Numerical simulation of transient forced convection in a square enclosure containing two heated circular cylinders

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Abstract

Purpose – Unsteady simulation of forced convection of two heated horizontal cylinders confined in a 2D squared enclosure. The paper aims to discuss this issue.

Design/methodology/approach – The finite-volume method is used to solve the transient heat transfer problem by employing quadrilateral mesh type. To solve the governing equations (conservations of mass, momentum and energy) on unstructured control volumes, a second-order quadratic upwind interpolation of convective kinematics scheme for the convection terms and the semi-implicit method for pressure-linked equations pressure correction algorithm were used.

Findings – The results indicate that the variation of the area-averaged Nusselt number strongly depends on the Reynolds number. On the contrary, the effect of cylinders’ space on heat transfer was found to be nearly negligible for Re < 460. It is also observed that steady state flow and heat transfer shift to periodical oscillation, and ultimately chaotic oscillation in non-dimensional cylinders distance of 0.1; however the sequence of appearing this route is completely different for higher cylinder spaces.

Research limitations/implications – Reynolds numbers between 380 and 550 and dimensionless horizontal distances of cylinders 0.1, 0.2 and 0.3.

Originality/value – Comprehensive knowledge of the effect of tube arrays flow regime on each other and in turn, heat transfer among them. Better understanding of convective heat transfer around an array of horizontal cylinders compared with from those around a single cylinder because of the mutual interaction of the buoyant plumes generated by the cylinders. Time-dependent phenomena of the problem including periodical oscillation or chaotic features.

Keywords Numerical simulation, Enclosure, Cylinder, Forced convection, Finite-volume method, 2D squared enclosure, Heated horizontal cylinders

Paper type Research paper

Nomenclature

\[ d \quad \text{diameter of cylinders, m} \]
\[ D \quad \text{dimensionless cylinders diameter} \]
\[ g \quad \text{gravitational acceleration, ms}^{-2} \]
\[ h \quad \text{heat transfer coefficient, Wm}^{-2}\text{K}^{-1} \]
1. Introduction

Natural, forced and mixed convections play key roles in many technological processes, such as solar collectors, energy storage systems, thermal design of buildings, air conditioning and electronics cooling (Forson et al., 2007; Bouhdjar and Harhad, 2002; Hung and Fu, 1999). Many researchers have studied natural, forced and mixed convection heat transfer in enclosures with the presence of a body and various thermal boundary conditions. Numerical investigation of combined free and forced convection in a square enclosure with a square heat conducting body inside the enclosure was conducted by Hsu and How (1999). The influences of the enclosure configurations and physical properties on flow and heat transfer were studied. Shuja et al. (2000) investigated the effect of exit port locations on the heat transfer characteristics and irreversibility generation in a square cavity with heat generating body with two extreme aspect ratios (0.25 and 4.0) and 15 different locations of the exit port. The two-dimensional, steady and laminar numerical simulations of mixed convection flow in a vented cavity with a heat conducting horizontal square cylinder was reported by Rahman et al. (2008). Using the finite element scheme based on the Galerkin method of weighted residuals, they investigated the effect of the inner cylinder position on the fluid flow and heat transfer in the cavity for different Richardson numbers (0-5). Rahman et al. (2013) also numerically studied the steady laminar mixed convection flow inside a vented square cavity with a heat conducting solid circular body placed at the center of the cavity. The effects of cylinder size (0.1-0.6) and thermal conductivity ratio range from 0.2 to 50 between solid and fluid on fluid flow and heat transfer performance were investigated. Rahman et al. (2009) investigated the combined free and forced convection in a two-dimensional rectangular cavity with a uniform heat source applied on the right vertical wall and a circular heat conducting horizontal cylinder within the cavity. The distributions of streamlines, isotherms and heat transfer coefficient were reported for wide ranges of the parameters, including Reynolds number, Richardson number, Prandtl number and some physical parameters. Rahman et al. (2012) numerically investigated mixed convective flow and heat transfer of a heated hollow cylinder (uniform heat flux) inside an open cavity attached with a horizontal channel for $10^3 < Ra < 10^5$, $0.7 < Pr < 7$ and ratio of solid to fluid
thermal conductivities from 0.2 to 50. Mamun et al. (2010) solved the mixed convection heat transfer characteristics within a ventilated square cavity with a heated hollow cylinder. They concluded that the cylinder diameter had significant effect on both the flow and thermal fields but the solid-fluid thermal conductivity ratio has significant effect only on the thermal field. Billah et al. (2011) analyzed fluid flow due to mixed convection in a lid-driven cavity having a heated circular hollow cylinder. The computation was carried out for wide ranges of the Richardson numbers, cylinder diameters and solid-fluid thermal conductivity ratios. It was concluded that the flow field and temperature distribution strongly depend on the cylinder diameter and also the solid-fluid thermal conductivity ratio at the three convective regimes.

