Numerical Simulation of Steady Mixed Convection Around Two Heated Circular Cylinders in a Square Enclosure

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In this paper, a numerical study has been carried out to investigate the steady-state mixed convection around two heated horizontal cylinders in a square two-dimensional enclosure. The cylinders are located at the middle of the enclosure height and the walls of the cavity are adiabatic. Streamlines and isotherms are produced and the effects of cylinder diameter, Reynolds number, and Richardson number on the heat transfer characteristics are numerically analyzed. The average Nusselt number over the surface of cylinders and average nondimensional temperature in the enclosure are also presented. The results show that both heat transfer rates from the heated cylinders and the dimensionless fluid temperature in the enclosure increase with increasing Richardson number and cylinder diameter. However, the trend of average Nusselt number and nondimensional temperature variation is completely opposite when Reynolds number increases. In addition, by increasing the cylinders diameter and Richardson number, the left cylinder is less affected by the inlet flow than right one.

INTRODUCTION

There are many applications for the mixed convection flow and heat transfer from horizontal heated cylinders confined in an enclosure, such as space heating, heat exchangers, solar collectors, chemical reactors, energy storage systems, electronic equipment, and so on [1–5]. A literature survey indicates that numerous experimental and numerical studies have been conducted on convection in enclosures with the presence of a body with various thermal conditions. One of the numerical investigations of this problem was conducted by Cesini et al. [6], who studied the effects of Rayleigh number and the cavity geometry on the heat transfer from a horizontal cylinder enclosed in a square cavity. The temperature distribution in the air and the heat transfer coefficients were measured by a holographic interferometer. Roychowdhury et al. [7] investigated the effects of different thermal boundary conditions of the enclosure wall, Prandtl number, and aspect ratio on the two-dimensional natural convective flow and heat transfer around a heated cylinder in a square enclosure. The high-accuracy benchmark solution for the natural convection flow around a horizontal circular cylinder with uniform surface temperature was reported by Saitoh et al. [8]. Typical isotherms, streamlines, vorticities, local Nusselt numbers, tangential and radial velocities, and temperature distributions were discussed in detail.

Angeli et al. [9] numerically investigated the buoyancy-induced flow regimes for a horizontal cylinder centered in a long coaxial square-sectioned cavity filled with air. Atayilmaz and Teke [10] studied natural convective heat transfer from a horizontal cylinder experimentally and numerically for two different diameters (4.8 and 9.45 mm). The average Nusselt numbers over the cylinder were obtained in the range of $74 < \text{Ra} < 3.4 \times 10^3$. Kim et al. [11] solved the two-dimensional unsteady natural convection between a cold outer square enclosure and a hot inner circular cylinder using an immersed boundary method (IBM). They investigated the effects of the inner cylinder location on the heat transfer and fluid flow with

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Rayleigh number ranged from $10^3$ to $10^6$. Hussain and Hussain [12] studied the two-dimensional steady-state natural convection fluid flow and heat transfer from an inner heated circular cylinder in a square enclosure. The effects of vertical cylinder locations and Rayleigh numbers ($10^3$ to $10^6$) were investigated.

De and Dalal [13] studied the natural convection around a tilted heated square cylinder in an enclosure for Rayleigh numbers in the range of $10^2$ to $10^6$. The flow and heat transfer for two different thermal boundary conditions (uniform wall temperature and uniform wall heat flux) were assessed for different aspect ratios and the locations of the square cylinder. Butler et al. [14] experimentally investigated the natural convection from a heat generating horizontal cylinder enclosed in a square cavity, where a temperature difference existed across its vertical walls for $2 \times 10^4 < Ra < 8 \times 10^4$ and a Prandtl number of 0.71. Sasaguchi et al. [15] reported the effect of the position of a cooled cylinder in a rectangular cavity on the cooling process of water around the cylinder. Lee et al. [16] obtained numerical solutions of natural convection induced by a temperature difference between a cold outer square cylinder and a hot inner circular cylinder. They used an immersed-boundary method (IBM) to model the inner circular cylinder based on the finite-volume method to study a two-dimensional natural convection for different Rayleigh numbers in the range of $10^2$ to $10^6$.

