NUMERICAL SIMULATION OF MELTING PROBLEMS USING THE LATTICE BOLTZMANN METHOD WITH THE INTERFACIAL TRACKING METHOD

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A lattice Boltzmann method with an interfacial tracking method is used to solve melting problem in an enclosure. Both conduction- and convection-controlled melting problems are solved to validate the proposed method. For the conduction-controlled melting problem, the results agreed very well with those from the analytical solution. The results for the convection-controlled melting problem also agreed with those in the literature. The proposed approach is valid for numerical simulation of the melting problem.

1. INTRODUCTION

Melting problems appear in different areas such as thermal energy storage, electronics cooling, and food processing. These problems always involve nonlinearities, strong couplings, and a moving boundary [1]. Analytical, experimental, and numerical methods can be used to solve melting problems. Some one-dimensional conduction-controlled melting problems can be solved analytically. Under limited conditions, the exact solutions of one-region and two-region problems can be obtained analytically [2], but the melting problems that can be solved by the exact solutions are very limited. The integral approximate method is another valid way to solve some melting problems analytically [3, 4].

Numerous experimental studies on the melting problem have been reported in the literature. Okada [5] reported experimental results for melting in an enclosure with constant temperature on the sidewall. Zhang and Bejan [6] investigated the melting problem in an enclosure at a constant heating rate. Zhang et al. [7] solved the melting problem in an enclosure with discrete heaters at a constant heating rate. Bertrand et al. compared the results from different numerical methods [8].

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The key point for the numerical simulation of a melting problem is the methods to obtain the location of the melting front. Basically they can be divided into the strong numerical solution group and the weak numerical solution group [9]. For the first group, transformed coordinate or deformed grid is employed to obtain the location of the interface, which is limited by its disadvantages of significant amount of computational time. For the second group, heat transfer in the liquid and solid phases is described by the same governing equations and its interface location is determined after the converged temperature field is obtained. Enthalpy method [10, 11] and equivalent heat capacity method [12,13] are the two important methods in this group. The equivalent heat capacity method can only solve the melting problem occurring in a range of temperatures, and when this range is small, it is difficult to reach the converged result using this method. Although the enthalpy method can solve the melting problem taking place at a fixed temperature or in a range of temperatures, it has difficulty in temperature oscillation.

Zhang and Chen [14] proposed an interfacial tracking method for melting and resolidification of a free-standing metal film under ultrafast laser heating. It is becoming a promising method for the melting problems with the following advantages: good numerical stability, high computing efficiency, applicability for fixed grid, and suitability of melting with a fixed melting or a range of temperatures. Chen et al. [15] and Li et al. [16] applied this method to solve natural convection-controlled melting in a rectangular enclosure under constant wall temperature and discrete heating, respectively. All of the above-mentioned applications of the interfacial tracking method are based on the finite volume method (FVM) [17].

The lattice Boltzmann method (LBM) has grown to be a powerful approach in solving the fluid flow and heat transfer problems [18–25]. LBM can be used alone or
along with hybrid methods with other macroscopic approaches. Several models for solving the heating transfer problem using LBM have been proposed [26–30]. LBM has been applied to solve the melting problems. Miller et al. [31] proposed an LBM model for anisotropic liquid–solid phase transition. Eshraghi and Felicelli [32] solved a conduction problem with phase change using an implicit LBM model. Chakraborty and Chatterjee [33] and Huber et al. [34] solved natural convection-governed melting problems with hybrid LBM–FDM and pure LBM, respectively. Gao and Chen [35] solved the melting process in a rectangular cavity filled with porous media with LBM. The above-mentioned Refs. [31–35] are all based on the enthalpy method. Developing an LBM model for melting problem is meaningful based on the interfacial tracking method regarding its advantages discussed above. Li et al. [36] succeeded in solving a melting problem with hybrid LBM–FVM based on the interfacial tracking method. In this paper, pure LBM using the interfacial tracking method is developed first for simulating transport phenomena during the melting process. This method treats melting front as a moving boundary. Velocity and temperature settings are different on that boundary. Both conduction and convection melting problems are solved for validation.