Saeidi and Khodadadi (2006) also reported a finite-volume-based computational study of steady laminar forced convection inside a square cavity with inlet and outlet ports at different locations. The case of periodical forced convective laminar flow and heat transfer of various rotating objects (circle, square and equilateral triangle) with different sizes in the middle of a square cavity was investigated by Shih et al. (2009). Guimaraes and Da Silva (2010) studied the forced convection in an enclosure with a tube bank composed of 18 stationary cylinders including one heat transferring wall. Temperature and velocity distributions for various Reynolds numbers were summarized and some recirculation worked as isolation layers; it was observed that some tubes had negligible contribution on heat transfer and might be taken out of the tube set. Saeidi and Khodadadi (2007) numerically studied transient laminar forced flow and heat transfer leading to periodic state within a square cavity with inlet and outlet ports due to an oscillating velocity at the inlet port. They summarized the effect of sinusoidally variation of inlet velocity and instantaneous Reynolds number (100-500) on streamlines, temperature contours and Nusselt numbers.

Recently, Karimi et al. (2014) studied the unsteady natural convection from two heated horizontal cylinders in a square enclosure for Rayleigh numbers between $10^3$ and $10^7$ and dimensionless horizontal distances of cylinders between 0.1 and 0.4. They concluded that the effect of cylinder spacing on heat transfer is nearly negligible when the Rayleigh number is between $10^4$ and $10^7$. In spite of numerous investigations on natural and mixed convective flow in cavities with internal obstructions of various configurations, more complicated mixed convection flow and heat transfer patterns inside a cavity with two horizontal heated cylinders has attracted little attention up to now.

The heat sources and fluid streams must be properly arranged to obtain the maximal heat transfer rate which is a key parameter for heat exchanger design. In this field, the comprehensive knowledge of the effect of tube arrays flow regime on each other and in turn, heat transfer among them, is essential. Another important point is that the convective heat transfer around an array of horizontal cylinders is different from those around a single cylinder because of the mutual interaction of the buoyant plumes generated by the cylinders. In addition to great number of engineering applications of this flow system, time-dependent phenomena of the problem including periodical oscillation or chaotic features is another attractive theoretical aspect of the present study. In this work, an unsteady state forced convection of water around two isothermal heated cylinders in a square enclosure with adiabatic walls will be simulated. The emphasis will be on the effects of Reynolds number and distance of cylinders on the heat transfer characteristics.
2. Physical model

The physical model of the problem under consideration with the coordinate system is shown in Figure 1. A two-dimensional square enclosure of equal height (H) and width (L) contains two horizontal cylinders with diameter (d) placed in the center of enclosure height and separated by a distance (s). The cavity walls are all adiabatic and the uniform temperature on the surfaces of the heated cylinders is $T_H$. The cavity is subjected to an external flow entering the cavity from the left-middle opening and leaving from the right-middle opening. The size of the inlet port is the same as the exit one which are equal to one tenth of the cavity height (0.1H), while the incoming flow velocity is uniformly equal to $u_i$ and inlet temperature is uniformly $T_i$ ($T_H > T_i$). The convective boundary condition for outgoing flow is assumed, i.e. zero diffusion flux for all dependent variables. All solid boundaries are assumed to be rigid no-slip walls.

