Freezing of Cylindrically Shaped Foodstuffs with Boundary Condition of the Third Kind

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Abstract

In this paper,integral approximate method was used to analyze the heat transfer process during freezing of a cylindrically shaped foodstuffs. By comparing with the experimental data of potato sample, it is shown that the results obtained by integral approximate method have a better agreement with the experimental results than the results obtained by improved enthalpy method in [3].

1. INTRODUCTION

Freezing of foodstuffs is widely used in food processing. From the viewpoint of heat transfer, freezing of foodstuffs is a conduction process with phase change. The rate of freezing, freezing time, temperature distribution, are all of prime importance both to the quality of products and the correct design of refrigeration equipment . So it is necessary to determine these parameters with reasonable accuracy.

The problem of freezing of foodstuffs is one kind of typical Neumann problems. Because of the strong nonlinearity of the problem, exact solutions exist only for a few very simple cases of Stefan problem [1,2]. For most practical problems, approximate method or numerical method is often used. Freezing of long cylindrically shaped foodstuffs has been investigated by applying an improved enthalpy method by Cui[3], his calculation was relatively complex. Furthermore, agreement between the calculated results and experimental results is not good enough. In the present paper, integral approximation method is applied to analyze this problem . The nonlinear equation was transformed into two ordinary differential equations, so the calculation becomes much simpler. Moreover, the results obtained by the present method have a better agreement with the experimental results than the calculated results obtained by Cui[3].

Integral approximate method is used to solve this problem. It is necessary to divide the freezing process of foodstuffs into three phases(precooling, freeziug, tempering), then to solve these phases in succession. Furthermore, precooling phase is divided again into the process before and the process after the thermal penetration layer reaches the geometric center of foodstuffs.

2. MATHEMATICAL DESCRIPTION OF THE PROCESS

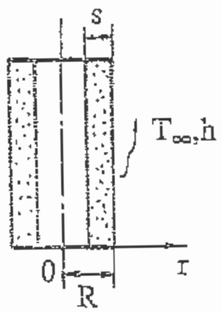


Fig.1 physical model

Consideration is given to the physical model shown in Fig.1. A long cylindrically shaped foodstuffs is initially at a superheating temperature $T_i(T_i > T_m)$. From the time $t=0$, surrounding fluid, which is at the temperature T_∞ ($T_\infty < T_m$), flows across the foodstuffs suddenly. Due to the external cooling of forced convection, freezing takes place inside the foodstuffs. The following analysis aims at the heat transfer during the entire freezing process,and the different phases experienced during the freezing process of the foodstuffs are analyzed in succession.

(1) PRECOOLING PHASE

As the initial temperature of the foodstuffs T_i is higher than its freezing point T_m, the foodstuffs will not be solidified until its surface temperature reaches T_m. Therefore a period of precooling is needed. During this period the process is simply a conduction problem (without phase change) with boundary condition of the third kind.

(a) BEFORE THE THERMAL PENETRATION LAYER REACHES THE GEOMETRIC CENTER OF FOODSTUFFS ($\eta_c < 1$).

Assuming the temperature distribution in the foodstuffs to be a quadratic form, from $\partial\theta / \partial\eta|_{\eta=\eta_c} = 0$; $\theta|_{\eta=\eta_c} = \theta_i$, and $\theta|_{\eta=0} = \theta_s$, one can obtain the temperature distribution

$$\theta = \begin{cases} \theta_s + a_1\eta - \dfrac{a_1^2}{4(\theta_i - \theta_s)}\eta^2, & 0 < \eta < \eta_c \\ \theta_i, & \eta_c < \eta < 1 \end{cases} \tag{1}$$

from the dimensionless boundary condition at $\eta = 0$, $\dfrac{\partial\theta}{\partial\eta} = Bi(1+\theta)$,one obtains