Numerical study of combined free and forced convection in a square enclosure with a finite-size heat-conducting body was carried out by Hsu and How [17]. They reported that both the heat transfer coefficient and the dimensionless temperature depend on the configuration of the system. Shuja et al. [18] studied the effect of cavity exit port locations on mixed convection heat transfer in a square cavity with a heat-generating rectangular body. Mixed convection in a vented enclosure with a heat-generating wall was studied numerically by Rahman et al. [19]. They used a finite-element method to study various inlet port locations and Reynolds and Richardson numbers. In another study, Rahman et al. [20] carried out numerical simulations for mixed convection flow in a vented cavity with a heat-conducting horizontal square cylinder. They reported that the flow field and temperature distributions inside the cavity strongly depended on the Richardson numbers and the position of the inner cylinder. At the same time, Rahman et al. [21] analyzed steady laminar mixed convection flow inside a vented square cavity with a heat-conducting horizontal solid circular cylinder. Rahman et al. [22] also numerically studied the combined free and forced convection in a two-dimensional rectangular cavity with a uniform heat source applied on the right vertical wall and a circular heat conducting horizontal cylinder placed within the cavity. Mamun et al. [23] numerically investigated mixed convection heat transfer within a ventilated square cavity having a heated hollow cylinder. It was observed that the cylinder diameter had a significant effect on both the flow and temperature fields but the solid–fluid thermal conductivity ratio had a significant effect only on the temperature field. Recently, Billah et al. [24] analyzed fluid flow due to mixed convection in a lid-driven cavity with a heated circular hollow cylinder.

For the case of two heated cylinders, a numerical study of natural convection of air from two vertically separated horizontal heated cylinders confined in a square enclosure was carried out by Lacroix and Joyeux [25]. The interactions between convection in the fluid and conduction in the vertical walls at Rayleigh number from $10^3$ to $10^6$, dimensionless wall–fluid thermal conductivity ratio in the range of 0.2–1000, and dimensionless wall thickness 0.25–1.375 were investigated. Lacroix [26] also studied the natural convection for air around two heated horizontal cylinders confined in a square enclosure cooled from above with two cavity widths and three different top cylinder positions. The local and average Nusselt numbers were determined over the range of Rayleigh numbers from $10^2$ to $10^6$.

Although there have been numerous investigations on mixed convective flow in cavities with internal obstructions of various configurations, the interactive effect of a pair of cylinders on the mixed convection flow and heat transfer patterns has attracted little attention to date. Moreover, the present study simulates a more practical system such as solar collectors, storage water tanks, and shell-and-tube heat exchangers, compared with assuming a cavity that included one object. All of these application examples need to be studied more in order to find practical aspects of improving heat transfer efficiency. The content of the present paper is also very attractive in the field of space thermal management.

In this article, steady mixed convection of water around two isothermal heated cylinders in a square enclosure with adiabatic walls is studied. The emphasis is on the effects of Richardson number, Reynolds number, and geometrical configuration of cylinders on the heat transfer characteristics.

**PHYSICAL MODEL**

Figure 1 shows a schematic of the case considered in the present study. The problem deals with a two-dimensional square enclosure of equal height ($H$) and width ($L$) that contains two horizontal cylinders with diameter ($d$) situated in the middle of enclosure height and separated by a distance ($s$). All the cavity walls are adiabatic and the uniform temperature on the surfaces of the heated cylinders is $T_H$. The cavity is subjected to an external flow entering the cavity from the left-bottom opening and leaving from the right-upper opening. The size of the inlet port is the same as that of the exit port and is equal to one-tenth of the cavity height (0.1$H$). It is also assumed that the incoming flow velocity and temperature are uniformly distributed with the values of $u_i$ and temperature $T_i$, respectively. The outgoing flow is assumed to have zero diffusion flux for all dependent variables, that is, convective boundary conditions (CBC). All solid boundaries are assumed to be rigid no-slip walls.
GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The governing equations describing the problem are based on the conservation of mass, momentum, and energy. The working fluid is water (Pr = 7.0) and the flow is laminar in all cases. The radiation effects are assumed to be negligible. It is further assumed that the water thermophysical properties are constants except for the density in the body force term of the y-momentum equation, which follows the Boussinesq approximation. The flow is assumed to be steady-state, and viscous dissipation and compressibility effects are considered negligible in the present study. The governing equations can be written in the dimensionless form as follows:

