Convection heat transfer with internal heat generation in porous media: Implementation of thermal lattice Boltzmann method

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ABSTRACT

Evaluation of lattice parameters for convection heat transfer in porous media with internal heat generation from physical and macroscale properties was described. A hierarchical process was defined to implement thermal Lattice Boltzmann Method (LBM) to investigate convection heat transfer with internal heat generation in different geometries; from a simple geometry (flow channel) to complex ones (porous media). In this regard, seven different without any obstacle cases with different geometries were designed and the detailed information about how thermal LBM should be implemented for these cases are addressed. Going from one case to the next, the cases with more complex physics and/or geometries were examined. The results showed that LBM is an appropriate method to predict heat transfer with internal heat generation in porous media.

1. Introduction

Lattice Boltzmann Method (LBM) has been developed to simulate the fluid flow, heat transfer, and chemical reactions at mesoscale. It is based on kinetic equations and statistical physics to simulate transport phenomena by tracking movements of molecule ensembles or the evolution of the distribution function [1]. In LBM, fluid motion is simulated at the level of distribution functions. Therefore, the microscopic physics of the fluid particles could be incorporated as easily as in other particle collision methods. Boltzmann transport equation can describe the statistical behaviors of particles that are not in thermodynamic equilibrium, in the form of density distribution function [2]:

\[
\frac{\partial f}{\partial t} + \gamma \cdot \frac{\partial f}{\partial \eta} + \sigma \cdot \frac{\partial f}{\partial \eta} = \Omega(f)_{\text{collision}}
\]  

(1)

where \( f \) is the density distribution, and \( \Omega \) is the collision operator that is dictated by the collision rules. \( \gamma \), \( \eta \), and \( \sigma \) are the particle’s velocity, location, and acceleration, respectively. To solve this Boltzmann transport equation, the collision term needs to be simplified. The Bhatnagar-Gross-Krook (BGK) model is a widely used model for such simplification. This model uses the Maxwell equilibrium distribution \( f^{eq} \) which can be written in terms of density \( \rho \), fluid velocity \( \mathbf{u} \), and temperature \( T \) as follow:

\[
f^{eq} = \frac{\rho}{(2 \pi R_g T)^{d/2}} e^{-\frac{\gamma - u^2}{2 R_g T}}
\]

(2)
where \( d \) is the number of spatial dimensions and \( R_g = k_B/m \) is the gas constant given by Boltzmann constant \( k_B \) and the particle mass \( m \). Equation (2) describes the density distribution when the system has reached the local equilibrium state. In the BGK model, the collision phase is assumed to be a linear relaxation of the density distribution toward an equilibrium state represented by the Maxwell equilibrium distribution. Assuming the relaxation time is \( \tau \), the Boltzmann transport equation under the BGK model then can be expressed as:

\[
\frac{\partial f}{\partial t} + \gamma \cdot \frac{\partial f}{\partial \eta} + \sigma \cdot \frac{\partial f}{\partial \eta'} = -\frac{1}{\tau} (f - f^{eq})
\]  

(3)

In the field of utilizing LBM for heat transfer, Shan [3] numerically simulated Rayleigh-Benard convection and found that LBM is an efficient, accurate, and numerically stable method for fluid flow and heat and mass transfer simulations. To simulate the 2D natural convection in a concentric horizontal annulus with a constant flux wall, Hu et al. [4] developed a novel thermal immersed boundary LBM. To study heat transfer of flow past a cylinder, Grucelski and Pozorski proposed a modified boundary scheme for curvilinear fluid-solid interface [5]. Yamamoto et al. [6] implemented a single distribution approach for combustion phenomena. Yuan and Schaefer [7] developed a thermal LBM for two-phase flow and showed that LBM is a stable, accurate, and numerically efficient scheme. LBM was also proposed for convection-diffusion equation [2, 8]. Savithiri et al. [9] carried out single-component nonhomogeneous LBM to examine natural convection heat transfer in Al\(_2\)O\(_3\) – water nanofluid considering the Brownian and thermophoretic diffusions. Biswas et al. [10] analyzed mixed convection heat transfer in a ribbed channel using 2D LBM and discussed about the effects of superimposed thermal buoyancy on flow and isotherm patterns, as well as Nusselt number. Rihab et al. [11] utilized an enthalpy-based LBM for convection-diffusion heat transfer problems in heterogeneous media and found good agreement between the results of LBM and CVM.