2. PROBLEM STATEMENT

Figure 1 shows the physical model of melting process in an enclosure filled with a phase change material (PCM). The left wall is kept at a constant temperature $T_h$, which is higher than the melting temperature $T_m$. The right wall is also kept at a constant $T_c$, which is lower than or equal to $T_m$. Meanwhile the top and the bottom of the enclosure are adiabatic. No slip conditions are applied to all the boundaries. The initial temperature of the system is at $T_c$. The temperature difference in the liquid

![Figure 1. Phase change model.](image-url)
phase can cause natural convection due to the buoyancy effect. The following additional assumptions are made for convection-controlled melting problem:

1. The PCM is pure and homogeneous.
2. The volume change due to the phase change is negligible.
3. The liquid phase of the PCM is Newtonian and incompressible.
4. Boussinesq approximation is applied to the liquid phase.
5. Natural convection of the liquid phase is laminar.

Then the liquid PCM can be described by the following governing equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g \beta (T - T_m) \tag{3}
\]

\[
\left( \rho c_p \right) \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}
\]

Equations (1)–(4) are subject to the following boundary and initial conditions:

\[
x = 0, \ u = 0, \ v = 0, \ T = T_h \tag{5}
\]

\[
x = W, \ u = 0, \ v = 0, \ T = T_c \tag{6}
\]

\[
y = 0, \ u = 0, \ v = 0, \ \partial T/\partial y = 0 \tag{7}
\]

\[
y = H, \ u = 0, \ v = 0, \ \partial T/\partial y = 0 \tag{8}
\]

Melting front:

\[
x = s, \ T = T_m \tag{9}
\]

\[
x = s, \left[ 1 + \left( \frac{\partial s}{\partial y} \right)^2 \right] \left( k_s \frac{\partial T_s}{\partial x} - k_l \frac{\partial T_l}{\partial x} \right) = \rho \dot{h} \frac{\partial s}{\partial t} \tag{10}
\]

Initial condition:

\[
t = 0, \ T = T_c \tag{11}
\]

For a two-dimensional problem, Eq. (10) represents the melting front moving velocity in the x -direction [37, 38].
Assuming $c_s$ is the speed of sound and defining the following nondimensional variables:

\[
\begin{align*}
X &= \frac{x}{H}, 
Y &= \frac{y}{H}, 
u_c &= \sqrt{\frac{g\beta(T_h - T_m)}{c_s^2} H}, 
Ma &= \frac{u_c}{c_s}, 
U &= \frac{u}{\sqrt{3}cs}, 
V &= \frac{v}{\sqrt{3}cs}, 
sc &= \frac{r(T_h - T_c)}{H}, 
Sc &= \frac{r(T_h - T_m)}{H}, 
Ste &= \frac{r(T_h - T_c)}{H}, 
Pr &= \frac{r^2}{\nu^2}, 
Ra &= \frac{r^3}{Pr^3} 
\end{align*}
\]

Equations (1)–(11) can be rewritten as

\[
\begin{align*}
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\frac{\partial P}{\partial X} + Ma \sqrt{\frac{Pr}{3Ra}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) 
\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial Y} + Ma \sqrt{\frac{Pr}{3Ra}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Ma^2\theta}{3} 
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= Ma \sqrt{\frac{1}{3Ra \cdot Pr}} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) 
\end{align*}
\]

\[
\begin{align*}
X &= S, 
0 &= 0 
\end{align*}
\]

\[
\begin{align*}
X &= S, 
Ma \cdot Ste \left[ 1 + \left( \frac{\partial S}{\partial Y} \right)^2 \right] \left[ k_x \frac{\partial \theta_x}{\partial X} - k_y \frac{\partial \theta_y}{\partial X} \right] &= \frac{\partial S}{\partial \tau} 
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= Ma \sqrt{\frac{1}{3Ra \cdot Pr}} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) 
X &= 0, 
U &= 0, 
V &= 0, 
\theta &= 1 
\end{align*}
\]

\[
\begin{align*}
X &= W/H, 
U &= 0, 
V &= 0, 
\theta &= -Sc 
Y &= 0, 
U &= 0, 
V &= 0, 
\partial \theta / \partial Y &= 0 
Y &= 1, 
U &= 0, 
U &= 0, 
\partial \theta / \partial Y &= 0 
\end{align*}
\]

Initial condition:

\[
\begin{align*}
\tau &= 0, 
\theta &= -Sc 
\end{align*}
\]

