Effects of slotted structures on the nonlinear characteristics of natural convection in a cylinder with an internal concentric slotted annulus

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ABSTRACT
Natural convection in a cylinder with an internally slotted annulus was solved by SIMPLE algorithm, and the effects of different slotted structures on the nonlinear characteristics of natural convection were investigated. The results show that the equivalent thermal conductivity $K_{eq}$ increases with Rayleigh number, and reaches the maximum in the vertical orientation. Nonlinear results were obtained by simulating the fluid flow at different conditions. With increasing Rayleigh number, heat transfer is intensified and the state of heat transfer changes from steady to unsteady. We investigated the effects of different slotted structures on natural convection and analyzed the corresponding nonlinear characteristics.

1. Introduction
Nonlinear dynamics of natural convection in closed cavities have attracted increasing interest in recent years. Quere et al. [1–3] studied natural convection nonlinear phenomena in square cavities using a spectral method. Li et al. [4–6] studied natural convection in a square cavity using various numerical methods. Liu and Tao [7] investigated periodic oscillations in a closed rectangular cavity, and their results showed that at $Ra = 2.0 \times 10^4$ the system was steady symmetric, at $Ra = 2.0 \times 10^5$ to $10^6$ the system performed nonsymmetrical oscillation, and at $Ra = 2.0 \times 10^6$ the flow turned into counter-periodical oscillation. Liaqat and Baytas [8] employed the SIMPLER algorithm to simulate natural convection in a square cavity with an internal heat source at a high Rayleigh number. Their results reached periodic oscillation for the case of $Ra < 10^7$ and non-periodic oscillation when $Ra > 10^8$. Fusegi et al. [9] used the SIMPLE algorithm with the QUICK scheme to simulate natural convection in a three-dimensional square cavity. Li et al. [10] discussed the nonlinear results of natural convection melting problems in a square cavity. Ishida et al. [11] and Benouaguef et al. [12] studied the nonlinear characteristics of natural convection in an inflatable square cavity using different numerical methods.

Bishop [13] and Powe et al. [14] experimentally studied natural convection in a closed helium-filled annular space. Yoo [15] reported that when the Rayleigh number was greater than a critical value, a stable solution was observed. Rao et al. [16] investigated heat convection in an annular space and obtained the critical Rayleigh number for the flow regime transition under different conditions. Net et al. [17] and Mizushima et al. [18] studied the development processes of flow and the change of heat transfer with time, and discussed the transition from steady state to chaos in detail. Fant et al. [19, 20] studied the flow and heat transfer in a two-dimensional annular space. Yoo [21, 22] studied the nonlinear phenomena occurring in a narrow annulus at different parameters.
Kuleek [23] studied heat transfer in a circle with an internal slotted annulus and proposed an approximate formula. Wang et al. [24] conducted experiments to investigate natural convection in a current bus. Zhang et al. [25,26] experimentally studied a horizontal cylinder with a slotted octagonal structure. Yang et al. [27] discussed multiple solutions for natural convection in the system. In this paper, we will investigate the effects of a slotted structure on flow and heat transfer, and analyze their influence on nonlinear characteristics.

2. Physical model

Figure 1 shows the physical model of the problem under consideration. It is assumed that the natural convection is two-dimensional, and the inner and outer surfaces are kept at constant temperatures of $T_i$ and $T_o (T_i > T_o)$. Moreover, $\phi$ is the slotted direction deflection angle, and $\phi = 90^\circ$ for the horizontal slot. The other geometric parameters are as follows: $\eta = r_i/r_o = 5/13$, $S = 2a/\pi = 0.1$, and $\delta = 0.1125$. The working fluid is incompressible, and Boussinesq assumption is employed. Nonslip boundary conditions are used for all solid walls, and viscous dissipation effect in the energy equation is neglected.

