Forced-Convection Heat Transfer of Microencapsulated Phase-Change Material Suspensions Flow in a Circular Tube Subject to External Convective Heating

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A numerical model for laminar forced-convection heat transfer of microencapsulated phase-change material (PCM) suspensions in a circular tube with external convection boundary condition is presented in this paper. Melting in the microcapsule was solved by using a temperature-transforming model and convection of the suspension was solved using a finite difference method. The results showed that heat transfer of PCM suspensions is controlled by the external convective conditions, as well as the latent heat of PCM and concentration. This research extends the capability of effectively evaluating the performance of engineering heat transfer devices based on PCM.

Key words: Heat transfer; Phase-change material; Suspensions; Numerical model; Convection

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1. INTRODUCTION

It is a promising new technology to utilize phase-change material (PCM) in thermal energy storage and control systems. In this method, the PCM is microencapsulated and suspended in a heat transfer fluid to form a phase-change suspension. The latent heat of the PCM particles provides an approach to store energy and enhance heat transfer when the particles experience phase change. The heat transfer problem of PCM suspensions in a circular tube was experimentally investigated by Goel et al. [1994]. Their results showed that the use of a suspension of n-eicosane microcapsules in water could reduce the rise in wall temperature by up to 50% as compared to a single-phase flow. These results agreed qualitatively with the theoretical prediction of Charunyakorn et al. [1991], but quantitative agreement was not good. Zhang and Faghri [1995] developed a numerical model, by accurately simulating the energy balance of the PCM particles and including the effects of the finite range of melting temperature and the initial subcooling, and achieved very good agreement between their modeling prediction and the experimental results of Goel et al. [1994].

Recently, characteristics and performances of microencapsulated PCM slurry flow in circular tubes have become a subject that raises extensive interest in the enhanced heat transfer community. For example, Hu and Zhang [2002] presented a mathematical model incorporating the effect of temperature-dependent specific heat in the phase-change temperature range; Ho et al. [2004] demonstrated the significant role of the tube wall conduction based on a numerical model.
PCM suspensions flow under turbulent conditions was also investigated [e.g., Roy and Avanic, 2001; Alvarado et al., 2007].

In the previous PCM suspension flow studies, it was widely assumed that the tube wall is subject to a condition of constant heat flux, for this is easy to achieve and can be accurately controlled in the laboratory. However, in the potential applications, the convective condition or boundary condition of the third kind is often encountered. For example, when PCM suspensions are used in various heat exchangers and energy storage devices, the PCM suspension may exchange energy across the tube wall with an external convective fluid. Therefore, developing numerical models incorporating various boundary conditions is not only helpful in studying the fundamental heat transfer characteristics of PCM suspensions, but also valuable in evaluating engineering devices that utilize PCM in real world technologies. Since boundary conditions of the first and second kinds can be viewed as two extreme cases of a third-kind condition when the heat transfer coefficient approaching infinity or zero [Sparrow and Patankar, 1977], a boundary condition of the third kind is actually a general one that represents various boundary conditions.

2. MODEL DEVELOPMENT

The physical model of the problem is schematically shown in Figure 1. A circular tube with internal radius \( R_d \) and external radius \( R_w \) is heated by the external fluid through convection. The external-convection heat transfer coefficient and fluid temperature are \( h_c \) and \( T_e \), respectively. The overall thermal resistance of the external convection and tube wall conduction can be expressed by

\[
\frac{1}{2\pi R_d h_{eff}} = \frac{1}{2\pi R_c h_c} + \frac{1}{2\pi k_w} \ln \left( \frac{R_w}{R_d} \right)
\]

where \( h_{eff} \) is the effective heat transfer coefficient that accounts for both the external convection and wall conduction effects and \( k_w \) is the thermal conductivity of the tube wall. Solving \( h_{eff} \) from Eq. (1) yields

\[
h_{eff} = \left[ \frac{R_d}{R_c h_c} + \frac{k_w}{k_w} \ln \left( \frac{R_w}{R_d} \right) \right]^{-1}
\]

The flow of PCM suspensions in the tube is assumed to be hydrodynamically fully developed, so that the velocity profile can be expressed by

\[
u = 2u_m [1 - (R/R_d)^2]
\]

