Numerical study of periodically fully-developed convection in channels with periodically grooved parts

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Abstract

The periodicity of convection heat transfer in channels with periodically grooved parts is studied numerically using an unsteady-state model. The governing equations are discretized using SIMPLE algorithm with a QUICK scheme. The computational results show that there exists a critical Reynolds number, above which the fluid flow and heat transfer will exhibit time-dependent behavior and will oscillate more frequently at higher Reynolds number. When the oscillation takes place, the instantaneous phenomenon of the convection is not periodically fully developed, but the average behaviors of the convective heat transfer show the characteristics of periodically fully-developed flow and heat transfer.

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1. Introduction

Flow interruption at periodic intervals is a well-known technique for heat transfer enhancement. A commonly used technique is to include periodic disturbance promoters along the streamwise direction. Some examples of this technique are offset fins, louvers, communicating channels, etc. These passages are called periodical channels because their cross-section shapes change periodically. Numerous research works have been carried out to study the convective heat transfer in various periodical channels. For example, Dejong and Jacobi [1] studied the effects of bounding walls on flow and heat transfer in louvered-fin arrays using the naphthalene sublimation technique and complementary flow visualization. Yucel and Dinler [2] investigated the effect of inner fins on laminar and turbulent flow in a pipe. The pressure drop and heat transfer characteristics in the fully-developed region of wavy channels were investigated by Savino et al. [3] and Comini et al. [4].

The computational domain of the periodical channel is usually very long. Some techniques must be developed in order to obtain a solution in a reasonable computational time. One approach is to use the concept of the periodically fully-developed flow and heat transfer. Many researchers studied the convective heat transfer in various periodical channels under the concept of the periodically fully-developed flow and heat transfer. Xin and Tao [5] performed numerical solution of laminar flow and heat transfer in wavy channels of uniform cross-sectional area. They found that both Nusselt number and friction factor were significantly affected by Reynolds number and geometric parameters of the channel. Bahaidarah et al. [6] studied a two-dimensional steady developing fluid flow and heat transfer through a periodic wavy passage. Wang and Tao [7] numerically analyzed heat transfer and fluid flow characteristics of plate-array aligned at angles to the flow direction. Prata and Sparrow [8] studied the heat transfer and fluid flow in an annulus with periodically varying cross-section. Zhang et al. [9] simulated the laminar forced convection of air ($Pr = 0.7$) in two-dimensional wavy-plate-fin channels with sinusoidal...
flow. Valencia [20] studied the laminar to oscillatory temperature field for an unsteady self-sustained laminar Stokes and energy equations. They obtained the unsteady analysis and integration of the time-dependent Navier–Stokes and energy equations. The numerical results revealed that the flow became unsteady when Reynolds number was greater than 200. Korichi and Oufer [21] simulated the heat transfer enhancement in oscillatory flow in a channel with periodically upper and lower walls mounted obstacles. They found that the flow was steady and stable at low Reynolds number but oscillatory flow occurred as Reynolds number is increased. Beale and Spalding [22] studied the transient flow in in-line-square and rotated-square tube banks in Reynolds number ranging between 30 and 3000. Both in-line and staggered tube banks exhibit transient behavior for Re > 100.

Due to the computational time constrain, many numerical works on convective heat transfer in periodic channels based on unsteady model used the concept of periodically fully-developed flow and heat transfer. Adachi [23] studied the correlation between heat transfer and pressure drop in channels with periodically grooved parts. The streamwise direction is investigated for various channel configurations by assuming periodically fully-developed flow and temperature fields. Xu and Min [24] numerically predicted unsteady flow characteristics in corrugated channels and suggested that the flow was periodically fully developed. Wang and Vanka [25] studied convective heat transfer in one module of periodic wavy passages. Islamoglu and Parmaksizoglu [26,27] numerically and experimentally investigated convective heat transfer in a corrugated channel. The studies were conducted for turbulent regime using the fully developed convection flow and the results are in good agreement with experimental results. Nonino and Comini [28] solved fully-developed convection problems in spatially periodic domains using the finite-element method. Liou et al. [29] numerically investigated spatially periodic turbulent fluid flow and heat transfer in a channel with slit rectangular ribs mounted on one wall. Najam et al.
[30] presented a numerical study of laminar unsteady mixed convection in a two-dimensional horizontal channel containing heating blocks periodically mounted on its lower wall while the upper wall was maintained at a constant temperature. The flow was assumed to be fully developed and periodic conditions were used in the longitudinal direction of the channel. Rokni and Gatski [31] investigated the turbulent flow and heat transfer in both straight and wavy ducts, with rectangular, trapezoidal and triangular cross-sections, under fully developed conditions. Valencia and Cid [32] analyzed the unsteady turbulent flow and heat transfer characteristics in a channel with streamwise periodically mounted square bars arranged side-by-side to the incoming flow.