LBM has been utilized for the simulation of fluid flow and heat transfer in porous media due to its simplicity and ability to handle complex geometry and boundary conditions [12]. On the same grid size, to reach the same accurate solutions in porous media, LBM takes less computational time than the finite difference method [13]. For incompressible thermal flows

### Nomenclature

- \( c_s \): speed of sound (m/s)
- \( C \): concentration
- \( CVM \): Control Volume Method
- \( d \): number of spatial dimensions
- \( D \): diffusion coefficient (m\(^2\)/s)
- \( e_i \): lattice velocity (m/s)
- \( f \): fluid flow distribution function
- \( FVM \): Finite Volume Method
- \( g \): thermal distribution function
- \( LBM \): Lattice Boltzmann Method
- \( MRT \): Multi Relaxation Time
- \( PCM \): Phase Change Material
- \( Q \): heat generation (LT/Lt)
- \( q \): heat generation (K/s)
- \( R_g \): gas constant
- \( S \): source term
- \( t \): time (s)
- \( T \): Temperature (K)
- \( u \): velocity (m/s)
- \( w \): weight factor

### Greek Symbols

- \( \rho \): density (kg/m\(^3\))
- \( \nu \): kinetic viscosity (m\(^2\)/s)
- \( \eta \): particle’s location
- \( \gamma \): particle’s velocity
- \( \sigma \): particle’s acceleration
- \( \alpha \): thermal diffusivity (m\(^2\)/s)
- \( \varepsilon \): porosity
- \( \rho \): density (kg/m\(^3\))
- \( \nu \): kinetic viscosity (m\(^2\)/s)
- \( \tau_f \): fluid flow relaxation time (s)
- \( \tau_g \): thermal relaxation time (s)
- \( \tau_{g,f} \): thermal relaxation time for fluid (electrolyte) (s)
- \( \tau_{g,s} \): thermal relaxation time for solid (electrode material) (s)
- \( \Omega \): collision operator

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in porous media, a modified BGK model was proposed by Wang et al. [14]. This model incorporates the shear rate and temperature gradient in the equilibrium distribution functions. Fluid flow and heat transfer were simulated in an anisotropic porous medium by Hu et al. [15] using a Multi Relaxation Time (MRT)-LBM model at the representative elementary volume scale. The conjugate natural convection problems of a porous layer occupying partially in the middle of the square cavity was studied by Guo and Zhao [16]. Convection heat transfer between parallel plates of a conduit partially filled with a porous media was investigated by Shokouhmand et al. [17]. Li et al. [18] utilized a LBM with an interfacial tracking to examine both conduction and convection melting problems. Their results agreed very well with analytical solutions as well as those in the literature. Mishra et al. [19] solved a 1D planar porous media with a localized volumetric heat generation using LBM and found good agreement comparing the results with FVM’s.

Furthermore, LBM is a promising numerical approach for studying the fluid flow and diffusion through porous media that involves single or multiple components [20] as well as chemical reactions [21]. LBM has been used to simulate the latent heat thermal energy storage performance of metal foams/paraffin phase change material (PCM) [22]. Gao et al. [23] proposed an improved lattice Boltzmann model for solid-liquid phase change in porous media. Their results agreed well with numerical solutions reported in the previous studies. Liu et al. [24] proposed a MRT-LBM which its stability is better than BGK model at low viscosities. LBM was utilized to investigate the effects of Al2O3-water nanofluid on thermal management of cylindrical batteries [25]. It also has been utilized to simulate multi-phase and multi-component fluid flows in porous electrodes [26–29].

To effectively utilize thermal LBM to predict fluid flow, and heat transfer with internal heat generation in different geometries, detailed information about how physical properties should be connected to the lattice parameters is needed. Furthermore, transition from macroscale units to lattice units have not been discussed in the literature very well. Therefore, in this study, a hierarchical process was defined to investigate convection heat transfer with internal heat generation in different geometries from a simple flow channel without any obstacle to complex porous medium geometries. In this regard, seven different cases with different geometries and/or physics were designed, and by going from one case to the next, the more complex physics and/or geometries were examined. Also, it was tried to address some detailed information about how thermal LBM should be implemented for each case.

