A Boundary Element Method for Evaluation of the Effective Thermal Conductivity of Packed Beds

The problem of evaluating the effective thermal conductivity of random packed beds is of great interest to a wide-range of engineers and scientists. This study presents a boundary element model (BEM) for the prediction of the effective thermal conductivity of a two-dimensional packed bed. The model accounts for four heat transfer mechanisms: (1) conduction through the solid; (2) conduction through the contact area between particles; (3) radiation between solid surfaces; and (4) conduction through the fluid phase. The radiation heat exchange between solid surfaces is simulated by the net-radiation method. Two regular packing configurations, square array and hexagonal array, are chosen as illustrative examples. The comparison between the results obtained by the present model and the existing predictions are made and the agreement is very good. The proposed BEM model provides a new tool for evaluating the effective thermal conductivity of the packed beds. [DOI: 10.1115/1.2430721]

1 Introduction

Heat transfer through a packed bed is of great interest in numerous industrial thermal systems, such as catalytic reactors [1], drying processes [2], processes involving transpiration cooling [3], high-performance cryogenic insulation [4], etc. Knowing the thermal properties of these materials, especially the effective thermal conductivity, is essential to allow correct design of these thermal systems. Many theoretical models for predicting the effective thermal conductivity of a packed bed have been reported. Most of the existing models are based on the so-called “unit cell method” [5–16], in which some unit cell is regarded as representative for the whole bed. Three heat transfer modes—conduction, convection and radiation—are arranged in series and/or in parallel in the unit cell. The unidirectional heat flow that leads to easy computation is usually assumed. For example, Yagi and Kunii [5] proposed a correlating formula for the effective thermal conductivity based on a unit cell, in which a portion of the heat is conducted by the solid, a portion by the fluid, and the rest of heat by a multi-layer system consisting successively of a layer of the solid and a layer of the fluid. Zehner and Schlunder [6] presented an analytical expression by choosing a cylindrical unit cell, in which heat is transferred by conduction through two parallel paths: conduction through the gas-filled voids in the outer cylinder and conduction through the solid and gas phases in the inner cylinder. The unit cell method can give satisfactory predictions only when the constituent thermal conductivities are of the similar magnitude [17,18].

In recent years, a new method based on particle-size scale is receiving more and more attention [19–21]. In this method, the geometrical details between two touching deformed particles are considered to a more or less extent, and a representative temperature—usually the temperature at each particle center—in the packed bed can be calculated. The constriction resistance to heat flow between two touching particles is calculated first by some methods, and then a linear system of simultaneous equations is established based on the thermal equilibrium requirement for each particle. These equations can be solved to obtain the temperatures at all particle centers. The effective thermal conductivity is finally determined according to the particle-center temperature distribution. For instance, Argento and Bouvard [19] calculated the constriction resistance between two touching spheres by using the finite element method and then established a linear system of simultaneous equations. Cheng et al. [20] employed the Voronoi polyhedron to represent the structure of random packing composed of monosized spheres. Based on this structure, they obtained the integral expressions of heat flux between two spheres and a linear system of simultaneous equations was established.

Theoretically, the temperature field in the whole packed bed can be obtained by solving the conduction differential equations in the solid and liquid phases with a continuous temperature and heat flux boundary condition at the solid–fluid interfaces [22]. The effective thermal conductivity can then be determined by the temperature field. This approach is subparticle scale in nature, which is expected to be able to give the most “exact” solutions since almost no assumption is introduced in the whole solution process. However, the irregular geometries at the solid–solid and solid–fluid interfaces necessitate very high computer time and storage. Among the existing numerical methods, the finite difference method (FDM) is not convenient for the establishing of such a model due to its poor adaptability to complex geometry. The finite element method (FEM) appears to be a suitable candidate since the grid-generation in FEM is flexible, but it is still a domain-based numerical method that requires discretizing the space everywhere inside the entire computation domain. Particularly, an extremely fine mesh is needed near the point of contact between particles. Accordingly, a very large computer memory is required to store the space-discretizing information. Therefore, both FDM and FEM are not appropriate to establish such a subparticle scale model.