$$a_1 = Bi(\theta_s + 1) \tag{2}$$

Integrating the heat diffusion equation with respect to η between $(0,\eta_c)$, and considering Eq.(1), one can obtain the variation of surface temperature θ_s with dimensionless time τ

$$\tau = -Ste[\frac{\Phi}{a_1}|_{\theta_s=\theta_i}^{\theta_s=\theta_s} + \int_{\theta_i}^{\theta_s} \frac{Bi}{a_1^2} \cdot \Phi \cdot d\theta_s] \tag{3}$$

where

$$\Phi = \frac{\theta_s}{2} + \frac{a_1}{6} + \frac{a_1}{24}\left[\frac{(1-\eta_c)^4 - 1}{\eta_c}\right] \tag{4}$$

$$\eta_c = \frac{2(\theta_i - \theta_s)}{a_1} \tag{5}$$

(b) AFTER THE THERMAL PENETRATION LAYER REACHES THE GEOMETRIC CENTER OF THE FOODSTUFFS.

Approximating the temperature profile in the foodstuffs as a quadratic equation and applying the conditions of $\partial\theta / \partial\eta|_{\eta=1} = 0$ and $\theta|_{\eta=0} = \theta_s$, one obtains the temperature distribution

$$\theta = \theta_s + b_1\eta - \frac{b_1}{2}\eta^2 \tag{6}$$

from the dimensionless condition at $\eta = 0$, $\frac{\partial\theta}{\partial\eta} = Bi(1+\theta)$, b_1 can be expressed as

$$b_1 = Bi(\theta_s + 1) \tag{7}$$

Integrating the heat diffusion equation with respect to η between (0,1) using Eq.(6), one obtains the relation between surface temperature θ_s and time τ

$$\tau = \tau^* - Ste\left[\int_{\theta_s^*}^{\theta_s} \frac{1}{2b_1} d\theta_s + \frac{1}{8} \ln b_1 \Big|_{\theta_s = \theta_s^*}^{\theta_s = \theta_s}\right] \tag{8}$$

τ^* and θ_s^* time and surface temperature when the thermal penetration layer reaches the geometric center of the foodstuffs.

(2) FREEZING PHASE

With the going of the cooling process, when surface temperature reaches the freezing point T_m, the foodstuffs begin to freeze. Since the foodstuffs have experienced the precooling period before freezing, certain initial non–uniform temperature distribution must exist in the foodstuffs when freezing begins. Thus the problem becomes a two–region problem, when solving this problem, the cylindrically shaped foodstuffs should be separated into two regions,the frozen region and the non–frozen region. The dimensionless mathematical description of the problem is

Frozen region:

$$K_a Ste \frac{\partial\theta_1}{\partial\tau} = \frac{1}{1-\eta}\frac{\partial}{\partial\eta}\left[(1-\eta)\frac{\partial\theta_1}{\partial\eta}\right], \quad 0 \leqslant \eta \leqslant S, \tau > \tau_f \tag{9}$$

$$\frac{\partial\theta_1}{\partial\eta} = BiK_r(\theta_1 + 1), \quad \eta = 0, \tau > \tau_f \tag{10}$$

Non–frozen region:

$$Ste\frac{\partial\theta_2}{\partial\tau}=\frac{1}{1-\eta}\frac{\partial}{\partial\eta}[(1-\eta)\frac{\partial\theta_2}{\partial\eta}],\quad S\leqslant\eta\leqslant1,\tau>\tau_f \tag{11}$$

$$\frac{\partial\theta_2}{\partial\eta}=0,\quad \eta=1,\tau>\tau_f \tag{12}$$

Frozen–non–frozen interface:

$$\theta_1=\theta_2=0,\quad \eta=S,\tau>\tau_f \tag{13}$$

$$\frac{\partial\theta_1}{\partial\eta}-K_r\frac{\partial\theta_2}{\partial\eta}=K_r\frac{dS}{d\tau},\quad \eta=S,\tau>\tau_f \tag{14}$$