Under the similar assumptions as made by Zhang and Faghri [1995], the energy equation of the PCM suspensions flow in the tube and its boundary conditions can be written as

\[
C_p U \frac{\partial T}{\partial x} = \frac{1}{R} \frac{\partial}{\partial R} \left( k_{eff} R \frac{\partial T}{\partial R} \right) + s
\]

\[
\frac{\partial T}{\partial R} \bigg|_{R=R_d} = \frac{h_{eff}}{k_{eff,w}} (T_c - T_w)
\]

\[
\frac{\partial T}{\partial R} \bigg|_{R=R_d} = 0
\]

\[
T \big|_{x=0} = T_i
\]

where \( s \) is the source term associated with heat exchange between PCM particles and the base fluid and \( k_{eff} \) is the effective thermal conductivity of flow slurry, evaluated by Charunyakorn et al. [1991]

\[
k_{eff} = k_b \left( 1 + BcPe^c \right)
\]

where \( c \) is the volumetric concentration of suspended particles and \( k_b \) is the thermal conductivity of static dilute suspensions, evaluated from Maxwell's relation.
with $k_f$ and $k_p$ being thermal conductivities of the base fluid and PCM particles, respectively. Because of the enhancement created by the particle-fluid interactions, the effective conductivity of flow slurry $k_{eff}$ is higher than that of the static suspensions $k_b$. In this study, the constants $B$ and $m$ in Eq. (8) adopt the values determined by Charunyakorn et al. [1991] for different ranges of the particle Peclet number, defined by

$$\Gamma = 1 + Bc \left( \frac{8 \text{Pe}_f r_p^2 \eta}{k_f R_d^2} \right)^m$$

which can be evaluated from the conduction model based on the effective thermal conductivity that includes the effects of molecular diffusion and eddy convection around the particles [Charunyakorn et al., 1991].
The temperature distribution of the bulk flow is not only dependent on external boundary conditions at the tube wall and inlet conditions, but also depends on the melting process of the PCM particles. The energy exchange between the bulk liquid and the PCM particles is indicated by the source term defined in Eq. (17), which clearly shows the coupling nature of the temperature of the bulk fluid $\theta$ and that of the PCM particle $\theta_p$.

The melting process in a particle schematized in Figure 2 is modeled following Zhang and Faghri [1995] using the temperature-transforming model [Cao and Faghri, 1990]. This model has the advantage of eliminating the time step and grid size limitations that are normally encountered in other fixed grid methods. It is assumed that melting takes place over a range of temperatures below $T_m$, where the width of that range is $\Delta T_p$. The dimensionless range of the phase-change temperature is defined as

$$
\varepsilon = \frac{\Delta T_p}{T_c - T_m}.
$$

The discretization form of Eqs. (12)-(15) can be obtained by an implicit finite difference scheme [Patankar, 1988] and solved by the tri-diagonal matrix algorithm (TDMA) method. Since the source term in Eq. (12) depends on the particle temperature solution, as indicated by Eq. (17), we need to solve the temperature distribution of the particles first. Meanwhile, the particle temperature depends on the bulk flow temperature in reverse [Zhang & Faghri, 1995]. Therefore, it is necessary to solve the coupling problems of particle temperature and bulk flow temperature by iteration.

3. RESULTS AND DISCUSSION

A problem with the same geometry and working fluids as in the experiments of Goel et al. [1994] but different boundary conditions is studied. Fully developed flow of water carrying PCM ($n$-eicosane) microcapsules in a circular tube with an internal diameter of 3.14 mm has a Reynolds number of $Re = 200$. The spherical PCM particles have a diameter of 0.1 mm with a crust that made up 30% of the microcapsules volume. The suspension enters the tube with an initial subcooling of $\theta_i = -0.07$, and the melting process of the PCM particles is assumed to be within a phase-
change temperature range characterized by $\varepsilon = 0.4$ [Zhang and Faghri, 1995]. The external convective heating conditions are specified by the heat transfer coefficient and external fluid temperature. Although both the external heat transfer coefficient and external fluid temperature could vary with axial locations in practical engineering applications, they are assumed to be constant in this paper for simplicity. It should be noted that there is no substantial difficulty for the numerical model and code developed in this research to solve a problem subject to axially varying external convective conditions.

Figure 3 shows the variation of the tube wall temperature along the axial direction, comparing the re-

![Graph](image)

**FIGURE 4.** Predicted temperature distributions for a reference model: (a) dimensionless temperature vs. dimensionless axial coordinate; (b) dimensionless temperature vs. dimensionless radial coordinate.
results obtained from boundary conditions of the third kind with that of the second kind, i.e., constant heat flux. For easy comparison, the results are presented with the same dimensionless wall temperatures and dimensionless axial distance along the tube as in Goel et al. [1994] and Zhang and Faghri [1995]. For the convection boundary conditions cases shown in Figure 3, the effective heat transfer coefficient is assumed to be constant (Bi = 1 or 2), i.e., the heat flux across the tube wall $q$ is proportional to the temperature difference between the external fluid and the wall. Since this temperature difference decreases towards the downstream direction as a result of the increasing wall temperature and assumed constant external fluid tem-

![Graph](image)

**FIGURE 5.** Effect of the Biot number: (a) dimensionless wall temperature vs. dimensionless axial coordinate; (b) dimensionless wall and bulk liquid temperatures as a function of the Biot number Bi.
perature (Ste = 1), the heat flux also decreases. Consequently, the wall temperature increases along the x-direction slower under the convection boundary conditions than that under a constant wall flux condition. This explains why the constant flux curve in Figure 3 has a steeper slope.