3. Mathematical formulation and boundary conditions

The governing equations describe conservations of mass, momentum and energy. The working fluid is water and the flow is laminar in all cases. To simplify analysis, the following assumptions are made for the two-dimensional forced convection:

1. the viscous dissipation and compressibility effects are negligible;
2. the radiation effects are assumed to be negligible; and
3. thermo-physical properties are constant.

Based on above assumptions, the governing equations can be expressed in the following dimensionless forms (Patankar, 1980):

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

Figure 1. Physical model
\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right)
\]  
\[
\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{RePr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

where the dimensionless parameters are defined as:

\[
\tau = \frac{u_i}{L}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_i}, \quad V = \frac{v}{u_i}, \quad P = \frac{p}{\rho u_i^2}, \quad \theta = \frac{T - T_i}{T_{hi} - T_i}, \quad \text{Re} = \frac{u d}{v}, \quad \text{Pr} = \frac{v}{\alpha}
\]

where \(u_i, v, g, \alpha\) and \(\beta\) are the inlet flow velocity, kinematic viscosity, gravitational acceleration, thermal diffusivity and volume expansion coefficient, respectively. The non-dimensional boundary conditions used for the present problem are specified as follows:

- at the inlet: \(U = 1, V = 0, \theta = 0\);
- on all solid cavity walls: \(U = 0, V = 0\), and \(\partial \theta/\partial n = 0\), where \(n\) is the unit normal coordinate on the cavity walls; and
- on the surface of the cylinders: \(U = 0, V = 0\) and \(\theta = 1\).

The non-dimensional diameter and cylinders distance are defined as:

\[
D = \frac{d}{L}, \quad S = \frac{s}{L}
\]

The local Nusselt number at the heated surface of cylinders is calculated from the following expression:

\[
Nu_{\varphi} = \frac{h_{\varphi} d}{\lambda} = -\frac{1}{\frac{\partial \theta}{\partial n}} \text{cylinders wall}
\]

where \(n\) is the unit normal coordinate to the surface of the cylinders.

The area-averaged Nusselt number of the cylinders is calculated by integrating the local Nusselt number along their walls as the following:

\[
\overline{Nu_s} = \frac{1}{2\pi} \int_0^{2\pi} Nu_{\varphi} d\varphi
\]

4. Numerical solution

The finite-volume method is used to solve the transient heat transfer problem by employing quadrilateral mesh type. To solve the governing equations on unstructured control volumes, a second-order quadratic upwind interpolation of convective kinematics scheme for the convection terms and the semi-implicit method for pressure-linked equations pressure correction algorithm were used (Patankar, 1980). The finite-volume method based CFD software package FLUENT (2010) is employed for computation. The convergence criteria of the steady state solutions is that the
maximum residual of continuity, X- and Y-momentum equations and energy equation are less than \(10^{-6}, 10^{-6}\) and \(10^{-9}\), respectively. The convergence of solutions is assumed when the relative error between consecutive iterations is recorded below the convergence criterion such that \(|\Psi_{k+1} - \Psi_k| \leq 10^{-3}\), where \(k\) is the number of iteration and \(\Psi\) can be \(\text{Nu}\) or \(\theta\).

### 4.1 Grid independency check

A schematic view of the grid distribution for the case of \(D = 0.2\) and \(S = 0.2\) is shown in Figure 2. The nodes arranged in a circular pattern near the cylinders and the grid sizes have been optimized. The grids near the cylinders walls uniformly distributed in order to get enough accuracy for the high temperature gradients. In an unsteady solution, the numerical accuracy strongly depends on both grid density and time-step intervals. For determining the quality of the mesh and time-step sizes, a grid-independence test is carried out for the case of \(D = 0.2, S = 0.2, \text{Ri} = 0\) at maximum studied Reynolds number, i.e. \(\text{Re} = 550\). Five combinations of different grid sizes (13,544, 18,346 and 23,701 cells) and non-dimensional time-steps (0.005522 and 0.011044) are considered as listed in Table I. The time-averaged Nusselt number over the surface of two cylinders is used as a parameter for comparing numerical accuracy. The relative deviation of the time-averaged Nusselt number for each case is