Initial condition:

$$\theta=Bi(\eta-\frac{\eta^2}{2}),\quad 0\leqslant\eta\leqslant1,\tau=\tau_f \tag{15}$$

Then the problem described above will be solved with integral approximate method. Assume the temperature distribution of the non–frozen region has the form

$$\theta_2=d_1(\eta-S)+d_2(\eta-S)^2,\quad S\leqslant\eta\leqslant1 \tag{16}$$

where $d_1$and $d_2$are functions of τ, it satisfies Eq.(13) automatically. From Eq.(12), one can derive

$$d_1=-2d_2(1-S) \tag{17}$$

Integrating Eq.(11) with respect to η between [S,1], one obtains

$$\frac{dP(\tau)}{P(\tau)}=\frac{-8}{Ste(1-S)^2}d\tau \tag{18}$$

where

$$P(\tau)=-d_2(1-S)^4 \tag{19}$$

From the initial condition Eq.(15), $P(\tau_f)=Bi/2$. Assume the temperature distribution of frozen region is of the form

$$\theta_1=C_1[1-\frac{ln(1-\eta)}{ln(1-S)}]+C_2[1-\frac{ln(1-\eta)}{ln(1-S)}]^2 \tag{20}$$

it satisfies Eq.(13) automatically. then from Eq.(10)

$$\frac{C_1+2C_2}{ln(1-S)}=BiK_r(1+C_1+C_2) \tag{21}$$

From Eqs.(9),(13),(14), C_2 can be expressed

$$C_2=\frac{-K_aSte}{2K_r}C_1[C_1-K_rd_1(1-S)ln(1-S)] \tag{22}$$

Substituting Eq.(22) into Eq.(21), we get

$$C_1 = \frac{K_r}{2K_a Ste}[B - \sqrt{B^2 - \frac{8K_a SteBiln(1-S)}{2 - BiK_r ln(1-S)}}] \tag{23}$$

where

$$B = K_a Sted_1(1-S)ln(1-S) + \frac{2 - 2BiK_r ln(1-S)}{2 - BiK_r ln(1-S)} \tag{24}$$

Substituting Eqs.(16) and (20) into Eq.(14)

$$\frac{dS}{d\tau} = \frac{C_1}{K_r}\frac{1}{(1-S)ln(1-S)} - d_1 \tag{25}$$

and

$$d_1 = \frac{2P(\tau)}{(1-S)^3} \tag{26}$$

the initial condition is $S(\tau_f) = 0$

The frozen–non–frozen interface location can be calculated by solving the simultaneous Eqs.(18) and (25) with the Runge–Kutta method. Then the time needed for frozen and the temperature distribution in the foodstuffs can be obtained.

(3) TEMPERING PHASE

The foodstuffs will be completely frozen when the frozen–non–frozen interface reaches the geometric center of the cylindrically shaped foodstuffs($S = 1$). If the foodstuffs continue to be cooled, this problem becomes a conduction problem without phase change. Assuming the temperature distribution in the foodstuffs is of a quadratic form and applying the conditions of $\partial\theta / \partial\eta|_{\eta=1} = 0$ and $\theta|_{\eta=0} = \theta_s$, the temperature distribution can be expressed

$$\theta = \theta_s + e_1\eta - \frac{e_1}{2}\eta^2 \tag{27}$$

from the dimensionless condition at $\eta = 0$, $\frac{\partial\theta}{\partial\eta} = BiK_r(1+\theta)$, e_1 can be expressed

$$e_1 = BiK_r(\theta_s + 1) \tag{28}$$

The relation between surface temperature θ_s and time τ is

$$\tau = \tau' - K_a Ste[\int_{\theta'_s}^{\theta_s} \frac{1}{2e_1} d\theta_s + \frac{1}{8} lne_1 |_{\theta_s = \theta'_s}^{\theta_s = \theta_f}] \tag{29}$$

where τ' Time when the foodstuffs are frozen completely.
θ'_s $\theta'_s = -BiK_r / (2+BiK_r)$

3. RESULTS AND DISCUSSION

The calculation is carried out for a sample of potato, which is taken as homogeneous. The properties of potato were determined in accordance with [4].