The boundary element method (BEM) [23–27] converts the partial differential equation into boundary integral equation and thus is referred to as the “boundary-only method.” Although the particle number is usually very large in a simulation, the computational cost is expected to be acceptable since only boundary discretization is required. Furthermore, the BEM has very strong

Jianhua Zhou1
Aibing Yu
School of Materials Science and Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

Yuwen Zhang2
Department of Mechanical and Aerospace Engineering, University of Missouri-Columbia, Columbia, MO 65211
e-mail: zhangyu@missouri.edu

1Current address: Dept. of Mechanical and AeroSpace Engineering, University of Missouri-Columbia, E2412 Lafferre Hall, Columbia, MO 65211.
2Corresponding author.
Contributed by the Heat Transfer Division of ASME for publication in the JOURNAL OF HEAT TRANSFER. Manuscript received December 9, 2005; final manuscript received June 6, 2006. Review conducted by Walter W. Yue.


Downloaded 11 Nov 2010 to 149.171.194.104. Redistribution subject to ASME license or copyright; see http://www.asme.org/terms/Terms_Use.cfm
adaptability to complex geometry involved in the packed bed. The unknowns in the BEM simulation are the temperatures and heat fluxes on the domain boundary, which allows us to combine the BEM with the thermal radiation in an efficient way because radiation is also a surface behavior for the case where the solid phase is opaque and the fluid phase is nonparticipating. The foregoing advantages of BEM make it an ideal tool to analyze the combined conduction/radiation heat transfer in a packed bed.

This study develops a boundary element method for the prediction of the effective thermal conductivity of a two-dimensional packed bed. The boundary element formulation, net-radiation method, and the iterative solution procedure for the coupled conductive/radiative heat transfer are described in detail for two regular packing configurations—square array and hexagonal array. The predicted results are compared with the existing theoretical predictions and good agreement is found.

2 Mathematical Formulation

2.1 Physical Model and Boundary Discretization. Figure 1 illustrates two regular packing structures, namely square array and hexagonal array (alternatively, the terms “in-line arrangement” and “staggered arrangement” have also been used, for example, in Ref. [14]) that will be studied. Figure 2 shows the Cartesian coordinate system and the discretization of the boundaries for a square array including 20 particles. In our real simulation, however, packed beds can include hundreds of particles in order to obtain physically meaningful results.

The whole packed bed is bounded by two horizontal plates and two vertical plates and some external compression loads are exerted on the bounding plates. As a result, all the particles in the packed bed are subject to deformation at the particle–particle contact point. Although the particle–particle contact areas can be calculated by the theory of contact mechanics [28], the details of contact mechanics will not be considered in this study for simplicity. It is worth noting that, in Fig. 2, the sizes of all the particle–particle contact areas, as well as the bounding plate–particle contact areas, are depicted in an exaggerated way for clarity (in practice, the ratios of contact size to particle size are usually very small, e.g., 0.01). Since the thermal conductivity of the solid phase is different from that of the fluid phase, the entire packed bed domain will be divided into two subregions: solid and fluid. In addition, the boundary of the solid region can be further divided into external boundary and internal boundaries due to the presence of the fluid-filled voids.

The heat transfer occurring in the fluid-filled voids immediately adjacent to the bounding plates (e.g., the void 3-4-5-6-7-3 in Fig. 2) is not considered in the heat transfer computation, which may cause the so-called “wall effect.” In this study, the wall effects are removed by increasing the number of particles, as is usually done in previous studies [19,20]. All the external boundary nodes of the solid region are subject to a zero heat flux boundary condition except those nodes located at the top or bottom bounding plates where a constant temperature boundary condition is applied ($T_t$ at top bounding plate and $T_b$ at bottom bounding plate).