2. Model description

In general, thermal LBM models can be categorized in three approaches: 1) the multispeed approach, 2) the hybrid approach, and 3) the double distribution approach [30]. The multispeed approach carries out one set of distribution functions for both flow and thermal fields; therefore, its applications are relatively limited to the severe numerical instability and the narrow temperature variation range [31]. The hybrid approach utilizes LBM for the flow filed, and another numerical approach (e.g. the finite difference method) for energy equation. Finally, the double-distribution method uses two sets of distribution functions: one for the flow field and one for the thermal field. Numerical stability of this method is significantly improved in comparison with the multispeed method, and its computational implement is more convenient than the hybrid approach [30]. Therefore, the double-distribution approach was carried out in this study. More details of thermal LBM models can be found in the literature [2, 32–35].

2.1. Lattice Boltzmann equation for flow field

In this study, the lattice BGK model was implemented to approximate the collision term. The discrete particle distribution functions $f_i$ are written as:
\[ f_i(x + e_i \Delta t, \ t + \Delta t) = f_i(x, t) - \frac{\Delta t}{\tau_f} [f_i(x, t) - f_i^{eq}(x, t)] \]  

where \( f_i(x, t) \) is the probability of finding a particle in the \( i \)th velocity \( e_i \) at \( x, t \), \( \Delta t \) is the time step, and \( \tau_f \) is the relaxation time that controls the tendency of the system to relax the local equilibrium and is related to the kinetic viscosity \( \nu \). \( f_i^{eq} \) is the equilibrium distribution and is given as follow:

\[ f_i^{eq} = \rho w_i \left( 1 + \frac{e_i \cdot u}{c_s^2} + \frac{(e_i \cdot u)^2}{2c_s^4} - \frac{u \cdot u}{2c_s^2} \right) \]  

where \( c_s \) is the speed of sound and \( w_i \) is the weight factor. For a two-dimensional problem, nine velocities (D2Q9) model was utilized for flow field in this study, \( w_i \) is defined as follows:

\[
w_i = \begin{cases} 
  \frac{4}{9} & i = 1 \\
  \frac{1}{9} & i = 2, 3, 4, 5 \\
  \frac{1}{36} & i = 6, 7, 8, 9 
\end{cases}
\]  

and the discrete velocity \( e_i \) defines as

\[
[e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9] = \begin{bmatrix}
  0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\
  0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 
\end{bmatrix}
\]  

Chapman-Enskog analysis determines the link between the lattice Boltzmann equation and the Navier Stokes equation. The discrete-velocity distribution function, \( f_i(x, t) \) is the basic quantity of LBM that shows the density of particles with velocity \( e_i(x, y) \) [2]. The mass density \( \rho \) and momentum density \( \rho u \) can be found as follow:

\[
\rho(x, t) = \sum_i f_i(x, t), \quad \rho u(x, t) = \sum_i e_i f_i(x, t)
\]  

With the kinematic shear viscosity given by the relaxation time \( \tau \), the Lattice Boltzmann equation results in macroscopic behavior according to the Navier Stokes equation, as

\[ \nu = c_s^2 \left( \tau_f - \frac{\Delta t}{2} \right) \]  

### 2.2. Lattice Boltzmann equation for thermal field

To solve the thermal field, advection-diffusion equation was employed. LBM is a powerful method to solve advection-diffusion problems [2]. Advection-diffusion equation defines as below.

\[
\frac{\partial C}{\partial t} + \nabla \cdot (C u) = \nabla \cdot (D \nabla C) + S
\]  

where \( C \) is a scalar that could be a concentration or temperature. The left-hand side of Eq. (10) shows the advection of \( C \) and the right-hand side describes the diffusion term and the source term \( S \). Advection-diffusion model is an appropriate model for convection heat transfer where advection and diffusion of heat are coupled to the dynamics of the ambient fluid, and the velocity \( u \) is provided by the flow field [2]. In this model, it is assumed that the thermal field does not affect the fluid dynamics and in this one-way coupling the temperature is a passive field. Energy equation can be extracted from the advection-diffusion equation using \( T \) and \( q \) for temperature and internal heat generation, respectively.
\[
\frac{\partial T}{\partial t} + \nabla \cdot (uT) = \nabla \cdot (\kappa \nabla T) + q
\]  
(11)

where \( T, \kappa, \) and \( q \) are temperature, thermal diffusivity, and internal heat generation, respectively.