![Figure 1](image-url)  
Figure 1. The closed circle within the slotted round mathematical model.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Greek symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p$ specific heat (J/kgK)</td>
<td>$\phi$ angle of slot from the vertical (rad)</td>
</tr>
<tr>
<td>$F$ nondimensional time</td>
<td>$\mu$ viscosity (Kg/ms)</td>
</tr>
<tr>
<td>$g$ gravitational acceleration (m/s$^2$)</td>
<td>$\rho$ density (Kg/m$^3$)</td>
</tr>
<tr>
<td>$p$ pressure (Pa)</td>
<td>$\Theta$ nondimensional temperature</td>
</tr>
<tr>
<td>$P$ nondimensional pressure</td>
<td>$\theta$ angular coordinate</td>
</tr>
<tr>
<td>$Pr$ Prandtl number</td>
<td>$\Delta t$ nondimensional time step</td>
</tr>
<tr>
<td>$Q$ heat generation by the heaters (W/m)</td>
<td>$\sigma$ thermal diffusivity (m$^2$/s)</td>
</tr>
<tr>
<td>$Ra$ Rayleigh number</td>
<td>$\beta$ thermal expansion coefficient (1/K)</td>
</tr>
<tr>
<td>$r$ radial coordinate</td>
<td>$\delta$ thickness of the inner cylinder</td>
</tr>
<tr>
<td>$R$ nondimensional radial coordinate</td>
<td></td>
</tr>
<tr>
<td>$r_i$ radius of inner slotted cylinder (m)</td>
<td></td>
</tr>
<tr>
<td>$r_o$ radius of outer cylinder (m)</td>
<td></td>
</tr>
<tr>
<td>$S$ nondimensional internal heat generation</td>
<td></td>
</tr>
<tr>
<td>$t$ time (s)</td>
<td></td>
</tr>
<tr>
<td>$T_o$ temperature of outer walls (K)</td>
<td></td>
</tr>
<tr>
<td>$T_i$ temperature of inner walls (K)</td>
<td></td>
</tr>
<tr>
<td>$U$ nondimensional velocity in the radial coordinate</td>
<td></td>
</tr>
<tr>
<td>$v_r$ radial velocity</td>
<td></td>
</tr>
<tr>
<td>$v_\theta$ tangential velocity</td>
<td></td>
</tr>
<tr>
<td>$V$ nondimensional velocity in the tangential coordinate</td>
<td></td>
</tr>
</tbody>
</table>
This problem can be described by the following equations:

Continuity equation:

\[
\frac{1}{r} \frac{\partial}{\partial r} (r \cdot v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0
\]  

Momentum equations:

\[
\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + v_\theta \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} = -\frac{1}{\rho} \frac{\partial p'}{\partial \theta} - \beta \cdot g \cdot (T - T_r) \cdot \sin \theta
\]

\[
+ \nu \cdot \frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} + \frac{v_\theta^2}{r^2} - \nu \frac{\partial v_\theta}{\partial r}
\]

\[
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_\theta \frac{\partial v_r}{\partial \theta} - \frac{v_\theta v_r}{r} = -\frac{1}{\rho} \frac{\partial p'}{\partial r} + \beta \cdot g \cdot (T - T_r) \cdot \cos \theta
\]

\[
+ \nu \cdot \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{v_\theta^2}{r^2} - \frac{v_\theta v_r}{r^2}
\]

Energy equation:

\[
\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_\theta \frac{\partial T}{\partial \theta} = a \cdot \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{\rho \cdot r^2} \frac{\partial^2 T}{\partial \theta^2} \right)
\]

Equations (1–4) are subject to the following boundary and initial conditions:

\[ r_1 - \delta \leq r \leq r_1, \alpha \leq 0 \leq \pi - \alpha \]

or

\[
\pi + \alpha \leq 0 \leq 2\pi - \alpha : v_r = v_\theta = 0, T = T_h
\]

\[ r = r_o : v_r = v_\theta = 0, T = T_c \]

Defining the following nondimensional variables:

\[
R = r/L, F = at/L^2, U = v_\theta/u_R, V = v_r/u_R, P' = P/(\rho u_R^2),
\]

\[
\Theta = (T - T_c)/(T_h - T_c), Ra = \beta g L^3 (T_h - T_c)/(\alpha \cdot \gamma), U_R = (Ra \cdot Pr)^{1/2} (a/L)
\]