For the case shown in Fig. 2, the boundary of the solid phase is discretized into 196 constant boundary elements (100 elements on the external boundary and 96 elements over the 12 internal boundaries). The boundary nodes, where the unknown values (temperature or heat flux) are to be solved, are located at the middle of each boundary element. In addition, special care is given to the numbering of the boundary nodes. As shown in Fig. 2, for the external boundary of the solid region, the numbering scheme is defined in the counterclockwise direction, whereas for the internal boundaries the numbering scheme is defined in the clockwise direction. For the boundary of the fluid region or “fluid-filled voids,” the numbering scheme is defined in the same direction as that of the internal boundary of the solid region. To capture the drastic temperature variation near the particle–particle contact point, fine mesh is generated in the vicinity of contact point and coarse mesh is generated elsewhere (this feature is not shown in Fig. 2 for the clearness of the figure).

2.2 Boundary Element Scheme. It is assumed that the solid and fluid phases in the packed bed are homogeneous and isotropic media. The two-dimensional steady heat conduction in the solid and fluid phases of the packed beds in Cartesian coordinates are described by the Laplace equations.
\[
\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0 \quad \text{in } \Omega_s
\]  
(1)

\[
\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} = 0 \quad \text{in } \Omega_f
\]  
(2)

where \( \Omega_s \) and \( \Omega_f \) are solid and fluid regions, respectively. The boundary conditions of the problem are as follows

\[
T_s = T_i \quad \text{on } \Gamma_{ei}
\]  
(3)

\[
T_s = T_b \quad \text{on } \Gamma_{eb}
\]  
(4)

\[-k_s \frac{\partial T_s}{\partial n_s} = 0 \quad \text{on } \Gamma_{es}\]

\[
T_f = \frac{\partial T_f}{\partial n_f} = q_i^* \quad \text{on } \Gamma_f
\]  
(5)

where \( \Gamma_{ei} \) is the external boundary of solid region located at the top bounding plate; \( \Gamma_{eb} \) is the external boundary of solid region located at the bottom bounding plate; \( \Gamma_{es} \) is the rest part of the external boundary of solid region excluding \( \Gamma_{ei} \) and \( \Gamma_{eb} \) (the total external boundary of the solid region is \( \Gamma_{s} = \Gamma_{ei} + \Gamma_{eb} + \Gamma_{es} \)); \( \Gamma_{f} \) is the internal boundary of the solid region (the internal boundary of the solid region is composed of many sub-boundaries, which constitute the solid–fluid interface; and \( \Gamma_{f} \) represents the sum of all these sub-boundaries); \( q_i^* \) is the radiative heat flux on the internal boundaries of the solid region. Equations (6) and (7) fulfill the requirement of the continuous temperature and heat flux condition at the solid–fluid interface. In the numerical solution procedure, iteration between solutions of solid and liquid phase is needed in order to satisfy Eqs. (6) and (7).

The boundary element method will be briefly described using the solid phase as an example. When the solid phase is the present computational domain, and the normal temperature derivatives on the solid–fluid interface, \( \partial T_s / \partial n_s \), are obtained from the solution of the fluid phase. Therefore, the boundary condition in Eq. (7) can be written as

\[
\frac{\partial T_s}{\partial n_s} = \bar{q}_i \quad \text{on } \Gamma_f
\]  
(8)

where \( \bar{q}_i \) is the normal temperature derivative on the solid–fluid interface

\[
\bar{q}_i = \frac{k_s \partial T_s}{\partial n_s} = q_i^* \frac{k_s}{k_f}
\]  
(9)

Equations (1), (3)–(5), and (8) form the closed mathematical description for the BEM simulation in the solid region. It can be seen that essential boundary conditions are imposed on \( \Gamma_{ei} + \Gamma_{eb} \), whereas natural boundary conditions are exerted on \( \Gamma_{es} + \Gamma_f \). The mathematical description for the BEM simulation in the solid region can be rewritten in a more compact form as follows

\[
\nabla^2 T_s = 0 \quad \text{in } \Omega_s
\]  
(10)

\[
T_s = \bar{T}_s \quad \text{in } \Gamma_{ei} + \Gamma_{eb}
\]  
(11)

\[
\frac{\partial T_s}{\partial n_s} = \bar{q}_i \quad \text{in } \Gamma_{es} + \Gamma_f
\]  
(12)

where \( \nabla^2 \) is the Laplace operator.