\[
\begin{align*}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \\
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ri \theta \\
\frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= \frac{1}{RePr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\end{align*}
\]

where dimensionless variables are defined as

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_i}, \quad V = \frac{v}{u_i}, \quad P = \frac{P}{\rho u_i^2}, \quad \theta = \frac{T - T_i}{T_H - T_i}
\]

Reynolds number (Re), Richardson number (Ri), and Prandtl number (Pr) are defined as

\[
Re = \frac{u_L}{v}, \quad Ri = \frac{g \beta L (T_H - T_i)}{u_i^2}, \quad Pr = \frac{\nu}{\alpha}
\]

where \(u_i, \nu, g, \) and \(\beta\) are the inlet flow velocity, kinematic viscosity, gravitational acceleration, and volume expansion coefficient, respectively. The dimensionless forms of the boundary conditions for the present problem are specified as follows:

- At the inlet: \(U = 1, V = 0, \theta = 0\).
- At the outlet: \(convective\ boundary\ condition\ (CBC), P = 0\).
- On all solid cavity walls: \(U = 0, V = 0, \) and \(\frac{\partial \theta}{\partial n} = 0\).
- On the surface of the cylinders: \(U = 0, V = 0, \) and \(\theta = 1\).

The non-dimensional diameter and cylinders distance are defined as

\[
D = \frac{d}{L}, \quad S = \frac{s}{L}
\]

The local Nusselt number at the surface of cylinders is obtained from the following expression:

\[
Nu = \frac{h_d d}{\lambda} = \left( \frac{\partial \theta}{\partial n} \right)_{cylinders\ wall}
\]

where \(n\) is the unit normal coordinate to the surface of the cylinders.

The surface-averaged Nusselt number of the cylinders is calculated by integrating the local Nusselt number along their walls:

\[
\overline{Nu} = \frac{h_{avg} d}{\lambda} = \frac{1}{2\pi} \int_0^{2\pi} Nu_{\phi} d\phi
\]

and the bulk average temperature is defined as

\[
\theta_{avg} = \frac{1}{V} \int_V \theta dV
\]

NUMERICAL METHOD

The solutions were carried out using a finite-volume method by employing quadrilateral mesh type. Figure 2 presents a schematic view of the grid distribution for the case of \(D = 0.2\) and \(S = 0.2\). The nodes are arranged in a circular pattern near the cylinders and the grid sizes have been optimized. The grids were uniformly distributed near the cylinder surfaces in order to account for the high temperature gradients. The governing equations were solved on unstructured control volumes using the quadratic upwind interpolation of convective kinematics (QUICK) scheme for the convection terms and by employing the semi-implicit method for pressure-linked equations (SIMPLE) pressure correction algorithm [27]. The finite-volume method-based computational fluid dynamics (CFD) software package FLUENT was employed for computation [28]. The unsteady-state governing equations were solved at the highest Reynolds and Richardson numbers considered in the present study, and it was observed that the results reach the steady state. The convergence criteria of the steady state solutions are that the maximum residual of continuity, X and Y momentum equations, and energy equation are less than \(10^{-6}\) and \(10^{-9}\), respectively. The
convergence of solutions is assumed when the relative error between consecutive iterations is recorded below the convergence criterion such that \(|\Psi_{k+1} - \Psi_k| \leq 10^{-3}\), where \(k\) is the number of the iteration and \(\Psi\) can be \(\text{Nu}\) or \(\theta\).