Equations (1–4) become

\[
\frac{\partial V}{\partial F} + V \frac{\partial V}{\partial R} + U \frac{\partial V}{\partial \theta} = \frac{\partial P'}{\partial R} + \frac{Pr}{(Ra Pr)^{1/2}} \frac{1}{R} \frac{\partial}{\partial \theta} \left[ \frac{1}{R} \frac{\partial V}{\partial R} + \frac{1}{R^2} \frac{\partial^2 V}{\partial \theta^2} \right] + S_R
\]

\[
\frac{\partial U}{\partial F} + V \frac{\partial U}{\partial R} + U \frac{\partial U}{\partial \theta} = -\frac{1}{R} \frac{\partial}{\partial \theta} \left[ \frac{1}{R} \frac{\partial U}{\partial R} + \frac{1}{R^2} \frac{\partial^2 U}{\partial \theta^2} \right] + S_0
\]

\[
\frac{\partial \Theta}{\partial F} + V \frac{\partial \Theta}{\partial R} + U \frac{\partial \Theta}{\partial \theta} = \frac{1}{(Ra Pr)^{1/2}} \frac{1}{R} \frac{\partial}{\partial \theta} \left[ \frac{1}{R} \frac{\partial \Theta}{\partial R} + \frac{1}{R^2} \frac{\partial^2 \Theta}{\partial \theta^2} \right]
\]

where

\[
S_\theta = -\frac{U V}{R} + \frac{Pr}{(Ra Pr)^{1/2}} \left(-\frac{U}{R^2} + \frac{2}{R^2} \frac{\partial V}{\partial \theta} \right) - \Theta \sin \theta
\]

\[
S_R = -\frac{U^2}{R} + \frac{Pr}{(Ra Pr)^{1/2}} \left(-\frac{V}{R^2} + \frac{2}{R^2} \frac{\partial U}{\partial \theta} \right) + \Theta \cos \theta
\]
The corresponding periodic boundary conditions are
\[ U(\theta = 0, R) = U(\theta = 2\pi, R), \quad V(\theta = 0, R) = V(\theta = 2\pi, R), \]
\[ \Theta(\theta = 0, R) = \Theta(\theta = 2\pi, R) \]  
(14)

For the inner slotted annulus, the following conditions must be satisfied:
\[ R_i - \delta / L \leq R \leq R_i, \quad \alpha \leq 0 \leq \pi - \alpha, \quad U = V = 0, \quad \Theta = 1 \]
\[ R_i - \delta / L \leq R \leq R_i, \quad \gamma + \pi \leq 0 \leq 2\pi - \alpha, \quad U = V = 0, \quad \Theta = 1 \]  
(15)

The initial conditions are
\[ F = 0, \quad U = V = \Theta = 0 \]  
(16)

In order to describe the strength of the heat transfer under different Rayleigh numbers, the average equivalent thermal conductivity is defined as
\[ K_{eq} = \frac{Q}{2\pi(T_i - T_o) \ln r_o / r_i} \]  
(17)

The whole heat transfer area includes the entire inner surface of the outer cylinder and two half-annular surfaces. To compare with the existing experimental results [21], the revised average thermal conductivity can be defined as
\[ K_{eqs} = \frac{K_{eq}}{1 - S}, \quad S = 2\alpha / \pi \]  
(18)

3. Results and discussions

3.1. Numerical method assessment

We can obtain the best grid number and the more accurate results through the grid-independent test. At the time step of \( \Delta t = 0.01 \), the \( K_{eq} \) obtained with different grid numbers and the results are shown in Table 1. It can be seen from Table 1 that on increasing the grid number, \( K_{eq} \) decreases. However, when the grid number reaches 100(\( \theta \)) \( \times \) 50(\( r \)), \( K_{eq} \) changes slowly and, therefore, we consider that the results reached grid number-independence; thus, the grid number used in the rest of the simulations will be 100(\( \theta \)) \( \times \) 50(\( r \)). The time step-independent test is then carried out with the grid number of 100(\( \theta \)) \( \times \) 50(\( r \)). Table 2 shows the relationship between time step and \( K_{eq} \) under different \( Ra \) values. It can be seen that \( K_{eq} \) decreases with decreasing time step; when \( \Delta t \) reaches 0.01, \( K_{eq} \) changes insignificantly with time step. Therefore, this time step will be used in this work.