Applying a weighted residual formulation to Eq. (10) yields

\[
\int_{\Omega_s} \left( \nabla^2 T_s \right) \nabla^2 d\Omega = -\int_{\Gamma_{ei}} (\bar{q}_i - q_i^*) \nabla^2 d\Gamma + \int_{\Gamma_{ei} + \Gamma_{eb}} (T_i - T_s) q_i^* d\Gamma
\]  
(13)

where \( q_i = \partial T_i / \partial n_s \); \( q_i^* = \partial T_i / \partial n_f \); and \( T_i ^* \) is the weighting function, which is the fundamental solution to the Laplace equation subject to a concentrated unit heat source, and is given by (for a two-dimensional problem)

\[
T_i ^* = \frac{1}{2\pi} \ln \left( \frac{1}{r} \right)
\]  
(14)

where \( r \) is the distance from a “source point” \( i \) (i.e., the point of application of the concentrated unit heat source) to any field point.

Integrating the left-hand side of Eq. (13) by parts twice gives

\[
\int_{\Omega_s} T_i \left( \nabla^2 T_i \right) d\Omega = -\int_{\Gamma_{ei}} \bar{q}_i \nabla^2 T_i d\Gamma - \int_{\Gamma_{ei} + \Gamma_{eb}} q_i \nabla^2 T_i d\Gamma
\]

\[
\quad + \int_{\Gamma_{ei} + \Gamma_{eb}} T_i q_i^* d\Gamma + \int_{\Gamma_{ei}} \bar{T}_i q_i^* d\Gamma
\]

\[
\quad - \int_{\Gamma_{ei}} q_i \nabla^2 T_i d\Gamma + \int_{\Gamma_{ei} + \Gamma_{eb}} T_i q_i^* d\Gamma
\]  
(15)

where \( \Gamma_i \) is the total boundary of the solid region, \( \Gamma_i = \Gamma_{ei} + \Gamma_{eb} + \Gamma_{es} + \Gamma_f \).

The properties of the fundamental solution \( T_i ^* \) lead to the following integral expression

\[
T_s + \int_{\Gamma_{ei} + \Gamma_{eb}} T_i q_i^* d\Gamma + \int_{\Gamma_{ei}} \bar{T}_i q_i^* d\Gamma
\]

\[
= \int_{\Gamma_{ei} + \Gamma_{eb}} \bar{q}_i T_i d\Gamma + \int_{\Gamma_{ei} + \Gamma_{eb}} q_i T_i d\Gamma
\]  
(16)

Equation (16) is valid for any point inside the domain \( \Omega_s \). If we take the point \( i \) to the boundary, the boundary integration equation, which only involves boundary integration, is obtained as follows

\[
c_i T_i + \int_{\Gamma_{ei} + \Gamma_{eb}} T_i q_i^* d\Gamma + \int_{\Gamma_{ei}} \bar{T}_i q_i^* d\Gamma
\]

\[
= \int_{\Gamma_{ei} + \Gamma_{eb}} \bar{q}_i T_i d\Gamma + \int_{\Gamma_{ei} + \Gamma_{eb}} q_i T_i d\Gamma
\]  
(17)

where the coefficient \( c_i \) is a function of the solid angle of the boundary at point \( i \); \( c_i \) takes the value of 0.5 on a smooth boundary, which is the case in this study since the constant boundary elements are adopted.

If we discretize the boundary into a series of elements, a linear system of equations will result. After solving these equations, the unknowns on the boundary can be obtained. Once all the normal derivatives and temperatures on the boundary are known, the temperature at any points inside the region \( \Omega_s \) can be computed using Eq. (16).