**Grid-Independent Solutions**

In order to determine proper resolutions, grid-independence tests are carried out for the case of \(D = 0.2\), \(S = 0.2\), \(\text{Re} = 100\), and \(\text{Re} = 100\). Five different grid sizes with 12,349, 16,081, 17,629, 22,802, and 24,826 cells are chosen to test the results independency with the grid variation, as shown in Table 1. The value of average Nusselt number over the surface of two cylinders and the nondimensional average temperature of the fluid in the cavity are used as measures for comparing numerical accuracy. Table 1 shows that average fluid temperatures do not change significantly with the grid sizes when mesh size is more than 16,081 cells. Also, when the total number of cells is greater than 22,802, further improvement on resolutions cannot result in desirable gain in accuracy and the numerical results of \(\text{Nu}_s\) become nearly independent of the grid size. Therefore, the trade-off between numerical accuracy and computational cost suggests using intermediate resolutions with 22,802 cells and 23,251 nodes.

In this study, when the diameter of the cylinders changes, the grid points are refined to obtain accurate results.

**Validation**

For the purpose of validating the present numerical method, simulation is carried out for mixed convection flow inside a square cavity with a heat-conducting horizontal solid circular cylinder placed at the center of the cavity. The calculated surface-averaged Nusselt numbers for the test case of \(D = 0.2\) and \(\text{Re} = 100\) are compared with the benchmark values by Rahman et al. [21] as shown in Table 2. As seen from the results, the surface-averaged Nusselt numbers obtained in the present study are in good agreement with the values of Rahman et al. [21].

**RESULTS AND DISCUSSION**

Since so many basic dimensionless parameters can affect the heat transfer characteristics inside the cavity, a comprehensive analysis of all parameters is not practical. In this paper, the influence of cylinder diameter, Richardson number, and Reynolds number on the flow field and heat transfer are discussed. The dimensionless governing parameters, Reynolds number (\(\text{Re}\)) and Richardson number (\(\text{Ri}\)), and related configuration parameters such as \(D\) and \(S\) are used to identify each case. Both cylinders are assumed to have the same diameters, and nondimensional cylinders distance is kept constant (\(S = 0.2\)) in all cases.

**Effects of the Cylinder Diameter**

The effects of cylinder diameters on the streamlines at different values of Richardson number are shown in Figure 3. The columns from left to right present the streamlines for Richardson number of 0, 1, and 5, respectively. By supposing \(\text{Ri} = 0\), the results illustrate the situation such that flow and heat transfer

<table>
<thead>
<tr>
<th>Ri</th>
<th>Average Nusselt number at heated wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present study</td>
</tr>
<tr>
<td>0</td>
<td>4.05</td>
</tr>
<tr>
<td>1</td>
<td>4.76</td>
</tr>
<tr>
<td>2</td>
<td>5.12</td>
</tr>
<tr>
<td>3</td>
<td>5.35</td>
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<tr>
<td>4</td>
<td>5.52</td>
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<td>5.64</td>
</tr>
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are completely governed by the forced convection and the effect of natural convection is negligible consequently. At Ri = 0 and D = 0.1, a cellular vortex appears just at the top of the inlet. This is due to sudden expansion of the flow and its inability to follow the contour of the enclosure. The size of the vortices decreases gradually with increasing D at the same Richardson number, because the presence of heat sources with large diameter reduces the available cross section. In addition, the fluid hardly passes through the space between the cylinders and cavity vertical walls at D = 0.3. In this case, most of the flow passes in the area between the cylinders compared with lower cylinder diameters. At Ri = 1, another vortex also appears at the top left of the cavity, which becomes smaller at larger cylinder sizes. By increasing Richardson number further from 1 to 5 at a constant D value, the size of the upper side vortex increases considerably. Also, another vortex appears below the outlet port for the cases where the value of the nondimensional diameter equals 0.1 and 0.2 at Ri = 5. The presence of vortices is caused by the pressure difference between the downstream and vicinity of heated cylinder surfaces. Increasing buoyancy force at higher Richardson number results in higher velocity near the cylinders. Hence, when the fluid passes over the cylinders, the velocity descends and the pressure gradient makes a circulating flow. The vortices become smaller at larger heat source sizes due to limitation of space between the cylinders and the enclosure walls.