LBM could be easily adapted to advection-diffusion problems due to strong similarities between the Advection-Diffusion equation and the Navier Stokes equation. The Navier Stokes equation could be reformulated as an Advection-Diffusion equation for the fluid momentum density vector \( \rho u \) [2]. Lattice Boltzmann equation for Advection-diffusion equation could be defined as:

\[
g_i(x + e_i \Delta t, \ t + \Delta t) = g_i(x, t) - \frac{\Delta t}{\tau_g} \left[ g_i(x, t) - g_i^{eq}(x, t) \right] + Q_i(x, t)
\]  
(12)

and diffusion coefficient could be related to the relaxation time \( \tau_g \) as:

\[
\kappa = c_s^2 \left( \tau_g - \frac{\Delta t}{2} \right)
\]  
(13)

which is similar to the relation between kinematic viscosity \( \nu \) and relaxation time \( \tau_f \) for the standard LBM. It should be mentioned that, in LBM for Advection-Diffusion equation, only \( C \) is conserved and the velocity \( u \) is not obtained from \( g_i \); it is rather imposed externally [2]. The equilibrium distribution function \( g_i^{eq} \) can be found as:

\[
g_i^{eq} = w_i T \left( 1 + \frac{e_i \cdot u}{c_s^2} + \frac{(e_i \cdot u)^2}{2c_s^4} - \frac{u \cdot u}{2c_s^2} \right)
\]  
(14)

and the temperature \( T \) is extracted as

\[
T(x, t) = \sum_i g_i(x, t)
\]  
(15)

The source term (heat generation) in Eq. (11) can be defined as:

\[
Q_i = w_i q_L \Delta t
\]  
(16)

where \( q_L \) is heat generation in lattice units \( [q_L] = LT/Lt \) (Lattice Temperature/Lattice time). The process of how to find \( q_L \) is described in detail, later.

### 2.3. Thermal LBM simulation rules

In this study, below rules from Ref. [2] were followed to define lattice parameters.

- \( \Delta x \) is the distance between neighboring lattice nodes in physical units, i.e. \( [\Delta x] = m \).
- \( \Delta t \) is the physical length of a time step, i.e. \( [\Delta t] = s \).
- The dimension of relaxation time is second \( [\tau] = s \). Also, it was chosen to write \( \tau \) for the physical relaxation time and \( \tau^* \) for the dimensionless relaxation parameter.
- The dimensionless fluid density is \( \rho^* \). Its average value is usually set to unity, \( \rho_0^* = 1 \).
- In the standard LBM, the lattice speed of sound \( c_s^* \) is equal to \( \sqrt{1/3} \).

By choosing \( \Delta x^* = 1, \Delta t^* = 1 \) and \( \rho_0^* = 1 \) to relate the physical parameters \( \Delta x, \Delta t, \tau, \rho \) and \( U \) to their lattice counterparts \( \Delta x^*, \Delta t^*, \tau^*, \rho_0^* \) and \( U^* \), conversion factors for length, time, density, and velocity would be as follow, respectively:

\[
C_l = \Delta x, \ C_t = \Delta t, \ C_\rho = \rho, \ C_u = \Delta x/\Delta t
\]  
(17)
Following the same process, the conversion factor for kinematic viscosity $\nu$ and thermal diffusivity $\alpha$ would be $C_\nu$ or $C_D = C_l^2 / C_t = \Delta x^2 / \Delta t$. Therefore, the kinematic viscosity $\nu$ and thermal diffusion $\alpha$ are related to the simulation parameters as follow:

$$\nu = C^2 \left( \frac{\tau^*_f - 1}{2} \right) \frac{\Delta x^2}{\Delta t} \quad (18)$$

$$\alpha = C^2 \left( \frac{\tau^*_g - 1}{2} \right) \frac{\Delta x^2}{\Delta t} \quad (19)$$

Here, it should be mentioned that $\tau^*$, $\Delta x$ and $\Delta t$ are not independent. Only two of them can be chosen freely.