3. LBM WITH THE INTERFACIAL TRACKING METHOD

3.1. Thermal Lattice Boltzmann Model

Simplified two distribution functions are selected for fluid flow and heat transfer in LBM [26, 27, 30]. The D2Q9 model is preferred for the velocity field and each
computing node has nine local particle velocities shown in Figure 2. These velocities are given by

\[ e_i = \begin{cases} 
(0, 0) & i = 1 \\
 c(-\cos \frac{\pi}{2}, -\sin \frac{\pi}{2}) & i = 2, 3, 4, 5 \\
 \sqrt{2} c \left(-\cos \frac{(2i+1)\pi}{4}, -\sin \frac{(2i+1)\pi}{4}\right) & i = 6, 7, 8, 9 
\end{cases} \] 

(24)

where \( c \) is the lattice speed. It relates to \( c_s \) as

\[ 3c_s^2 = c^2 \] 

(25)

In the single relaxation time model, the equation for the density distribution \( f_i \) is

\[ f_i(r + e_i \Delta t, t + \Delta t) - f_i(r, t) = \frac{1}{\tau_v} \left( f_i^{eq}(r, t) - f_i(r, t) \right) + F_i, \ i = 1, 2, \ldots 9 \] 

(26)

where \( \Delta t \) is the time step and \( f_i^{eq} \) is the equilibrium distribution function:

\[ f_i^{eq} = \rho \omega_i \left[ 1 + \frac{e_i \cdot V}{c_s^2} + \frac{(e_i \cdot V)^2}{2c_s^4} - \frac{V \cdot V}{2c_s^2} \right] \] 

(27)

where

\[ \omega_i = \begin{cases} 
\frac{4}{9} & i = 1 \\
\frac{1}{9} & i = 2, 3, 4, 5 \\
\frac{1}{36} & i = 6, 7, 8, 9 
\end{cases} \] 

(28)

Figure 2. Nine directions in the D2Q9 model.
The force in Eq. (26) can be obtained as

$$F_i = \Delta t G \cdot \frac{(e_i - V)}{p} f_i^{eq}$$  \hspace{1cm} (29)$$

where the pressure, \( p \), equals \( \rho c_s^2 \) and \( G \) is the effective gravitational force

$$G = -\beta(0 - \theta_m)g$$  \hspace{1cm} (30)$$

Then density and momentum can be obtained by

$$\rho = \sum_{i=1}^{9} f_i$$  \hspace{1cm} (31)$$

$$\rho V = \sum_{i=1}^{9} e_i f_i$$  \hspace{1cm} (32)$$

Then the Navier–Stokes equation can be obtained through the Chapman–Enskog expansion [19] when the relaxation time \( \tau_v \) in Eq. (26) is related to the kinematic viscosity \( \nu \) by

$$\tau_v = \frac{\nu}{c_s^2 \Delta t} + \frac{1}{2}$$  \hspace{1cm} (33)$$

The D2Q5 model is used to obtain the temperature field. Each computing node has five discrete velocities as shown in Figure 3:

$$u_i = \begin{cases} (0, 0) & i = 1 \\ c(-\cos \frac{\pi i}{2}, -\sin \frac{\pi i}{2}) & i = 2, 3, 4, 5 \end{cases}$$  \hspace{1cm} (34)$$

Similar to the density distribution, the energy distribution can be obtained by

$$g_i(r + u_i \Delta t + \Delta t) - g_i(r, t) = \frac{1}{\tau_T} (g_i^{eq}(r, t) - g_i(r, t)), i = 1, 2, \ldots 5$$  \hspace{1cm} (35)$$

The equilibrium energy distribution in Eq. (35) is

$$g_i^{eq} = \theta \omega_i^T \left( 1 + \frac{e_i \cdot V}{c_s^2} \right)$$  \hspace{1cm} (36)$$

where

$$\omega_i^T = \begin{cases} \frac{1}{3} & i = 1 \\ \frac{1}{6} & i = 2, 3, 4, 5 \end{cases}$$  \hspace{1cm} (37)$$
Then the relaxation time $\tau_T$ is related to the thermal diffusivity $\alpha$ by

$$\tau_T = \frac{\alpha}{c^2 \Delta t} + \frac{1}{2}$$  \hspace{1cm} (38)

The temperature at each computing node can be obtained as

$$\theta = \sum_{i=1}^{5} g_i$$  \hspace{1cm} (39)

With these settings, the temperature field satisfies the macroscopic energy equation regarding the Chapman–Enskog expansion. And the density and energy distributions are related by the effective gravitational force in Eq. (29).