The effect of cylinder diameter on the isotherms at three different values of Richardson number is shown in Figure 4. In all cases, thermal plumes around the cylinders appear and combine with each other. At Ri = 0, thermal plumes at smaller cylinder size (D = 0.1, 0.2) are directed completely toward the outlet port. They incline more to the vertical direction with increasing Richardson number, especially at the left cylinder, which is less affected by the cold inlet fluid flow. The reason is that the buoyancy-driven flow and heat transfer become more

Figure 3 Distribution of streamlines at different cylinder diameters (D = 0.1, 0.2, and 0.3) and Richardson numbers (Ri = 0, 1 and 5) at Re = 100.
intense when Richardson number increases. At $D = 0.3$, an increase of the Richardson number implies that the isothermal lines become denser at the top of the left cylinder, and therefore the heat transfer is enhanced due to strong thermal gradients in that region. In addition, the direction of thermal plumes in Figure 4 indicates that generally natural convection affects the flow around the left cylinder more than that around the right one at higher Richardson numbers, since the right cylinder is influenced more by the inlet fluid flow at higher Richardson numbers and cylinder diameters.

Figure 5 presents the average Nusselt number and nondimensional temperature as a function of Richardson number for different cylinder sizes. It appears clearly that both $\overline{\text{Nu}}$ and $\text{Nu}_{\text{avg}}$ increase slightly with increasing Richardson number in all cases and their values are the largest at $D = 0.3$. The amount of $\overline{\text{Nu}}$ at $D = 0.3$ is around two times larger than that of $D = 0.1$ for all Richardson numbers. On the other hand, at constant cylinder diameter, the decreasing slope of $\overline{\text{Nu}}$ curve at higher Richardson numbers reveals that the effect of intensifying natural convection on heat transfer is not as much as that for the smaller Ri values.

The distributions of local Nusselt number along the hot surface of the cylinders are shown in polar graphs of Figure 6 for the case of $S = 0.2$, $Ri = 1$, and $Re = 100$ and different $D$ values. The distribution of local Nusselt numbers has a relatively large variation along the surface of the heated cylinders from less than 2 to about 8. As expected, the distribution of local Nusselt number is not symmetric for left and right half parts of every cylinder. All local Nusselt number distributions present that the natural convection heat transfer from the left cylinder is not significantly affected by the inlet flow, since the minimum and maximum Nusselt numbers occur at around $\phi = 90^\circ$ and $\phi$...
Figure 5 Effect of cylinders diameter on the average Nusselt number and average temperature at $Re = 100$.

$\phi = 270^\circ$, respectively. The variation is completely different for the right cylinder, where the maximum Nusselt number occurs at $200 < \phi < 220^\circ$ in different cylinder sizes, which specify the point facing to the flow stream accordingly. Considering related isotherms at Figure 4, cylinders surface regions with denser isotherms are the indication of higher Nusselt numbers accordingly.