### 2.4. Definition of different cases

To have a comprehensive implementation of thermal LBM for prediction of convection heat transfer with heat generation in different geometries, in this study seven different cases as illustrated in Figure 1 were defined. These cases were selected in such a way that in each case some issues of implementing LBM for thermal problems were examined. Case #1 was defined to describe how to implement thermal LBM for a simple channel. In Case #2, two circular obstacles were added to the channel to show how heat transfer inside the obstacles should be defined in thermal LBM. In Case #3, internal heat generation inside the channel was investigated. In this case, detailed description of how internal heat generation should be defined in thermal LBM was explained. In Case #4 it was tried to show that changing the grid sizes of the geometry changes all the lattice parameters. Also, in this case capability of thermal LBM to predict fluid flow, and heat transfer with heat generation in complex geometries was examined. The results of Cases #1 to #4 were compared with the results of FVM to validate them. In the following, three different porous geometries as Case #5 ($\varepsilon = 0.9$, $d_p = 6.5 \ mm$), Case #6 ($\varepsilon = 0.76$, $d_p = 4.7 \ mm$), and Case #7 ($\varepsilon = 0.65$, $d_p = 4.1 \ mm$) were defined to investigate the process of fluid flow with heat transfer and heat generation in porous media.

### 2.5. Numerical solution procedure

In literature, the application of LBM to porous flows mostly categorized in two scales: the pore scale and the representative elementary volume (REV) scale. The pore scale uses the standard lattice Boltzmann equation to simulate fluid flows in pores, and the local information of the fluid flow is obtained directly. It should be mentioned that for the pore scale modeling, a detailed
geometric information of the pores must be provided. However, for the REV approach, an additional term based on some semi-empirical models is added to the standard lattice Boltzmann equation to consider the presence of a porous medium [14].

In this study, the pore scale was carried out. As shown in Figure 2, the two-dimensional domain was selected to simulate the process of heat transfer with heat generation in different cases. Bounce-back boundary condition was applied on the solid-liquid interface and top and bottom channel walls. Velocity inlet boundary condition was set on the left side and outflow boundary condition was utilized on the right side. Also, isothermal boundary condition was chosen for the solid-liquid interface.

To generate porous mediums with different porosities, a random porous medium generator code was utilized. To define the porous geometry, a binary matrix in which "1" demonstrate a solid particle and "0" demonstrate a fluid particle is utilized. At the first, the matrix is filled by ones (solid part) and then the program changes the ones to zeros (solid to pore) at several fixed starting points. This change process continues by creating more zeros randomly around the previous pores, until a desired porosity of the matrix has been reached. Then, the average pore size of the porous medium was evaluated [36].

3. Results and discussions

As discussed earlier, $\tau^*, \Delta x$ and $\Delta t$ are related to each other and only two of them could be chosen independently. Since double-distribution thermal LBM needs two density distribution functions (one for fluid flow and the other one for temperature) in a same geometry, $\Delta x$ and $\Delta t$ are the parameters that should be the same on both side. Therefore, at the first $\tau_f^*$ and $\Delta x$ would be chosen independently and after finding $\Delta t$ from Eq. (17), $\Delta x$ and $\Delta t$ should be utilized to find $\tau_g^*$ from Eq. (18).

3.1. Case #1

At the first, we start with a very simple problem in which colder liquid water with constant inlet velocity $u_{in} = 5 \times 10^{-5} m/s$ and inlet temperature $T_{in} = 300K$ enters to a rectangular channel ($5cm \times 1cm$) which an initial temperature of $T_\text{ini} = 315K$. By considering a $500LU \times 100LU$ grid for the channel, we have $\Delta x = 1cm/100 = 10^{-4}m$. Using $\tau_f^* = 1$ and kinematic viscosity of $\nu_\text{water} = 8.56 \times 10^{-7} m^2/s$, we can find that $\Delta t = 1.95 \times 10^{-3}s$. Now, utilizing $\Delta x = 10^{-4}m$ and $\Delta t = 1.95 \times 10^{-3}s$ and thermal diffusivity of $D_\text{water} = 1.4 \times 10^{-7}m^2/s$, we can find that $\tau_g^* = 0.582$.