The assumption of periodically fully-developed flow and heat transfer was first developed in Patankar et al. [33] and was used under steady-state condition. More studies are needed to justify whether it is still valid on an unsteady model or on what condition it is valid. In this work, the entrance fluid flow and heat transfer in channels with periodically grooved parts will be investigated using an unsteady model to study the periodically fully-developed convection. The dynamic characteristics of the fluid flow and heat transfer will also be analyzed.

2. Physical model

The physical model of a two-dimensional channel under consideration is shown in Fig. 1. The channel consists of two parallel plates at constant temperature $T_w$ and with periodically grooved parts on them. The grooved parts are equal in size and are arranged equidistance along the flow direction. Fluid enters from an inlet with a given temperature $T_i$ and velocity $u_i$. The $x$-axis is taken in the flow direction along the lower plate and the $y$-axis is perpendicular to the flow direction. It is assumed that the flow is incompressible. The dimensionless continuity, momentum equations are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

(1)

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)$$

(2)

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right)$$

(3)

where the dimensionless variables are defined in the nomenclature. The energy equation in term of excess temperature, $\theta$, is

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re \cdot Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)$$

(4)

where

$$\theta = T - T_w$$

(5)

The fluid temperature in the fully-developed periodic flow with a constant wall temperature does not simply repeat itself but the normalized temperature profile defined by $(T - T_w)/(T_h - T_w)$ repeats from module to module. In order to analyze the periodically fully developed characteristics, the following dimensionless temperature is defined:

$$\Theta = \frac{T - T_w}{T_b(X) - T_w} = \frac{\theta}{\theta_b(X)}$$

(6)

where $T_b(X)$ is the bulk temperature, $\theta_b(X)$ is the cross-sectional local bulk temperature and is defined as [21]

$$\theta_b(X) = \frac{\int |U| \theta \, dy}{\int |U| \, dy}$$

(7)

The Reynolds number $Re$, Nusselt number $Nu$ and friction factor $f$ are defined as

$$Re = \frac{u_i H}{v}$$

(8)

$$Nu = \frac{-\frac{\partial \theta}{\partial Y}|_{Y=0} \cdot H}{T_b - T_w} = \frac{1}{\theta_b \frac{\partial Y}{\partial X}}|_{Y=0}$$

(9)

$$f = \frac{(- \frac{\partial \theta}{\partial X}) \cdot H}{\frac{1}{2} \rho u^2}$$

(10)

At the outlet of the channel, the assumption of the local unilateralism is adopted [34,35], i.e., the flow at out section is one direction locally and the node on the out section will not affect the first inner node. Thus, the boundary conditions on the plates are given as

$$X = 0 : \quad U = \frac{u_i}{u_m} = U_i, \quad V = 0, \quad \theta = 1$$

(11)

$$X = L : \quad \frac{\partial U}{\partial X} = 0, \quad V = 0, \quad \frac{\partial \theta}{\partial X} = 0$$

(12)

$$Y = 0 : \quad U = 0, \quad V = 0, \quad \theta = 0$$

(13)

$$Y = D_0 : \quad U = 0, \quad V = 0, \quad \theta = 0$$

(14)
The initial conditions are
$$\tau = 0: \quad U = 0, \quad V = 0, \quad \theta = 1 \quad (15)$$

3. Numerical Solution

The governing equations are discretized using SIMPLE algorithm with a QUICK scheme [34,35]. The blocked-off technique is adopted with the following conditions in the periodically grooved parts: $U = 0, \quad V = 0, \quad \theta = 0.$ The computational domain begins with the entrance and includes the whole channel. The computations are carried out using two different lengths of the channel (37.2 or 62.8, corresponding to 6 or 10 modules). The dimensionless height of the total channel $D_0$ is 1.6, the dimensionless height of the grooved parts is 0.6 and the length of the grooved parts $A$ is 2.0. The Prandtl number is 0.7 which approximates the properties of air.

