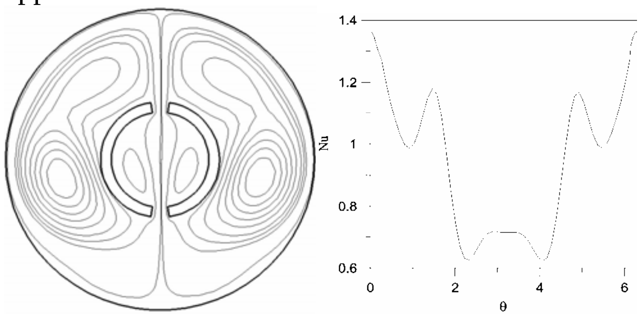
Figure 2 Sample of computational mesh

RESULTS AND DISCUSSION

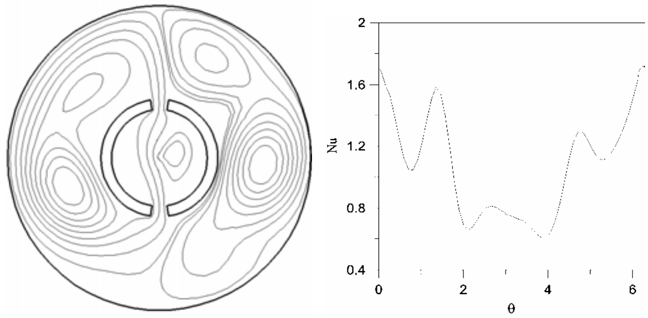
Calculations were performed to simulate the evolution of the natural convection heat transfer developing from the zero initial fields with a time step 10^{-2}. The parameters used in this paper are: $r_o / L = 1.625$, $r_i / L = 0.625$, $\delta / L = 0.1125$, $\Pr = 0.01$, $s = 0.0573$.

The numerical solutions were obtained for several Rayleigh numbers from a low value to 700. A sample point is selected in order to analysis the characteristics of periodic oscillations. There is no special reason to select particularly it since if the flow is periodic it will be so at every point in the enveloped cylinder. However, some points near boundary or in the middle may not be suitable due to small amplitude.

For the case of Ra≤700, the numerical investigation revealed that the unsteady process approached steady symmetrical field. The streamline and temperature field of such solution were shown in Figure 3 (a). But it can be seen from Figure 3 (b) that the solutions of streamline and temperature field can reach to steady and asymmetric. This limit point losses its stability and allows the periodic solution to appear at Ra=2000.



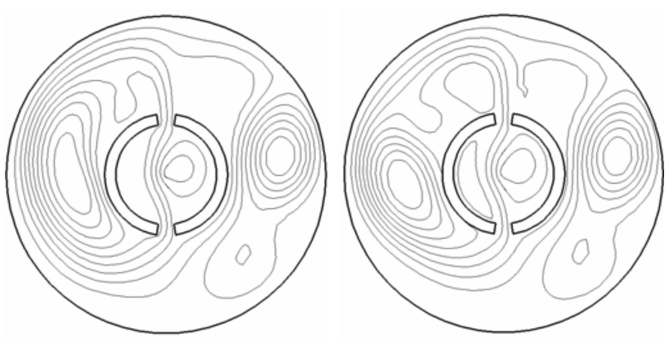
(a)



(b)

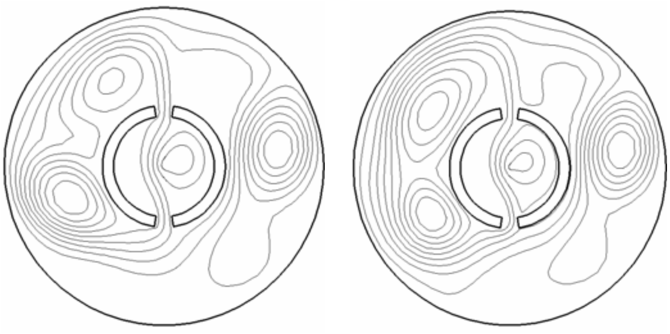
Figure 3 Steady streamline and temperature field for (a) Ra=700 and (b) Ra=1000.

A simulation of the natural convection from the zero initial fields was performed for Ra=2000. When time approaches infinite, a periodical solution was obtained. It can be shown in Figure 4 and Figure 5 that the flow pattern and temperature change at different steps of a cycle. The maximum value of Keq appear at the dimensionless time F=402.00 and F=404.01 respectively, and the minimum are at F=403.01 and F=405.65. From the whole point of view, the streamline and isotherm are of single periodicity, and the oscillating period is 3.65.



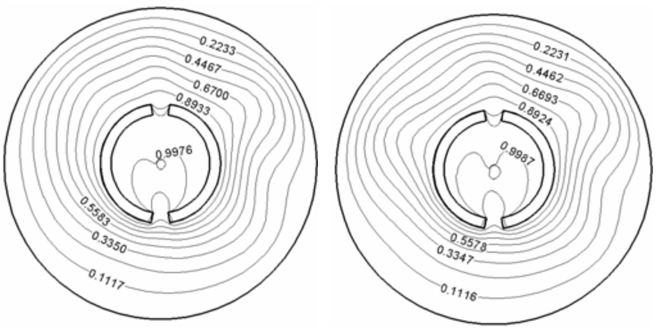
(a) F=402.00, 407.28 (b) F=403.01

equal to 1000. For a Rayleigh number ranged between 1500 and 3000, an asymmetric periodical solution is obtained although the initial field and boundary conditions were symmetric. As the Rayleigh numbers increase further, a quasi-periodic solution of the system is achieved at Ra=2000. There is one more bifurcation and period doubling at successive critical values of Rayleigh numbers from to. It is ascertained that periodicity is lost at Ra=20000. The results show that the oscillatory flow undergoes several bifurcations and ultimately evolves to a chaotic flow.