Effects of Reynolds Number

The flow and thermal fields for different Reynolds numbers (50, 100, 150, and 200) at $Ri = 1$ have been illustrated in Figure 7, where the cylinder diameter is kept constant ($D = 0.2$). It is found from Figure 7(a) that the streamlines are almost antisymmetric about the cavity diagonal line from inlet to outlet and there is not any circular vortex at the case of $Re = 50$. However, by increasing $Re$ to 100, the flow patterns become asymmetric and two separated vortices appear on the top left and bottom left of the cavity, respectively. The effect of forced convection is intensified by increasing $Re$ to 150, and consequently the vortices become stronger compared with the flow fields at $Re = 100$. Further increasing Reynolds number to 200 results in strengthening the vortex above the inlet port, and simultaneously weakening the top left one in the cavity. In addition, a circulating vortex can be observed at the bottom right of the cavity. At $Re > 100$, the streamline configuration reveals that the majority of inlet stream passes around the right cylinder. The isotherms in Figure 7b show that the thermal boundary layers near the heated surface of the cylinders become thinner with increasing Reynolds number. Figure 7 indicates that the point of denser isotherms in the left cylinder is shifted to higher clockwise degrees by increasing Reynolds number, although the
Figure 7  Streamline and isotherm distributions at different Reynolds numbers and Ri = 1, D = 0.2: (a) streamlines and (b) isotherms.

The corresponding region of the right cylinder does not change considerably. In addition, the thermal plumes are combined with each other at Re = 50, and they tend to be more separated with increasing Reynolds number. Also, the directions of thermal plumes of both cylinders are toward the outlet at Re = 50 and 100, although thermal plumes of the left cylinder completely tend to the left at Re = 200.

The average Nusselt numbers at the heated cylinder surfaces and the average nondimensional temperatures are affected by Richardson number and are presented in Figure 8 at four Reynolds numbers. The values of $\overline{\text{Nu}}_c$ are larger for higher Reynolds numbers at a constant value of Richardson number.

The corresponding trend of $\overline{\text{avg}}$ is obviously opposite, that is, the maximum nondimensional temperature occurs at Re = 50. As can be seen from Figure 8, the ascending slope of $\overline{\text{Nu}}_c$ at Re = 50 is lower than those for higher Reynolds numbers, since the natural convection heat transfer at Re = 50 in the studied range of Richardson numbers is very weak. Moreover, increasing Reynolds number results in lower affection of natural convection heat transfer at larger Richardson numbers.

Figure 9 shows the distribution of local Nusselt numbers over the cylinders surface at D = 0.2, Ri = 1, and various Reynolds numbers (Re = 50, 100, 150, and 200). The local Nusselt numbers of the cylinders are varied from about 1 to 8.5, which is a considerably wide range. By comparing the local Nusselt number variation over cylinder surfaces with isotherm lines of Figure 7b, it is seen that the maximum Nu occurs at around $\phi = 280^\circ$ and $\phi = 215^\circ$ for left and right cylinders in all Re values. In addition, the minimum local Nusselt numbers happened at the top of the cylinders where the boundary layers are thicker and temperature gradients are smaller accordingly.

**Figure 8** Effect of Reynolds number on average Nusselt number and average temperature at D = 0.2.

**Figure 10** shows the streamlines and isotherms at D = 0.2 and Re = 100 with Richardson number varying from 0 to 10. At Ri = 0, there is just a small vortex close to the top of the inlet port, and streamlines are almost symmetric about the diagonal line from the inlet to outlet. By increasing Ri from 0 to 1, the forced convection dominant flow pattern starts to be

**Effects of Richardson Number**

Figure 10 shows the streamlines and isotherms at D = 0.2 and Re = 100 with Richardson number varying from 0 to 10. At Ri = 0, there is just a small vortex close to the top of the inlet port, and streamlines are almost symmetric about the diagonal line from the inlet to outlet. By increasing Ri from 0 to 1, the forced convection dominant flow pattern starts to be...
affected by the buoyancy force. Therefore, the vortices become stronger and another vortex develops at the top left corner of the cavity. The signs of more dominating natural convection heat transfer mechanism can be observed at larger Richardson numbers, which indicates that the top left vortices become larger and another small vortex appeared below the exit port. It can be seen from Figure 10b that the plume-shaped isothermal lines become more vertical in their direction with increasing Richardson number. The thermal boundary layer around the heated cylinders becomes thinner, which implies that natural convection heat transfer is more significant at higher Richardson number due to stronger temperature gradient.