The flow and heat transfer in a passage composed of two parallel plates are calculated first to validate the computer codes. The characteristic length in the Reynolds and Nusselt numbers becomes the height of the channel, $H_0$, because $H_0 = H$, when periodically grooved part is not present (see Fig. 1). The dimensionless length of the channel is 23.25. A grid number of 390 $\times$ 40 and a time step of 0.005 were used. The Nusselt number of fully-developed convection heat transfer in parallel channel obtained by the analytical solution [36] is 3.77 (converted from a value of 7.54 based on hydraulic diameter of $D_h = 2H_0$). When $Re$ is 320, the Nusselt number in fully-developed region obtained by the present method using a grid number of 930 $\times$ 40 and a time step of 0.005 is 3.768. When Reynolds number is 480, the Nusselt number is 3.803. The relative errors on the Nusselt numbers for both Reynolds numbers are less than 1%.

The grid size and time step independent test for the configuration shown in Fig. 1 is then carried out when Reynolds number is 200. The average Nusselt number along the whole channel, obtained using a grid number of 1860 $\times$ 80 and a dimensionless time step of 0.001, is 4.808. When the grid number is reduced to 930 $\times$ 40 and the time step is increased to 0.005, the average Nusselt number becomes 4.771. The relative error between these two sets of grid size and time step is less than 1%. Consequently, the results reported below are obtained by using the grid number of 930 $\times$ 40 and a time step of 0.005 for six modules and 1570 $\times$ 40 with a time step of 0.005 for 10 modules.

4. Results and discussion

4.1. Oscillation of the fluid flow and heat transfer

The flow and heat transfer in the periodic channels under some conditions may be unsteady due to the nonlinearity of the problem. In this work, numerical simulations are carried out for Reynolds number ranging from 50 to 1250, which covers the stable, transitional and the unstable states. According to the calculation results, the fluid flow and heat transfer are steady and stable when $Re = 50$ and will begin to exhibit time-dependent behavior as $Re$ gradually increases. At high $Re$, the disturbance will be more significant and the fluid flow will oscillate more frequently, which leads to the oscillation of the heat transfer. The numerical results will not be absolutely the same all the time even for the stable state. The value of critical Reynolds number at which the flow begins to oscillate will depend on how this critical Reynolds number is defined. If the critical oscillating Reynolds number is defined as a Reynolds number at which the relative error of the average Nusselt in channel is greater than 0.2%, the numerical simulation indicated that the critical Reynolds number is about 100. When $Re$ is greater than or equal to the critical Reynolds number, fluid flow and heat transfer become unsteady. Figs. 2 and 3, respectively, show the variation of the average Nusselt number and the average friction factor with dimensionless time $\tau$ when $Re$ is 375 and the length of the channel along the $x$-direction is 37.2.

In order to clarify whether the oscillation of the flow and heat transfer are caused by the channel’s physical geometrical-structure or by the computational method, the flow and heat transfer in channel of two parallel plates are studied to make a comparison. When the depth of the grooved parts is zero, the passage will become a channel of two parallel plates. With the same computational method and same parameters the convective heat transfer in parallel channel is simulated numerically. It is found that the fluid flow does not oscillate when Reynolds number is under 640 (this value will be changed if different characteristic length, say, hydraulic diameter, is used to define the Reynolds number). When $Re$ equals 800 and 960, the convective heat transfer slightly oscillates with time. The variation of average Nusselt number (along the whole channel) for the case without grooved parts for $Re = 320, 640$ and 1120 and the length of channel is 37.2 are shown in Fig. 4.