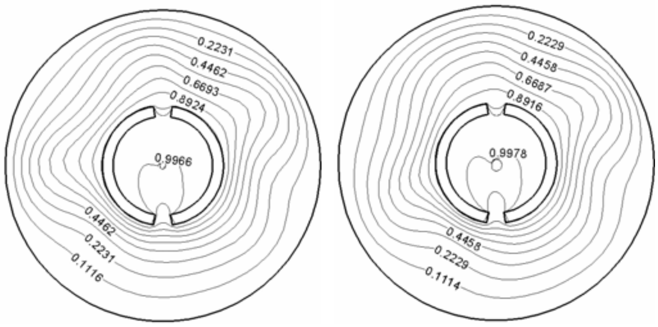


(c)F=404.01 (d) 405.65

Figure 4 Periodic evolution of streamlines in a cycle at Ra=2000



(a)F=402.00, 407.28 (b) F=403.01



(c) F=404.01 (d) 405.65

Figure 5 Periodic evolution of isotherms lines in a cycle at Ra=2000.

Figure 6 and 7 indicate that four different numerical solutions are achieved with the increasing Rayleigh numbers. There is one more bifurcation and period doubling at successive critical values of Rayleigh numbers from 700 to20000. It is ascertained in Figure 9 that periodicity is lost at Ra=20000. The results show that the oscillatory flow undergoes several bifurcations and ultimately evolves to a chaotic flow after the first HOPF bifurcation.

CONCLUSION

Calculations were performed on certain parameters with a Rayleigh number varying from 700 to 20000. The effect of the Rayleigh number on the route to the chaos of the system was analyzed by the phase space of velocity at the sample point. The results show that the system can reach to steady state and symmetric when the Rayleigh number is below 700, and to steady state and asymmetric when the Rayleigh number is

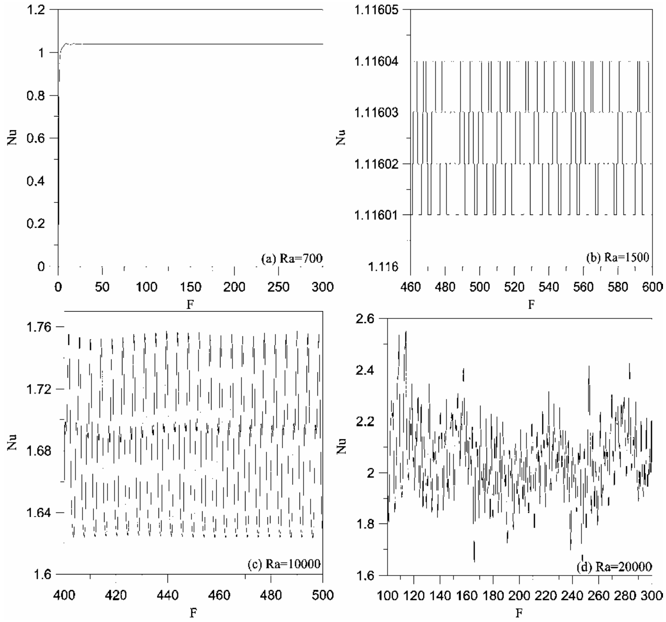


Figure 6 Time signal of Nusselt number at different Rayleigh numbers.

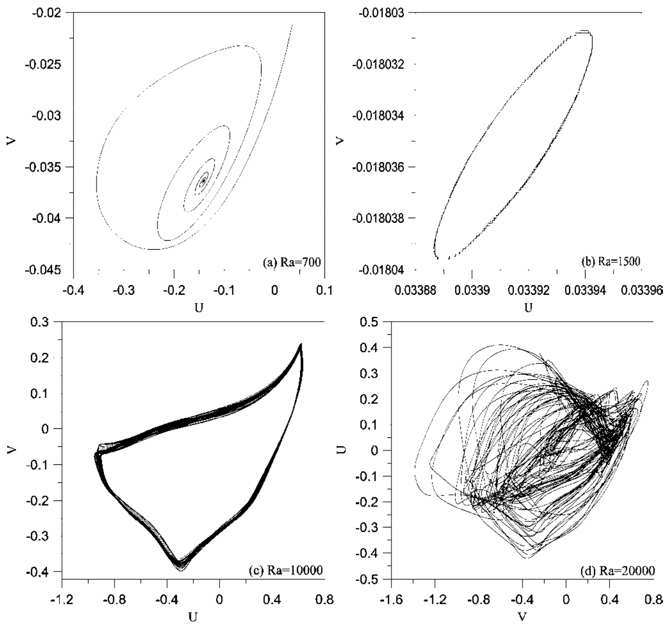


Figure 7 Velocity time trace at different Rayleigh numbers.

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