CONCLUSIONS

Numerical simulation of mixed convection around two heated cylinders confined in a two-dimensional square enclosure with adiabatic walls has been reported in this paper. The effects of cylinder diameter, Reynolds number, and Richardson number on mixed convection flow and heat transfer inside an enclosure with equal length and width containing two cylinders with equal diameter were studied numerically. In all cases, nondimensional distance of cylinders was kept constant ($S = 0.2$) and nondimensional diameter of cylinders was varied from 0.1 to 0.3, and the results were investigated for $0 < Ri < 10$ and $50 < Re < 200$.

The results show that the size of the cylinders has a considerable influence on the shape of streamlines. With increasing cylinder diameter at constant Richardson number, the vortices become smaller due to reduction of available cross section. At the highest diameter, $D = 0.3$, the majority of flow pass through the area between the cylinders. The isotherms have also shown that the thermal plumes of the cylinders combine with each other in all studied cases and tend to incline toward the vertical direction with increasing Richardson number. The effect of natural convection on the left cylinder is more than that of the right one at higher Richardson numbers, since the right cylinder is influenced more by inlet flow at higher Richardson numbers and cylinder diameters. The average Nusselt number over the cylinder surfaces and nondimensional temperature inside the enclosure are also increased gradually by enlarging the size of the cylinders. Local Nusselt number distribution at different cylinder sizes also reveals that the left cylinder is influenced less by incoming flow than the right one.
Moreover, it is also concluded that increasing Reynolds number from 50 to 200 at constant cylinder diameter ($D = 0.2$) is conducive to stronger circulating vortices in the flow domain and higher temperature gradient near the heated surfaces. The area-averaged Nusselt number at $Re = 50$ for various Richardson numbers does not change too much, which indicates the fact that natural convection heat transfer is very weak at this Reynolds number.

Extensive work can be done by increasing Reynolds and Richardson numbers to change the solutions to be time-dependent and comparing the influence of various parameters on the heat transfer indices.

**FUNDING**

This project was financially supported by National Natural Science Foundation of China (51276117, 51276118), Public Welfare Scientific Research projects of Shanghai Municipal Bureau of Quality and Technical Supervision (2012–12), and Shanghai Science and Technology Innovation Action Plan of Shanghai Municipal Science and Technology Commission (12DZ1200100).

**NOMENCLATURE**

- $d$: diameter of cylinders, m
- $D$: dimensionless diameter of cylinders, $d/L$
- $g$: gravitational acceleration, m s$^{-2}$
- $h_\phi$: local heat transfer coefficient, w m$^{-2}$ K$^{-1}$
- $H$: enclosure height, m
- $L$: enclosure width, m
- $n$: unit normal coordinate to the surface of the cylinders
- $Nu_\phi$: local Nusselt number
- $Nu_s$: average Nusselt number
- $p$: pressure, Pa
- $P$: dimensionless pressure
- $Pr$: Prandtl number
- $Re$: Reynolds number
- $Ri$: Richardson number
- $s$: distance between cylinders, m
- $S$: dimensionless distance between cylinders, $s/L$
- $T_H$: temperature of heated cylinders, K
- $T_i$: temperature of inlet fluid, K
- $u, v$: velocity components in the x and y directions, m s$^{-1}$
- $u_i$: velocity of inlet fluid, m s$^{-1}$
- $U, V$: dimensionless velocities
- $V$: volume of enclosure, m$^3$
- $x, y$: Cartesian coordinates
- $X, Y$: dimensionless coordinates

**Greek Symbols**

- $\rho$: density, kg m$^{-3}$
- $\alpha$: thermal diffusivity, m$^2$ s$^{-1}$
- $\beta$: thermal expansion coefficient, K$^{-1}$
- $\nu$: kinematic viscosity, kg m$^{-1}$ s$^{-1}$
- $\theta$: dimensionless temperature
- $\lambda$: thermal conductivity of the fluid, W m$^{-1}$ K$^{-1}$
- $\phi$: angular displacement from the front stagnation point, degree
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