![Figure 2. Schematic view of the enclosure and the grid generated for \(D = 0.2\) and \(S = 0.2\)](image)

| Table I. Independence test of grid and time-step intervals for \(\text{Re} = 550, D = 0.2\) and \(S = 0.2\) |
|-----------------------------------------------|-------------------------------|---------------|-----------------|
| Number of cells                          | Non-dimensional time-step size | \(\text{Nu}_t\) | Deviation (%)   |
| 13,544                                   | 0.011044                      | 21.18         | 3.55            |
| 13,544                                   | 0.005522                      | 21.27         | 3.14            |
| 18,346                                   | 0.011044                      | 21.68         | 1.28            |
| 18,346                                   | 0.005522                      | 21.71         | 1.14            |
| 23,701                                   | 0.005522                      | 21.96         | 0.00            |
calculated as well with respect to the case of the smallest grid and time-step intervals, i.e. total number of 23,701 cells and non-dimensional time-step 0.005522.

It can be seen in Table I that the maximum deviation between two the extreme cases is less than 4 percent. The highest relative deviation between two different time-step sizes is only 0.41 percent when the number of cells is 13,544. In addition, increasing the number of cells from 18,346 to 23,701 at dimensionless time-step of 0.005522 leads to a deviation less than 1.5 percent. Therefore, when the total number of cells is greater than 18,346 and time-step size is smaller than 0.011044, further improvement on temporal and spatial resolutions would not result in desirable gain in accuracy and the numerical results of $\overline{\text{Nu}}$ become nearly independent of the grid size and time-step intervals. Therefore, the trade-off between numerical accuracy and computational cost suggests using intermediate resolutions, i.e. a total number of 18,346 cells and time-step size of 0.011044. To ensure the accuracy of the numerical simulation, the grid sensitivity was also checked by a series of grid sizes of 1,746, 3,872, 8,966, 13,544, 18,346 and 23,701 cells. The local Nusselt number at the lowest point of the left cylinder ($\varphi = 270^\circ$) was calculated for grid independency test at Re = 550, D = 0.2 and $S = 0.2$. Figure 3 presented the results of grid independency check and the numerical grid of 18,346 cells was selected in this work. In this study, when the geometry of the problem cases changes, the grid resolution is refined to obtain accurate results.

4.2 Validation
In order to validating the numerical method, the forced convection heat transfer from a horizontal cylinder placed in a free air stream was tested for Reynolds numbers from 2 to 200. The calculated surface-averaged Nusselt numbers for the test case were compared with the following empirical correlation (Cengel, 1998) and the experimental formulas suggested by McAdams (1954), Kramers (1946) and Jain and Goel (1976) as shown in Table II:

$$\text{Nu} = C \text{Re}^n \text{Pr}^{1/3}$$

(9)

where C and n are constants which are determined for different Reynolds numbers. As seen from the results, the computed area-averaged Nusselt numbers obtained in the present study are in good agreement with the values extracted from the empirical correlation.
In addition, simulations were carried out to study the forced convection in a rectangular enclosure with isothermal walls. The predicted results were compared with Saeidi and Khodadadi (2006). Detailed comparisons for streamlines and isotherms were presented in Figure 4. The results showed good agreement between the present program and the previous work (Saeidi and Khodadadi, 2006).

5. Results and discussions

The numerical study of forced convection heat transfer from horizontally aligned cylinders in the enclosure is performed in a wide range of Reynolds numbers from

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Figure 4.
Streamlines and isotherms for Re = 500

Source: (a) Saeidi and Khodadadi (2006) and (b) present
380 to 550 in which the steady state mode is transmitted to the chaotic regime. In this paper, the effects of Reynolds number and the distance of cylinders on the flow field and heat transfer are investigated. The non-dimensional diameter of the cylinders is kept constant (D = 0.2).

5.1 Unsteady flow and heat transfer characteristics

Among the several methods to recognize the route to chaos such as analyzing time history, phase space trajectory and power spectral method (Kantz, 2003), the time histories of the area-averaged Nusselt number and phase space of velocity provide an easy way to identify the route to chaos.