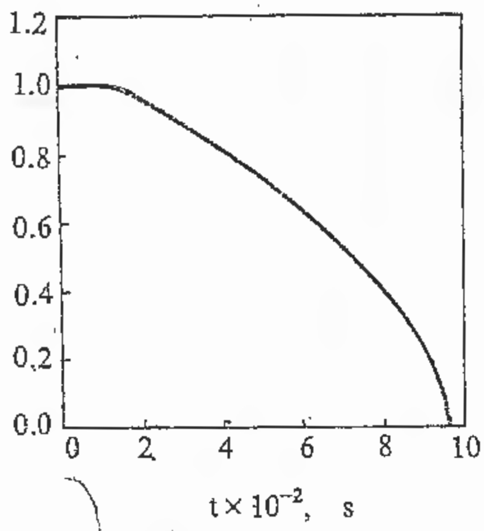


Fig.2 The frozen–non–frozen interface

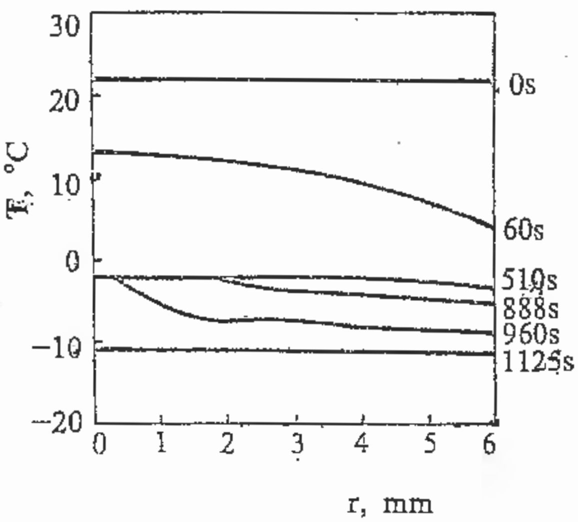


Fig.3 Temperature distribution of the foodstuffs

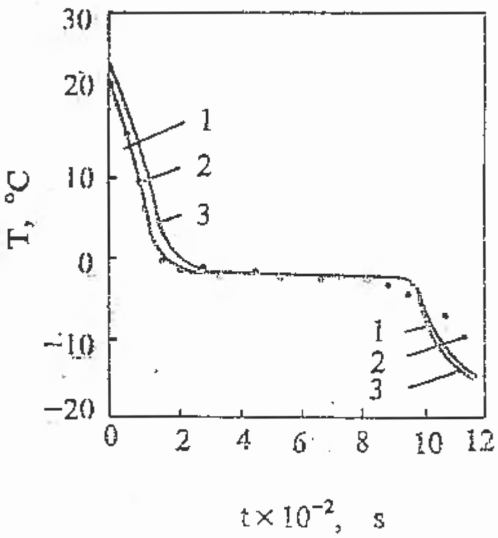
$$\psi_1 = p\psi_{w1} + (1-p)\psi_d \tag{30}$$

$$\psi_2 = p\psi_{w2} + (1-p)\psi_d \tag{31}$$

where ψ indicates specific heat capacity c or thermal conductivity k. c_d and k_d represent specific heat capacity and thermal conductivity of the dry solid, while c_{w1}, $k_{w1}$and c_{w2}, k_{w2}indicate thermal properties of ice and water. A good choice of values for the above mentioued set of properties is as follows [4]: $c_{wl}=4187$ J / kg · K, $c_{ws}=2093$ J / kg · K, $c_d=1256$ J / kg · K, $k_{wl}=0.59$W / m · K, $k_{ws}=2.44$ W / m · K, $k_d=0.26$ W / m · K. $p=0.8$ is the mass fraction of water in potato [6].The latent heat effect can be evaluated as: $\lambda=p\lambda_w$, in which $\lambda_w=335$kJ / kg is the heat of fusion / solidification of water. In order to compare the results predicted by the present method with the experiuental data [3], parameters used in our calculation are the same as in [3]: radius of the cylinder 6mm,initial temperature 21°C, environment temperature −16°C, convection heat transfer coefficient h 80.1W / (m^2 · K), freezing temperature −2°C.