The relationship between the depth of the grooved parts and the oscillation is also studied by changing the dimensionless height of grooved parts as 0.6, 0.4, 0.3, 0.2 and
The channel length is 37.2. It is found that the oscillation will be more easily to occur when the grooved parts is deeper. Fig. 5 shows the variation of the velocity at the representation point (the center of each module, i.e., its \(x\)-coordinate is average value of inlet and outlet of each module and its \(y\) coordinate is half of height of channel) in the fourth module with time when \(Re\) is 470 and the dimensionless depth of the grooved parts is 0.6 and 0.2, respectively. The velocity of the representation point oscillates more significantly for the case with the depth of 0.6 than that with the depth is 0.2. It is apparent from the above results that the oscillation of the flow and heat transfer is caused by the physical structure of the channel, not by computational method. There are also some research works showing that fluid flow oscillation occurred with different Reynolds number for different geometrical configuration. Herman and Kang [37] comparatively evaluated three heat transfer enhancement strategies in a grooved channel. Their experiments results showed that the flow oscillations were first observed between \(Re = 1050\) and 1320 for the basic grooved channel, and around \(Re = 350\) and 450 for the grooved channels equipped with cylinders and vanes.

When Reynolds number is increased, the flow and heat transfer will oscillate but the oscillation is not uniform along the channel. At the entrance the flow is stable and after several modules it begins to oscillate slightly and then the amplitude of oscillation increases gradually along the channel. After several modules the average value of the amplitude of oscillation will not change any more. At different Reynolds number, the module at which oscillation begins is also different. As Reynolds number increases, oscillation begins at the module near the entrance of the channel. Figs. 6 and 7 show the variation of the average Nusselt number of the second, fourth, sixth module with time \((Re = 375)\). It can be seen from the Figs. 6 and 7 that the amplitude of the oscillation for the fluid flow and heat transfer increases along the channel. Fig. 8 shows the variation of the velocity at the representative point for the fourth module when \(Re\) is 250, 375, 470. The length of the channel along the streamwise direction is 62.8. The computational results for different Reynolds number indicated that the amplitude of oscillation for the same module increases with increasing Reynolds number.

### 4.2. Periodicity of the flow and heat transfer

The computations performed at different Reynolds number show the characteristics of the periodicity of the
fluid flow when \( Re \) is less than 250. Fig. 9 shows the flow field when the length of the channel is 37.2 and \( Re \) is 100 and 200, respectively. It can be seen that the flow field of each period is almost same after two or three period module. In order to make a quantitative analysis, the velocity of representative point and the friction factor in each module for \( Re = 200 \) are shown in Table 1. It can be seen from Table 1 that the velocity of the representative point in the third, the fourth and the fifth modules are almost the same, which indicated that the fluid flow is periodically fully developed. When Reynolds number is less than 250, the fluid flow is in the state of oscillation but the amplitude of each module will not change with time. Consequently the average flow in each module will not change and the average flow is periodically fully developed.

The periodicity is also found in heat transfer when the Reynolds number is less than 250. Fig. 10 shows the dimensionless temperature field for \( Re = 100 \) and 200 and the length of streamwise direction is 37.2. It can be seen that the dimensionless temperature (\( \Theta \)) fields of each module are also almost same after two or three period module. According to Table 1, the average values of Nusselt number of the third, the fourth and the fifth module are also almost the same. Therefore, the heat transfer is also periodically fully developed.

Table 1

<table>
<thead>
<tr>
<th>Period</th>
<th>( U )</th>
<th>( V )</th>
<th>( f )</th>
<th>( Nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2\text{nd}</td>
<td>1.4198</td>
<td>0.5277</td>
<td>0.2054</td>
<td>3.5663</td>
</tr>
<tr>
<td>3\text{rd}</td>
<td>1.4315</td>
<td>0.5311</td>
<td>0.2051</td>
<td>3.5550</td>
</tr>
<tr>
<td>4\text{th}</td>
<td>1.4324</td>
<td>0.5314</td>
<td>0.2051</td>
<td>3.5574</td>
</tr>
<tr>
<td>5\text{th}</td>
<td>1.4327</td>
<td>0.5317</td>
<td>0.2050</td>
<td>3.5468</td>
</tr>
</tbody>
</table>

Fig. 7. Variation of the average velocity of representative point in the second, fourth, sixth module with time (\( Re = 375 \)).