Now, we need to define temperature domain based on lattice temperature (LT). The temperature domain is defined to be between 0 and 1. Therefore, we define 0 $LT$ as 273.15 K and 1 $LT$
as 373.15 K. By this definition any temperature in between could be found by interpolation. i.e. 300 K \( \equiv 0.2685 \) LT and 315 K \( \equiv 0.4185 \) LT.

Now we have all the parameters to solve fluid flow with heat transfer in a channel without any obstacle. Based on the above information 1 s \( \equiv 30816 \) Lt (time step). Figure 3 shows a comparison between the LBM and the FVM results. Good agreement between the results shows that LBM has solved the problem very well. Paying attention to the results, one obtains that with the passage of time higher amount of coolant (water) is entered to the channel that decreases the temperature of the channel. Furthermore, the temperature in the middle of the channel is lower than the temperature close to the walls (in a vertical line). It happened because of viscous boundary layer close to the walls that reduces the flow velocity which leads to higher temperatures on the area close to the walls compare to the middle of the channel.

3.2. Case #2

At Case #2, two circular obstacles were added to the geometry of Case #1 to investigate the temperature distribution in both solid and fluid. In this case, again we are not allowed to change \( \Delta x \) and \( \Delta t \). However, we need to define \( \tau_{s}^{*} \) for the solid obstacles. Assuming that the solid circles are aluminum, using same \( \Delta x \) and \( \Delta t \) and thermal diffusivity of \( D_{\text{aluminum}} = 9.7 \times 10^{-5} m^{2} / s \), one obtains \( \tau_{s}^{*} = 57.16 \).

Figure 4 shows the comparison between the LBM and the FVM results for this case. This figure shows good agreement between the results. It confirms that LBM is capable to solve the fluid flow with heat transfer in the channels with obstacles. Following the temperature contours from \( t = 60 \) s to \( t = 180 \) s, it can be found that because the flow mostly passes from the bottom of the first obstacle (the circle on the left), the temperature below the obstacle is lower than the temperature inside it at \( t = 60 \) s. The same occurrence for the second obstacle and the fluid over it (the obstacle on the right) is obvious at \( t = 300 \) s.

3.3. Case #3

Case #3 has the exact geometry of Case #2, but heat generation has been added. Based on Eq. (10), it can be found that \( q[K/s] = \dot{q}[W/m^3] / \left( \rho [kg/m^3] c_p [j/kg K] \right) \). Assuming that \( \dot{q} = 10^5 \) W/m\(^3\), \( \rho = 998.2 \) kg/m\(^3\) and \( c_p = 4182 \) j/kg K, it can be found that \( q = 0.024 \) K/s. Now,
we need to change the heat generation dimension from \( [K/s] \) to \( [LT/Lt] \) by \( q_L[LT/Lt] = q[K/s]/(30816 \, Lt/s \times 100 \, K/LT) \) since we knew 1 s = 30,816 Lt and 100 K = 1 LT. Following this process, one obtains \( q_L = 4.66 \times 10^{-7} [LT/Lt] \). Here, it should be mentioned that same heat generation was assumed for fluid flow and solid circles.

The results of Case #3 using LBM and FVM are shown in Figure 5. This figure shows good agreement between two different numerical results which clearly shows the ability of LBM to solve fluid flow and heat transfer with heat generation in the channels with obstacles. All matters were observed for Case #2 are visible for this case too. However, paying attention to the legends of both figures (Figures 4 and 5), it can be found that the maximum temperature inside the channel have increased from 315 K = 0.4185 LT to 324 K = 0.5085 LT. Also, Figure 5 shows that with the passage of time heat generation inside the channel shows itself more vivid. Paying attention to the end of the channel, one obtains that the temperature has been changed from 316 K at \( t = 60 \, s \) to 324 K at \( t = 420 \, s \).
3.4. Case #4

At Case #4 a bit complicated geometry was defined to switch from simple geometries to real porous media. Also, it was tried to highlight that by changing the grid size, all the lattice parameters should be calculated again. In Case #4 the grid size of 1500 \( LU \times 300 \) \( LU \) was chosen. Therefore \( \Delta x = \frac{1 \text{ cm}}{300} = 3.33 \times 10^{-5} \text{ m} \). Using the same process explained above, other parameters would be \( \Delta t = 2.16 \times 10^{-4} \text{ s}, \tau_{f,s}^* = 0.582, \tau_{g,s}^* = 57.16, \) and \( q_L = 5.18 \times 10^{-8}[\text{LT}/\text{Lt}] \).