The effective thermal conductivities of the packed bed are computed by

\[
k_e = \frac{Q}{A \cdot (T_i - T_b)}
\]  
(18)

After the temperatures on the internal boundary of the solid region are obtained, the BEM simulation is performed in each fluid-filled void. It should be pointed out that, in these regions, special attention should be paid to the calculation of the normal to the boundary since the boundary-node numbering scheme is de-
2.3 Radiation Heat Transfer. Radiation becomes important at high temperatures. The present BEM model takes the radiation between solid surfaces into account. It is assumed that the solid phase is opaque and its surface is diffuse and gray. The fluid in the voids is considered a nonparticipating medium. The radiation heat exchange occurs among the solid surfaces surrounding the fluid-filled voids. For two-dimensional packed beds, the solid surfaces surrounding a fluid-filled void usually constitute a closed cavity and therefore the radiation problem becomes one occurring in a closed cavity. Figure 3 shows the two geometries of the fluid-filled voids for the two regular packings depicted in Fig. 1. For random packing, the problem will become more complicated because the solid surfaces surrounding a fluid-filled void may not form a closed cavity (there are openings on the boundary). This means that the radiation rays can travel from the one fluid region to another, which is considered a long range effect [29]. The extension of the present work to random packing structures is underway.

To illustrate the key features of the present radiation model, the case shown in Fig. 3(a) will be considered. The boundary of the fluid-filled void is composed of four convex solid surfaces and there are eight boundary elements on the boundary. For simplicity, it is assumed that the enclosure (i.e., the fluid-filled void) is composed of four radiation surfaces (i.e., surfaces AB, BC, CD, and DA) disregarding how many boundary elements there are on the boundary in the BEM simulation. With this assumption, the view factors between the four surfaces can be calculated by Hottel’s crossed-string method [30] with considerable ease. For instance, the view factor from surface AB to BC is calculated by

$$ F_{AB-BC} = \frac{AB + BC - AC}{2AB} $$

where $AB$, $BC$ are, respectively, the path lengths of the curves $AB$ and $BC$; and $AC$ is the length of the line joining the points $A$ and $C$. The view factor from surface $AB$ to $CD$ is computed by

$$ F_{AB-CD} = \frac{AC + BC - BD}{2AB} $$

where $DA$ is the path length of the curve $DA$; and $BD$ is the length of the line joining the points $B$ and $D$. Since four surfaces of the void are all convex, the view factor between a surface and itself is zero.

The radiative heat exchange between the solid surfaces is computed by using the net-radiation method [30]. By applying this method, the following system of equations can be obtained

$$ J_i = (1 - e_i) \sum_j F_{ij} J_j = e_i \sigma T_i^4 $$

where $e_i$ is the emissivity of surface $i$.

In Eq. (21), $T_i$ is calculated in the following way. Once the BEM simulation is performed in the solid region, all the temperatures on the solid boundary surrounding a fluid-filled void are known. The temperatures of the surfaces $AB$, $BC$, $CD$, and $DA$ are taken as the arithmetic mean of the temperatures of the boundary elements on that surface. For instance, the temperature of surface $AB$ is calculated by

$$ T_{AB} = \frac{T_1 + T_2}{2} $$

where $T_1$ and $T_2$ are the temperatures of the boundary elements 1 and 2 in Fig. 3.

After all the outgoing radiative flux, $J_i$, are obtained from Eq. (21), the net heat loss of each surface is

$$ q_i'' = J_i - \sum_j F_{ij} J_j $$

where a positive $q''$ means a certain amount of heat should be supplied to the surface in order to maintain the present temperature of this surface. Conversely, a negative $q''$ implies a certain amount of heat should be removed from the surface in order to maintain its temperature. It is worth noting that $q_i''$ in Eq. (7) is identical to $q''$ in Eq. (23).

2.4 Iteration Solution Procedure. Due to the presence of a nonlinear radiative boundary condition, it is necessary to perform an iterative solution procedure in order to obtain the solution of Eqs. (1)–(7). The iteration solution procedure is as follows:

1. Make an initial guess for the normal temperature derivatives on the solid–fluid interface;
2. Perform the boundary element simulation in the solid region;
3. After step 2, the temperatures on the solid–fluid interface are known. The radiative heat flux $q''_i$ can thus be calculated by the net-radiation method; the conductive heat flux $-k_i \partial T_i / \partial n_i$ at the solid–fluid interface can be computed by performing boundary element simulation in the fluid-filled voids;
4. Using the most recently obtained value of the combined heat flux (i.e., $-k_i \partial T_i / \partial n_i - q''_i$, obtained by step 3) as the input heat flux on the solid–fluid interface, go back to step 2 and perform the boundary element simulation in the solid region again; and
5. Repeat the foregoing solution process until the changes of the combined heat flux on the solid–fluid interface (i.e., $-k_i \partial T_i / \partial n_i - q''_i$) become smaller than a prescribed minor tolerance (e.g., $10^{-6}$).