The results will also be compared with the experiment results under the same conditions. To simulate the experimental results [24] using the SIMPLE method, the geometry settings are as follows: \( r_o = 130mm, \quad r_i = 49mm, \quad \delta = 4.5mm, \quad S = 0.1 \), and \( r_o - r_i = 40.5 \). The corresponding dimensionless parameters are as follows: \( R_o = 1.625, \quad R_i = 0.625, \quad \) and \( \delta = 0.1125 \). The empirical correlation of \( K_{eqs} \) is [24]
\[ K_{eqs} = 1 + (0.0894Ra^{0.3478})^{7.884} \]  
(19)
\[ K_{eqs} = 0.181Ra^{0.281}, \quad 4.5 \times 10^4 \leq Ra \leq 10^6 \]  
(20)

<table>
<thead>
<tr>
<th>grid number</th>
<th>60 × 40</th>
<th>80 × 40</th>
<th>100 × 50</th>
<th>150 × 50</th>
<th>190 × 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \times 10^5 )</td>
<td>4.862</td>
<td>4.840</td>
<td>4.801</td>
<td>4.798</td>
<td>4.796</td>
</tr>
<tr>
<td>( 1 \times 10^6 )</td>
<td>9.107</td>
<td>9.006</td>
<td>8.945</td>
<td>8.939</td>
<td>8.936</td>
</tr>
</tbody>
</table>
Figure 2 shows the comparison between simulation and experimental results. It can be seen that the simulation results and the experimental results generally agreed well. Especially, when the Rayleigh number is less than $6 \times 10^5$, the results from simulation and experiments are consistent. Moreover, the Rayleigh number is higher than $10^6$, and the deviation is less than 6%. Therefore, the natural convection characteristics can be obtained through the simulation results.

Table 2. The assessment result of time step.

<table>
<thead>
<tr>
<th>$\Delta t$ (time step)</th>
<th>0.001</th>
<th>0.005</th>
<th>0.008</th>
<th>0.01</th>
<th>0.05</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^5$</td>
<td>4.794</td>
<td>4.796</td>
<td>4.801</td>
<td>4.811</td>
<td>4.816</td>
<td>4.823</td>
</tr>
<tr>
<td>$1 \times 10^6$</td>
<td>8.889</td>
<td>8.93</td>
<td>8.939</td>
<td>8.945</td>
<td>8.948</td>
<td>0.954</td>
</tr>
</tbody>
</table>

Figure 3. $K_{eq}$ changes with $Ra$ values.
3.2. Effects of slot directions

The effective thermal conductivities at different slotted directions are calculated, and the results are shown in Figure 3. It can be seen that \( K_{eq} \) increases with \( Ra \) under the same slot direction. At higher Rayleigh number, especially above \( 6 \times 10^5 \), \( K_{eq} \) cannot reach a stable solution and the flow showed nonlinear oscillation. Figure 4 shows \( K_{eq} \) changing with time at different \( Ra \), reflecting the state of heat transfer in the system under different \( Ra \). At \( Ra = 10^5 \), \( K_{eq} \) increases with time. When

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{Variation of \( K_{eq} \) in different \( Ra \) values.}
\end{figure}
dimensionless time $F$ is 160.0, $K_{eq}$ is 3.46. When $Ra$ increases to $10^6$, $K_{eq}$ increases significantly before $F = 150$. When $F = 300$, $K_{eq}$ shows stable periodic fluctuations. On increasing the Rayleigh number, the periodic fluctuations disappear and begin to show non-periodic oscillation. Finally, when the Rayleigh number goes beyond $5 \times 10^6$, the flow in the system becomes completely irregular and chaotic.

Under the same parameters, numerical solutions can reach two or more different results. As shown in Figure 5, when $Ra$ is $10^6$, there are three different temperature fields in the annular space. This is

Figure 5. Static bifurcation phenomenon.
called the static bifurcation phenomenon, which is an important characterization of nonlinear characteristics; these results are longitudinal axisymmetric.