The frozen–non–frozen interface location with time τ is shown in Fig.2. The flat part indicates the precooling period. During this period, the temperature of the foodstuffs drops continuously, but as long as its temperature is higher than the freezing point, it will not be frozen. When surface temperature of the foodstuffs reduces to the freezing point, the foodstuffs begin freeze. The frozen–non–frozen interface moves toward the geometric center of the foodstuffs. From Fig.2, we can see that the interface location moves faster and faster. It is due to the area of the interface becoming smaller and smaller as the frozen–non–frozen interface moves inward.

Temperature distribution within the foodstuffs at various time during the freezing process is shown in Fig.3.



1: Results by the present authors
2: Data of experiment [3]
3: Results predicted by Cui[3]

Fig.4 Time–temperature variation at the geometric center

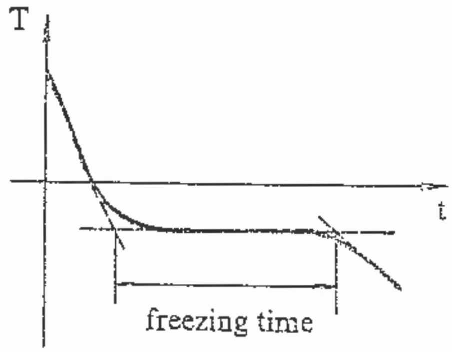


Fig.5 Definition of freezing time

As for freezing of foodstuffs, the time–temperature variation at its geometric center is of prime importance, and is shown in Fig.4. As compared to the experimental data in [3],it is obvious that the temperature history predicted by the present integral method is better than that from [3],especially during the precooling and the tempering phase.

During the tempering phase, there exists relatively larger difference between the calculating results and the experimental data. It is because during freezing process, water will be solidified and removed, thus water content in the non–frozen part of the foodstuffs declines continuously, which leads to the drop of freezing point, but in the present calculation,the freezing temperature is assumed to be constant, which is responsible for the difference.

Flat part of the curve in Fig.4 stands for freezing time(Fig.5)[5], which is an important parameter affecting the frozen quality of foodstuffs. It is obvious that the freezing time determined by the present method (curve 1 in Fig.4) is more accurate than that by Cui (curve 3 in Fig.4).

Conclusively, in this paper,a simple but reliable method is presented for freezing of cylindrically shaped foodstuffs, which can be used to predict the freezing process of foodstuffs with fair accuracy.

NOMENCLATURE

α	thermal diffusivity
Bi	Biot number, hR / k_2
c_p	specific heat
h	convection heat transfer coefficient
h_{sf}	latent heat
k	thermal conductivity
K_α	α_2 / α_1
K_r	k_2 / k_1
r	coordinate
R	radius of the cylindrically shaped foodstuffs
r_c	thermal penetration layer
s	frozen–non–frozen interface
S	dimensionless frozen–non–frozen interface, s / R
Ste	Stefan number, $c_{p1}(T_m - T_\infty) / h_{sf}$
t	time
T	temperature
θ	dimensionless temperature, $(T-T_m) / (T_m - T_\infty)$
η	dimensionless coordinate, $(R-r) / R$
η_c	dimensionless thermal penetration layer, $(R-r_c) / R$
τ	dimensionless time, $Ste\alpha_1 t / R^2$
τ_f	dimensionless precooling time

SUBSCRIPT

i	initial
f	freezing point
s	surface
1	frozen region
2	non–frozen region
∞	environment

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