3 Results and Discussion

A BEM computer code has been developed based on the model described in the preceding section. For simplicity, it is assumed that the deformations at all contact points are identical [31]. For the square array and hexagonal array packings, the void fraction is...
found to be a function of the ratio of contact radius to particle radius. Because of the periodicity of the packing structure, the void fractions for these two packing systems can be obtained from the unit cells as illustrated Fig. 4, where \( r_c \) and \( r_p \) are the contact radius and particle radius, respectively. The void fraction for the square array packing is calculated by

\[
\phi = 1 - \frac{\gamma(1 - \gamma^2)^{\frac{1}{2}} - \frac{\pi}{4} - \arcsin(\gamma)}{1 - \gamma^2}
\]

(24)

where \( \gamma \) is the ratio of contact radius to particle radius, \( \gamma = r_c/r_p \). The void fraction for the hexagonal array packing is computed by

\[
\phi = \frac{\sqrt{3}}{2} \left( \frac{\gamma(1 - \gamma^2)^{\frac{1}{2}} - \frac{\pi}{6} - \arcsin(\gamma)}{1 - \gamma^2} \right)
\]

(25)

The relationship between the ratio of contact radius to particle radius and the void fraction can be seen more clearly in Table 1. For the same deformation, the hexagonal array is a more closed packing than the square array. The void fraction corresponding to the case \( \gamma = 0.01 \) is very close to that of point contact, therefore the simulation case \( \gamma = 0.01 \) can be approximately regarded as the case of point contact.

As stated earlier, heat transfer occurring in the fluid-filled voids immediately adjacent to the bounding plates is not considered, so something should be done first to remove the "wall effects." Figure 5 shows the effect of particle number on the effective thermal conductivity. Similar results are obtained for other \( r_c/r_p \) and \( k_s/k_f \) values. It is seen that, to obtain stable calculating results, 20 particles are sufficient for square array packing and 120 particles are required for hexagonal array packing. This is because the wall effect for the hexagonal array is more notable than that for the square array packing. It is should be pointed out that, in this study, all the calculations are performed on a personal computer (Intel Pentium® IV 2.4 GHz with 256 MB memory). For the square array packing including 20 particles, one numerical case costs about 30 min of CPU time; for the hexagonal array packing including 120 particles, the CPU time is about 200 min.

One of the advantages of the present BEM method is that the temperature at any point within the packed bed can be determined accurately according to the calculated boundary values using Eq. (16). Figures 6 and 7 show the temperature distributions in local regions (to be more representative, these local regions are deliberately chosen to be located at the whole-bed center) for square array and hexagonal array. It is seen that the isotherm lines are much denser in the vicinity of the particle-to-particle contact area. This indicates that the heat flux constriction effect due to particle contact can be well captured by the present boundary element model. In addition, the symmetry of these temperature distributions agrees well with the regular packing features of the two packed beds.

Figure 8 presents a comparison of the results obtained by the present model with those obtained by previous models (radiation is not considered in Fig. 8). The maximum and minimum values of the thermal conductivity of a solid–fluid two phase packing system are also shown in this figure. According to Ref. [22], the maximum thermal conductivity of a two-phase packing system is attained when the media are arranged in alternate layers separated by planes parallel to the direction of heat flow; the minimum conductivity of such a system is attained when the media are separated by planes running perpendicular to the direction of heat flow. As can be seen from Fig. 8, all the previous models underestimate the effective thermal conductivity when the solid/liquid thermal conductivity ratio is high. This is in good agreement with the conclusions made in Refs. [17,18]. Since almost no assumption is made during the model formulation and solution process, the results obtained by the present BEM model might be the closest to the "exact solution" under the simulation condition.