Figure 6 shows the comparison between the LBM and the FVM results for Case #4. Comparing the results of LBM and FVM shows good agreement between them which denotes that LBM can solve fluid flow and heat transfer with heat generation in complex geometries.

Something interesting about this figure could be found paying attention to the two first obstacles (obstacles on the left). Since the obstacle in the bottom encounters with the cooler fluid sooner than the obstacle on the top, one obtains that in all time steps the temperature inside the obstacle in the bottom is lower than the temperature inside the obstacle on the top. Also, with the passage of time internal heat generation is more obvious considering the temperatures at the outlet side. Furthermore, because the fluid flows from the bottom of the last obstacle (obstacle on the right) always the temperature inside the obstacle is higher than the temperature of the fluid flow below it after \( t = 60 \text{ s} \). It clearly shows that fluid flow has transferred the internal generated heat by convection.

3.5. Cases #5–#7: Convection heat transfer with heat generation in porous media

For Cases #5–#7, a grid of 500\( LU \times 100LU \) was chosen for all porous geometries; therefore, parameters are the same as Case #3. It should be mentioned that preparing these geometries in FVM would be much harder than LBM. Consequently, LBM could be an appropriate method to simulate heat transfer with heat generation inside the porous media. Figure 7 shows the results for Case #6. This figure clearly shows that in the case of “With Heat Generation”, temperature inside the channel is higher than the case of “Without Heat Generation”, and this temperature difference becomes more visible as time goes by. Also, it can be found that on the area close to the pores with higher velocities (mostly, bottom of the channel) temperature is lower than the other area. It is because in the area close to pores with higher velocities both convection and
conduction transfer heat; however, in the area close to the pores with low velocities or even no fluid flow, it is thermal diffusion (conduction) that is dominant.

Figure 8 shows the temperature and velocity contours for Cases #5–#7. This figure clearly shows that on the area close to the pores where the velocity is higher and higher amount of mass flux happens, higher heat transfer occurs. In Case #6, higher velocity inside the pores occurs in the bottom of the channel; therefore, higher amount of heat transfers from the bottom of the channel. In addition, as illustrated in Case #5, there are almost uniform flows with same velocities in the flow channels through the pores which leads to almost uniform heat transfer inside the channel. Furthermore, results for Case #7 clearly show that without any fluid flow inside the porous medium, heat transfer would not be significant. It is because in a porous medium with fluid flow passing through the pores (Cases #5 and #6), both convection and conduction help to transfer heat; however, in a porous medium without any fluid flow passing through the pores (Case #7), just conduction transfers heat which would be too weaker.

Another interesting point of this figure is the comparison between the results of Case #5 and Case #6. As defined, the porosity of Case #5 and Case #6 are 0.9 and 0.76, respectively. In the
first impression, it comes to mind that Case #5 with higher porosity should have higher heat transfer compare to Case #6. However, the results show that heat transfer in Case #6 with lower porosity is higher than Case #5 with higher porosity. It happens because in Case #6 only one dominant flow channel through the pores transfer fluid flow from inlet to the outlet; However, in Case #5 there are three dominant channels through the pores that leads fluid flow from inlet to the outlet. Therefore, mass flux in the only dominant flow channel in Case #6 is higher that mass flux in the three dominant flow channels in Case #5. Higher mass flux leads to higher convection and causes higher heat transfer in Case #6 compare to Case #5.

4. Conclusions

In this study, a hierarchical process was defined to implement two-dimensional thermal LBM for investigation of convection heat transfer with heat generation in different geometries; from a simple flow channel to complex geometries such as porous media. Seven different cases with different geometries were designed and it was tried to explain how lattice parameters should be evaluated from physical and macroscale properties. By going from one case to the next, the more complex physics and/or geometries were examined. The results showed that LBM is an appropriate method to predict heat transfer with internal heat generation in porous media in the pore scale approach. Furthermore, it was found that heat transfer in porous media is not only related to the porosity and average pore size, but also related to the shape and construction of the porous medium. In the comparison between two random porous medium with different porosities and pore sizes, it is possible to have higher heat transfer for the porous medium with lower porosity and smaller average pore size.

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References