Fig. 8. Variation of velocity of representative point in the fourth module with time.

Fig. 9. Flow field.
When Reynolds number is increased further, the fluid flow will oscillate significantly. The flow pattern along the channel will change from stable state to unstable state and then oscillating state, similar to the phenomena of flow in parallel plate. At the entrance, the fluid flow is stable and begins to oscillate slightly along the flow direction after several modules. Then the oscillation will become more and more significant and the amplitude will increase gradually along the channel. After another several modules the oscillation tends to be steady and the amplitude will not increase any further. At this period the flow field and dimensionless temperature field of each module will not be same at all time and the transient value of the Nu and velocity of the representative point will not be equal. However, the average value of the Nusselt number and the friction factor of each module in a period of time will be almost same after several modules. Table 2 gives the average Nusselt number, friction factor and the velocity of representative point in each module are approximately equal after several modules. Consequently, even using an unsteady model, the average number within a period will be almost the same.

4.3. Computational results under the concept of the periodically fully-developed flow

The numerical simulation is then performed based on the assumption that fluid flow and heat transfer are periodically fully developed with the following periodic conditions:

\[
\begin{align*}
U(X, Y) &= U(X + S_p, Y) \\
V(X, Y) &= V(X + S_p, Y) \\
\Theta &= \Theta(X + S_p, Y)
\end{align*}
\]

where \(S_p\) is the dimensionless length of a module. Fig. 11 shows the flow field and dimensionless temperature field obtained under the concept of periodically fully-developed flow when \(Re\) is 100 and the length of streamwise direction

<table>
<thead>
<tr>
<th>(Re)</th>
<th>Fifth</th>
<th>Sixth</th>
<th>Seventh</th>
<th>Eighth</th>
<th>Ninth</th>
</tr>
</thead>
<tbody>
<tr>
<td>375</td>
<td>4.833</td>
<td>5.004</td>
<td>5.059</td>
<td>5.112</td>
<td>5.137</td>
</tr>
<tr>
<td>(f)</td>
<td>0.0821</td>
<td>0.0777</td>
<td>0.0771</td>
<td>0.0802</td>
<td>0.0772</td>
</tr>
<tr>
<td>(U)</td>
<td>1.434</td>
<td>1.428</td>
<td>1.427</td>
<td>1.425</td>
<td>1.425</td>
</tr>
<tr>
<td>(f)</td>
<td>0.0605</td>
<td>0.0566</td>
<td>0.0633</td>
<td>0.0585</td>
<td>0.0599</td>
</tr>
<tr>
<td>(U)</td>
<td>1.506</td>
<td>1.504</td>
<td>1.516</td>
<td>1.491</td>
<td>1.512</td>
</tr>
</tbody>
</table>
is 6.4 as an example, which are almost same as those shown in Figs. 9 and 10. The average Nusselt number and the velocity are also consistent with the result considering the effect of the entrance on flow and heat transfer. Consequently, the unsteady model based on the concept of periodically fully developed can be used to study the flow and heat transfer in periodic channels to get good results with substantially lower computational cost.

5. Conclusion

The flow and heat transfer in channels with periodically rectangular-grooved parts are investigated numerically. The results show that the convection heat transfer will change from steady-state to unsteady-state as Reynolds number is increased. At high Reynolds number, oscillation occurs and the fluid flow is stable in entrance and begins to oscillate after several modules. The amplitude increase gradually along the channel but after a few modules the amplitude will not change significantly. Although the flow field and dimensionless temperature field of each module will not be same, and the transient Nusselt numbers and velocities of the representative points are not equal, the average value of Nu and velocity of each module will be almost equal after several modules. When the oscillation takes place, the fluid flow and heat transfer still show the characteristics of the periodically fully developed flow and heat transfer.

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