Area-averaged Nusselt numbers $\overline{Nu}$ on cylinders with different values of $S$ are presented in Figures 5-7 for Reynolds numbers from 380 to 550. In order to better visual presenting of every case, some limited parts labeled as "details" which show the enlarged view of a portion of whole scale figures. For instance, Figure 5(a) reveals the variation of $\overline{Nu}$ for $Re = 500$ and 550 in the range of $8000 \leq \tau \leq 8014$. Moreover, the period and amplitude for periodical oscillation cases, such as Figure 5(b), can be clearly tracked. Generally, the magnitude of $\overline{Nu}$ is the minimum at $Re = 380$ and the maximum at $Re = 550$. As can be seen from Figure 6, the value of $\overline{Nu}$ for $Re = 460$ is higher than that of $Re = 500$ which makes this case as special one compared with other cylinder distances.

For the purpose of analyzing the unsteady characteristics of the problem, the sample point of the cavity domain is selected. Selection of a particular point is completely random since the results would be steady, fluctuated periodically or non-periodically at every point in the entire domain if the results are steady state, oscillatory or chaotic. Consequently, the numerical solutions of velocity at selected sample point (0.6, 0.7) were obtained at $S = 0.1$ and different Reynolds numbers and phase space results are presented in Figure 8.

It is observed from Figure 5 that the area-averaged Nusselt number at $Re = 380$ is constant over time, and the related phase trajectory of velocity at the sample point is a path that eventually reaches a point as shown in Figure 8(a). In the next stage, a periodic oscillation occurs at $Re = 460$ which leads to a closed loop trajectory path in Figure 8(b). However, by losing the periodical characteristic at $Re = 500$, chaotic solution finally appears as shown in Figure 8(c).

Qualitative view of the long-term results from numerical simulations is summarized in Table III. The flow regimes are categorized by steady, unsteady periodic oscillation and unsteady chaotic oscillation. Considering thermal and flow patterns, steady ones are those that the thermal and flow fields appear to be constants over time. In contrast, regular fluctuation of unsteady results is referred to as unsteady periodical oscillation and the thermal patterns are labeled as unsteady chaotic oscillation in case of complicated non-regular oscillating flows. The interesting point of Table III is that the route of steady state-periodical oscillation-chaotic oscillation cannot be tracked for dimensionless cylinders’ distances 0.2 and 0.3.

Another parameter that is considered in this paper is the time-averaged Nusselt number that is calculated over a period of time as below:

$$\overline{Nu_t} = \frac{1}{T} \int_{t}^{t+T} \overline{Nu_s} dt \quad (10)$$

Based on the time-dependent area-averaged Nusselt numbers obtained for various horizontal distances between heated cylinders and Reynolds numbers, the
time-averaged Nusselt numbers are obtained and shown in Figure 9. It is important to mention that $\overline{Nu_t}$ for the periodic oscillating cases is calculated for the interval time of one oscillation period from time-varied Nusselt number data. In addition, in case of non-regular oscillating of Nusselt number, two equal periods of time randomly selected
by the aim of that the maximum relative difference of corresponding time-averaged Nusselt numbers were lower than 3 percent.