Radiation is known to be an important mechanism of heat transfer in packed beds, particularly at high temperatures. The models that have been proposed to describe simultaneous radiation and conduction can be classified into three types: unit cell models [7–9], pseudo-homogeneous models [32,33], and the ray tracing method [34,35]. No matter which method is used, the contribution of radiation to heat transfer is usually expressed as an equivalent conductivity–radiative conductivity

\[
k_r = 4Fd_r^2T_m^3
\]

(26)

where \( d_r \) is the particle diameter; and \( F \) is the radiation exchange factor. Many analytical expressions have been proposed to calculate this parameter, as summarized in Table 2.

As Vortmeyer [29] pointed out, the radiation contribution in packed beds depends not only on the void fraction and emissivity of the solid surface, but also on the solid thermal conductivity. To date, only Schotte’s model [8] considers the influence of the solid conductivity on the radiation contribution. In Schotte’s model, the radiative conductivity \( k_r \) is obtained by the following two steps

\[
k_r = 4Fd_r^2T_m^3
\]

(27)

Table 1 Relationship between the void fraction and ratio of contact radius to particle radius

<table>
<thead>
<tr>
<th>Ratio of contact radius to particle radius ( \gamma = r_c/r_p )</th>
<th>Void fraction ( \phi )</th>
<th>Square array</th>
<th>Hexagonal array</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.1573</td>
<td>0.0386</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.1874</td>
<td>0.0650</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.2073</td>
<td>0.0851</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.2127</td>
<td>0.0910</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.2145</td>
<td>0.0930</td>
<td></td>
</tr>
<tr>
<td>Point contact</td>
<td>0.2146</td>
<td>0.0931</td>
<td></td>
</tr>
</tbody>
</table>
The radiation exchange factor in the equation is

\[ F = \frac{1 - \phi}{\phi + \frac{1}{k_e}} \quad (28) \]

The radiation exchange factor in Eq. (27) is \( F = \varepsilon \).

Once the radiative conductivity is known, the effective of thermal conductivity \( k_r \) of the packed bed can be calculated as the sum of the effective conductivity without radiation contribution \( k_c \) and the radiative conductivity \( k_r \)

\[ k_r = k_c + k_r \quad (29) \]

In this study, the radiative conductivity is obtained using the following procedure. The effective thermal conductivity at real temperature level \( k_e \) is first calculated using the combined conduction/radiation BEM model. Then, the pure conduction BEM model is employed to compute the effective conductivity without radiation contributions \( k_c \). Finally, the radiative conductivity \( k_r \) is obtained by: \( k_r = k_e - k_c \).

Figure 9 shows a comparison of the results with the previous theoretical predictions when the radiation contribution is considered. As is seen, there are large variations among the results ob-
tained by different thermal radiation models. The $k_r$ values predicted by the present model fall in the range of the previous ones. Since only Schotte’s model [8] considers the influence of the solid conductivity on the radiation contribution, the present results are closer to those obtained by Schotte’s model. It is also observed from Fig. 9 that the radiative conductivity increases with increasing solid conductivity. When the solid conductivity exceeds some value, further increase of radiative conductivity becomes less significant (see Fig. 9, $k_f/k_r$ increases from 600 to 2000). This can be explained as follows. When the solid conductivity is low, the heat conduction in solids dominates the heat transfer process, so even a small increase of solid conductivity can give rise to the increase of radiative conductivity. However, when the solid conductivity is high, the total energy transfer process is dominated by radiation, so further increase of solid conductivity cannot lead to a pronounced increase of the radiative conductivity.

Figure 10 shows the ratio $k_f/k_r$ plotted against the ratio $k_f/k_f$ with $\gamma$ (the ratio of contact radius to particle radius) as a parameter. It is plotted for the bed mean temperature 55°C, where the radiation contribution can be safely neglected. It is seen that the ratio $k_f/k_f$ increases with the increase of $k_f/k_f$. For the same value of $\gamma$, the effective thermal conductivity for the hexagonal array packing is higher than that for the square array packing since the hexagonal array is a more closed packing when the same deformation is considered.