On increasing the Rayleigh number, the state of heat transfer changes constantly. Figure 6 shows a temperature field evolution process for the case that $Ra = 10^6$. After a complete period, the temperature field returns to the initial state. In other words, the temperature field is the same for $F = 699.9$ and $F = 166.6$. Using the numerical approximation method, we obtained the critical Rayleigh numbers corresponding to the orientation of the annulus, and the results are summarized in Figure 7. It can be seen that the critical Rayleigh numbers are maximum, both for steady-state to period state and for period state to aperiodic state, for vertical orientation, indicating that heat transfer reaches the maximum, which is consistent with the results depicted in Figure 3. The critical Rayleigh numbers are minimum at $\phi = 45^\circ$, indicating that at this time the thermal state is the easiest to change.

### 3.3. Effects of slotted degrees

Figure 8 shows isotherms at different slotted degrees. The primary vortex of the temperature field changes with increasing slotted degree. However, when the slotted degree exceeds a specific value, it resembles a pair of counterclockwise vortices at the top of the annular. The emergence of the vortex improves the heat convection in the circle, and hence the slotted cylinder can enhance the heat

![Figure 6. Periodic oscillation phenomenon.](image)
transfer in the system. However, this improvement is not a monotonic function of the slotted degree. Because a greater slotted degree would lead to more cold fluid entering the cylinder, which prevents heat dissipation, the heat transfer will decrease as well. Under certain radius ratio and Prandtl number, there is an optimal slotted degree at which $K_{eq}$ reaches the maximum and heat transfer is also the strongest.

Figure 9 shows the change of $K_{eq}$ with critical $S_f$. It can be seen that when the annular space is at $S_f = 0.13$, natural convection is the strongest. When $Ra$ is $5 \times 10^5$, the $K_{eq}$ without slot ($S_f = 0$) is 5.31. If $S_f$ is 0.13, $K_{eq}$ reaches 7.62. In other words, the heat transfer is increased by 43%.

Figure 7. Critical $Ra$ for different slotted directions.

Figure 8. Temperature fields in different slotted degrees.
Figure 9. $K_{eq}$ in critical $S_f$.

Figure 10. $K_{eq}$ changes with the dimensionless time (F).
Figure 10 shows the natural convection development process for $S_f = 0.1$. The dashed line and the dotted line, respectively, represent the $K_{eq}$ development curves of the left and right half of the annulus. It can be seen from Figure 10a that the $K_{eq}$ no longer changes after a certain time, and eventually heat transfer reaches steady state for the case that $Ra$ is $10^6$. On the contrary, $K_{eq}$ changes to periodic oscillation as shown in Figure 10b when $Ra$ is $2 \times 10^6$. Figure 11 shows the critical Rayleigh number at which the convection changes from period to aperiodicity for different slotted degrees. Owing to the nonlinearity of the system, we obtained just one of the oscillation solutions. When the state of natural convection changes, the critical Rayleigh number in different slotted degrees becomes random, and the solutions of the system showed nonlinear features.

Meanwhile, as the Rayleigh number continues to increase, the temperature field became asymmetric in the annular space. Under unsteady oscillation, the semicircle vortex begins to appear alternately. Using the numerical approximation method, we obtained the critical Rayleigh number.

![Figure 11](image1.png)  
Figure 11. Critical $Ra$ with slotted degrees.

![Figure 12](image2.png)  
Figure 12. Critical $Ra$ in different $S_f$ values.
that the “secondary vortex” appears in different slotted degrees, as shown in Figure 12. When the “secondary vortex” occurred, the greater the slotted degree, the larger was the critical Rayleigh number. In other words, appearance of the secondary vortex delays as the slotted degree increases.

4. Conclusions

In this paper, natural convection in a cylinder with an internally slotted annulus is simulated, and the related nonlinear problems are analyzed. The effective thermal conductivity $K_{eqs}$ increases with increasing Rayleigh number, and reaches a maximum in the vertical orientation. Under certain geometric parameters, natural convection in a slotted circular annulus exhibits the bifurcation phenomenon. With increasing Rayleigh number, natural convection continues to strengthen, the heat transfer state transforms from the steady to unsteady state, and finally becomes chaotic. The slotted inner cylinder can enhance heat dissipation, but the greater slotted degree does not mean a better heat transfer. There is an optimal slotted degree where $K_{eq}$ reaches the maximum, and the system heat transfer is the strongest.

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References