Figure 9(a) reveals that $N_u$ ascends smoothly from $Re = 380$ to 420, and then sharply increases to the range of 16-19, at $Re = 460$. It can also be seen that various cylinders distances result in different values of time-averaged Nusselt number at $Re = 500$, but the
Figure 7. Time history of area-averaged Nusselt number over the cylinders for different Reynolds numbers, $S = 0.3$
value of $\overline{\kappa u_t}$ is almost the same at the highest Reynolds number ($Re = 550$). This means that $\overline{\kappa u_t}$ is increased by enlarging the cylinders distance at low Reynolds numbers $380 < Re < 460$. However, in cases of higher Reynolds numbers, the effect of distance between the cylinders on the heat transfer characteristics should be studied case by case. Figure 9(b) shows that greater cylinders distance results in slightly higher Nusselt
number in Re = 380 and 420, but this trend cannot be simply tracked for higher Reynolds numbers. Moreover, $\overline{Nu}$ decreases at first in some cases and then increase again as can be seen for the corresponding line of Re = 500 and 550. The detailed reasons will be discovered from flow patterns and isotherms in the following subsections.
5.2 Flow features
In this section, attention is focussed on the effects of convective flow patterns on the overall heat transfer from the cylinder surfaces. Following the purpose of discovering the reasons of $\overline{\text{Nu}}_c$ variation, flow patterns inside the enclosure are presented by means of streamlines. Due to the transient inherence of the solutions, the snapshots of streamlines and isotherms should be carefully analyzed and compared in different solution time to find the exact reason of $\overline{\text{Nu}}_c$ variation. In this section, the streamlines of the cavity containing two horizontal heated cylinders at different distances and Reynolds numbers at $\tau = 8,500$ are presented in Figure 10. For each set of the individual cavities shown, two arrows are drawn to help the reader to clearly identify the inlet and outlet ports. Generally, the cold inlet fluid move over the left heated cylinder until it reaches the second one before leaving the cavity. Figure 10(a) shows that the incoming flow mainly passes over the left cylinder and produces symmetric flow pattern corresponding to the inlet-outlet axis line. In addition, it is clearly observed that the outer and inner rotating vortices occupy a considerable space. By increasing $S$ from 0.1 to 0.3, the circulating vortices near the inlet port become smaller, and the right hands’ vortices changed their position toward the top and down of the right cylinder. When the Reynolds number is increased to 420 (see Figure 10(b)), one can observe that the flow fields of $S = 0.1$ and 0.3 at $\text{Re} = 420$ are appeared very similar to those of $\text{Re} = 380$ (e.g. Figure 10(a)); however, the size of inner vortices become smaller. The symmetry of flow patterns is lost in case of $S = 0.2$ since the flow turned to oscillating as presented in Figure 6.

When Reynolds number increases further to 460, as shown in Figure 10(c), the flow features turn to unsteady mode which results in losing their symmetric shape in all cases of Figure 10(b). The inner vortices move to the space between the cylinders and the outer ones move to the right side of the enclosure in cases of $S = 0.1$ and 0.2. In the case of $S = 0.3$, on the contrary, there are four inner vortices that are positioned between the cylinders and right hand’s one. Figure 10(d) indicates that the inner vortices almost disappear at $S = 0.1$. Also, the flow becomes weaker at the upper part of the cavity. Moreover, at $S > 0.1$, the inner rotating vortices are valid and still strong. Considering Figures 10(c)-(d), it is clear that downward main stream take distance from the right cylinder in $\text{Re} = 500$ compared with those of $\text{Re} = 460$. The streamlines become more complex at $\text{Re} = 550$ as shown in Figure 10(e). The flow passes over the surface of both cylinders in cases $S = 0.1$ and 0.2, although the main streamlines concentrate at the bottom part of the cavity after passing over the left cylinder at $S = 0.3$ and do not surround the right cylinder anymore.

5.3 Heat transfer results
Figure 11 shows the snapshots of isotherms for different cylinders distances and Reynolds numbers at $\tau = 8,500$. It is observed from Figure 11(a) that isotherms are symmetric in all $S$ values because of steady state behavior of the flow at $\text{Re} = 380$. The temperature fields lose their symmetry at $\text{Re} = 420$, $S = 0.2$ and transform to asymmetric structure because of periodic oscillation. Increasing Reynolds number to $\text{Re} = 460$, it is evident from Figure 11(c) that the isothermal lines become denser at $S = 0.1$ and 0.2. In these cases, the inlet flow just makes effect on very small area of the cavity, so an intensive heat exchange takes place in such a small area near the cylinders. Although the isotherms of the left hand cylinder at $S = 0.2$ are almost similar for $\text{Re} = 460$ and 500, they are denser in right one of the case of $\text{Re} = 460$ which is why $\overline{\text{Nu}}_c$ at $\text{Re} = 460$ is higher than $\text{Re} = 500$. 