Figure 4 shows dimensionless temperature distribution as a function of dimensionless axial and radial coordinates for a reference model with Bi = 1, Ste = 1, and c = 0.01. The temperature profiles clearly show the thermally developing process of the PCM suspension flow, from which the thermal entrance length can be estimated.

The effect of the Biot number Bi on the wall and bulk liquid temperatures is shown in Figure 5. Two observations can be made from the figure. First, the

![Figure 6](https://example.com/figure6.png)

**FIGURE 6.** Effect of the Stefan number: (a) dimensionless wall temperature and flux vs. dimensionless axial coordinate; (b) dimensionless wall and bulk liquid temperatures as a function of the Stefan number Ste.
wall temperature increases with Bi, as a result of the increase in the effective external heat transfer coefficient $h_{\text{eff}}$ and thus in heat flux transported into the tube. Second, it is indicated that the boundary condition of the third kind is a general boundary condition, with boundary conditions of the first and second kinds as its extreme cases. From Eq. (11), Bi is defined as a ratio between the external conductance $h_{\text{eff}}$ and the internal conductance ($-k_0/R_d$). When the external thermal contact is superior (Bi $>>$ 1), the wall temperature reaches the external fluid temperature within a very short entrance length, approaching a boundary condition of the first kind. On the contrary, when the external thermal conductance is very poor (Bi $<<$ 1), the

![Graph](image)

**FIGURE 7.** Effect of PCM concentrations: (a) dimensionless wall temperature and flux vs. dimensionless axial coordinate; (b) dimensionless wall and bulk liquid temperatures as a function of PCM concentration $c$. 
wall heat flux is throttled and controlled by the external resistance, which is independent of the internal flow, approaching a boundary condition of the second kind. In the case that a constant heat flux is given, the wall temperature varies slowly and linearly in the flow direction. The extreme trends for both $\text{Bi} > 1$ and $\text{Bi} << 1$ are clearly shown in Figure 5. This is an example showing that a model for a problem with a boundary condition of the third kind can be used for general applications subject to various boundary conditions.

Figure 6 shows the effect of the Stefan number, $\text{Ste}$, which positively relates to the external fluid temperature $T_e$ and inversely to PCM latent heat $L$, as can be seen from Eq. (11). For the whole tube length shown in the figure, the wall temperature, as well as the bulk fluid temperature, increases consistently with increasing $\text{Ste}$. The results shown in Figure 6, in terms of dimensionless quantities, reflect both the effect of PCM latent heat and that of the external fluid temperature. The influence of the PCM latent heat can be readily observed from Figure 6: the higher the latent heat (the smaller the Stefan number), the lower the wall temperature, and the higher the convective heat flux at a wall. On the other hand, if we consider the $T_e$ variation as a result of external fluid temperature variation (keeping the PCM latent heat constant), Figure 6 shows how the wall and bulk temperatures as well as the wall flux increase with the external fluid temperature. When Figure 6 is viewed as results showing the effect of the external fluid temperature, caution should be taken to realize that the dimensionless temperatures and wall flux are scaled by $T_e - T_m$ and thus also functions of the external fluid temperature $T_e$. Consequently, although the dimensionless wall flux decreases with $T_e$, as shown in Figure 6a, the actual dimensional heat flux increases with $T_e$.

The advantages of adding PCM microcapsules in forced-convection flow can be found from Figure 7, in which the effect of PCM particle volumetric concentration $c$ on wall and bulk temperatures is shown. Since the external convection boundary condition is completely specified through the given $\text{Bi}$ and $\text{Ste}$, a lower wall temperature always corresponds to a higher heat flux that is transferred to and carried by the flow in the tube. It is clear from Figure 7 that the temperature of the tube wall is significantly lower in the case of adding 10 percent of PCM particles in the base liquid, as compared with the much lower PCM concentration case ($c = 0.001$), which is essentially identical to a pure fluid flow. Therefore, a PCM suspension flow has a higher energy storage density and experiences smaller temperature variation. These characteristics make the PCM suspensions very attractive for applications in thermal energy storage and transportation.

4. CONCLUSIONS

Forced-convection heat transfer of microencapsulated phase-change material suspensions flow in a circular tube subject to external convective heating is modeled in this paper. This extends the capability of thoroughly investigating the heat transfer characteristics of PCM suspensions and effectively evaluating the performance of real world heat transfer devices based on PCM. In terms of the numerical solution to an illustrative example, effects of the heat transfer coefficient, external fluid temperature as well as PCM latent heat and concentration on the tube wall and bulk liquid temperatures are investigated. The numerical model presented here can be further improved by incorporating radiation boundary condition so that it can be employed in evaluating various devices utilizing PCM suspensions for energy storage and exchange, e.g., solar energy storage devices.

REFERENCES