The radiation contribution depends on the bed mean temperature as well as the particle size. Figure 11 shows the effects of bed mean temperature and particle size on the effective thermal conductivity. The effective thermal conductivity increases with the increasing mean temperature and particle size, indicating that the radiation contribution becomes more significant for higher mean temperature and larger particle radius. This is in good agreement with the experimental observation of Yagi and Kunni [5]. In addition, when the ratio of contact radius to particle radius is low (e.g., $r_c/r_p=0.01$, this means a small contact deformation), the effective thermal conductivity increases more quickly as the mean temperature increases. This result arises from the fact that, when the contact deformation is small, more solid surface can be used for radiation heat exchange. Furthermore, it is observed that, for closed packing of small-size particles (e.g., curve 4 in Fig. 11), the values of $k_f/k_f$ are nearly constant, which indicates that thermal radiation can be neglected in such cases. The effective thermal conductivity for the square array can exceed that for the hexagonal array when the mean temperature is high. This suggests that the effective thermal conductivity of hexagonal array packing is not necessarily larger than that of square array packing when the radiation is the main heat transfer mechanism, depending on the mean temperature of packed beds.

Figure 12 presents the effect of the solid surface emissivity on the effective thermal conductivity. As expected, the effective thermal conductivity of the packed bed goes up as the surface emissivity increases. The influence of the surface emissivity is more notable for the case of square array. This is because under the

### Table 2: Analytical expressions for radiation exchange factor

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Radiation exchange factor, $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argo and Smith</td>
<td>$\frac{1}{2}\left(1 - \frac{1}{\sqrt{\phi}}\right)$</td>
</tr>
<tr>
<td>Kunni and Smith</td>
<td>$2\left(1 - \frac{1}{\sqrt{\phi}}\right)^{-1}$</td>
</tr>
<tr>
<td>Wakao and Kato</td>
<td>$\frac{2}{2\pi r - 0.264\varepsilon}$</td>
</tr>
<tr>
<td>Schotte</td>
<td>$\epsilon\phi$</td>
</tr>
<tr>
<td>Godbee and Ziegler</td>
<td>$\frac{1}{1 - \phi}$</td>
</tr>
</tbody>
</table>

See Ref. [7];
See Ref. [9];
See Ref. [11];
See Ref. [8];
See Ref. [36].

---

**Fig. 8** Comparison of the present model with previous theoretical models (square array, $r_p=2.5$ mm, $r_c/r_p=0.1$, $T_m=55^\circ$C)

**Fig. 9** Comparison of the calculating results with the previous predictions when radiation contribution is considered (square array, $r_p=2.5$ mm, $r_c/r_p=0.1$, $k_f=0.029$, $r=0.9$)

**Fig. 10** The effective thermal conductivity for the square array and hexagonal array packings ($T_m=55^\circ$C)
same particle deformation ratio \( (r_c/r_p = 0.1) \) the void fraction of the square array is much higher than that of the hexagonal array and, this in turn, leads to a stronger radiative exchange effect.

4 Conclusions

A boundary element model is developed to predict the effective thermal conductivity of two-dimensional packing structures. The radiation heat exchange between solid surfaces is calculated by the net-radiation method. The problem is a coupled conductive/radiative heat transfer problem and was solved using an iterative procedure. The effective thermal conductivity of the packed bed can then be computed by using the temperature field. The proposed model is tested for two regular packing configurations: square array and hexagonal array. The comparison between the calculated results and the previous predictions are made to verify the coupled conduction/radiation model proposed in this study, and good agreements are obtained. The effective thermal conductivities for various solid/fluid conductivity ratios under different contact deformation are given. The effects of mean temperature, particle size, and surface emissivity on the effective thermal conductivity are examined. The numerical scheme formulated in this study is quite general in nature, and can be directly applied to random packing including multi-sized particles as long as the packing structure has been detected.

Nomenclature

- \( A \) = total area of the packed bed in a direction perpendicular to the heat flow (m²)
- \( d_p \) = particle diameter (m)

Fig. 12 The effect of solid surface emissivity on the effective thermal conductivity: \( (r_p=2.5 \text{ mm}, \ r_c/r_p=0.1, \ k_s/k_f=600, \ T_m =1000^\circ \text{C}) \)
References