By increasing the Reynolds number to its highest value, Re = 550, the isotherms of every cylinder become more separated especially in cylinder distances of 0.2 and 0.3. More information of the described features can be obtained from the distributions of local Nusselt number over both cylinders as shown in Figure 12 where cases are chosen...
for $S = 0.1$, 0.2 and 0.3 and Reynolds numbers 420 and 500. The maximum local Nusselt number occurs at the stagnation point of the left cylinder ($\phi = 180^\circ$) for all cases due to encountering the coldest fluid to the left heated cylinder, and hence, the maximum heat transfer occurs at this point. The local Nusselt number distribution over the left

**Figure 11.** Snapshots of isotherms for different cylinder distances and Reynolds numbers at $\tau = 8,500$: (a) $Re = 380$, (b) $Re = 420$, (c) $Re = 460$, (d) $Re = 500$ and (e) $Re = 550$
cylinder is symmetric for Re = 420 in all S values; however the same is not true for the right cylinder. Moreover, the distribution of dimensionless heat transfer coefficient over the surface of left and right cylinders clearly shows that heat transfer is completely asymmetric at Re = 500. The values of maximum local Nusselt number of left cylinders for both Reynolds numbers of S = 0.3 are more than those of S = 0.1 and 0.2.

Figure 12.
Comparison of local Nusselt number over the left horizontal cylinder (left column) and right horizontal cylinder (right column) for different Reynolds numbers: (a) S = 0.1, (b) S = 0.2 and (c) S = 0.3; Results are plotted at \( \tau = 8,500 \).
In contrary, as can be seen from Figure 12(c), the values of $Nu_\phi$ on the right cylinder of $S = 0.3$ case are lower than those of other cylinder distances. To identify the reason, referring to Figure 10(b), it can be noticed that the inner vortices move to up and down space of the right cylinder at $S = 0.3$, while they concentrate at the right side of that cylinder at cases of $S = 0.1$ and 0.2. The presence of vortices leads to increasing the local Nusselt number at $\phi = 0$ at $S < 0.3$ and $Re = 420$.

6. Conclusions
Numerical simulation of the unsteady forced convection from two heated cylinders confined in a two-dimensional square enclosure with adiabatic walls was performed. For the non-dimensional cylinders diameter ($D$) of 0.2, Reynolds number was varied from 380 to 550 and the effects of cylinders distance were investigated. Development of convective flow and heat transfer was discussed by analyzing the time variation of area-averaged Nusselt number over the walls of heated cylinders and different heat transfer phases including steady state, periodic oscillation and chaotic oscillation. Moreover, phase space of velocity at sample points for various solution phases was observed.

It is concluded that heat transfer mechanism in a cavity with two heated horizontally aligned cylinders is a complex function of Reynolds number and cylinders distance. The route of steady state, periodical oscillation and chaotic oscillation is not exactly traceable for $S = 0.2$ and 0.3 in the studied problem. It is observed that by increasing the Reynolds number in case of $S = 0.2$, steady state flow and heat transfer at $Re = 380$ undergoes chaotic oscillation at $Re = 460$ and finally turns to periodical oscillation at $Re = 500$. The time-averaged Nusselt number have a smooth ascending trend by enlarging cylinders distance at low Reynolds numbers ($Re < 460$). Increasing Reynolds number to 460 and higher, the variation of $Nu_t$ is different from case to case, especially for $Re = 500$ that the value of $Nu_t$ is the minimum at $S = 0.2$ compared with $S = 0.1$ and 0.3. The symmetric flow patterns in the cavity at $Re = 380$ becomes asymmetric by increasing Reynolds number. Moreover, outer vortices which are placed at the left part of the enclosure, shift to right hand at higher Reynolds numbers, and influence the flow stream over the right horizontal cylinder as well. At Reynolds numbers lower than 550, thermal plumes of the cylinders combines with each other at low cylinders’ spacing ($S = 0.1, 0.2$), but every cylinder has a separated plume at cylinder distances of 0.2 and 0.3 when $Re = 550$.

References


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