LBM is an explicit algorithm requiring an initial setting to start. The following settings are used in this paper:

$$\begin{align*}
\rho_I &= 1, \quad U = 0, \quad V = 0, \\
\theta_I &= \theta_0 \text{ for } X \neq 0, \\
\theta_I &= 1 \text{ for } X = 0, \\
f_I &= f_I^{eq}, \quad g_I = g_I^{eq}
\end{align*}$$  \hspace{1cm} (40)

### 3.2. Interfacial Tracking Method

The melting front location is obtained by the interfacial tracking method. The melting front location in the fixed grid $X_n$ satisfies the following equation:

$$-\frac{\Delta X}{2} \leq S - X_n < \frac{\Delta X}{2}$$  \hspace{1cm} (41)
where $S$ is the melting front location and $X_{n-1}$ is the location of the computing nodes beside the melting front (in the liquid side) shown in Figure 4. Then the liquid fraction $f_p$ in the control volume is related to the location of the solid–liquid interface by

$$ f_p = \frac{S - X_{n-1} - 0.5 \times \Delta X}{\Delta X} \tag{42} $$

where $0 \leq f_p < 1$. The following equation expresses the melting front moving velocity at $Y_m$ when the temperature field is known:

$$ \frac{\partial S}{\partial \tau} = \frac{Ma \cdot Ste}{\sqrt{3RaPr}} \left[ 1 + \left( \frac{S_{Y_m} - S_{Y_{m-1}}}{\Delta Y} \right)^2 \right] \left[ \frac{k_s}{k_l} \left( \frac{\theta_{X_{n+1}} - \theta_m}{\Delta X \cdot (1.5 - f_p)} - \frac{\theta_m - \theta_{X_{n-1}}}{\Delta X \cdot (0.5 + f_p)} \right) \right] \tag{43} $$

With these settings, the interfacial tracking model can be used in simulating the melting process. Then the interface location can be obtained by

$$ S = S^0 + \frac{\partial S}{\partial \tau} \Delta \tau \tag{44} $$

where $S^0$ is the interface location in the last time step and $\Delta \tau$ is the time step. Once $f_p$ is no less than 1, the melting front moves to the next grid. Fixed LBM lattice grids are employed in the solving process.

### 3.3. Boundary Condition

No slip boundary condition is applied to the fixed boundaries and melting front. The macroscopic variables on them are known from the problem statement. Fixed grid is employed in the model; therefore, $X_n$ in Figure 4 is the right moving boundary in LBM. Its velocity $U_{X_n}$ and temperature $\theta_{X_n}$ can be obtained by the following steps:

The velocity and temperature on the melting front are

$$ U_m = 0, \quad \theta_m = 0 \tag{45} $$

![Figure 4. Melting front.](image)
Then the velocity at $X_n$ is

$$U_{X_n} = \frac{f_p - 0.5}{f_p + 0.5} U_{X_{n-1}}$$

(46)

Similar to the velocity field, the temperature at $X_n$ is also not zero:

$$\theta_{X_n} = \frac{f_p - 0.5}{f_p + 0.5} \theta_{X_{n-1}}$$

(47)

If Eq. (47) is applied to the solving process, there might be negative values for $\theta_{X_n}$ in relation to the $f_p$ value. The force in Eq. (30) changes direction when $\theta_{X_n}$ turns to a negative value from the positive value. This leads to the divergence after testing. Then we assume the moving boundary $X_n$ temperature is $\theta_m$ for the whole solving process. This setting does not have any effect on the melting front moving velocity $\partial S/\partial t$. Directly regarding Eq. (43). But it may lead to a higher $\theta_{X_{n-1}}$ when $f_p$ is lower than 0.5. Then it will predict a higher melting front moving velocity $\partial S/\partial t$. Similarly, when $f_p$ is higher than 0.5, it has a lower $\partial S/\partial t$. These two errors tend to cancel each other out since the possibilities for $f_p$ being higher or lower than 0.5 are the same. Although there is still error due to this assumption, the test case results indicate that error is acceptable. Then the velocity and temperature on three fixed boundaries and the moving melting front are known. It is necessary to pay attention to the computing nodes that newly change from solid to liquid. The ways to obtain their velocity and temperature have been described above. Their densities are set as the initial density in LBM.

Nonequilibrium extrapolation scheme [27] is applied to both velocity and temperature fields in the solving process. Assuming $x_b$ is the boundary node and $x_f$ is its nearby inner mode, the density and energy distribution at the boundaries are

$$f_i(x_b, t) = f_i^{eq}(x_b, t) + f_i(x_f, t) - f_i^{eq}(x_f, t), \ i = 1, 2, \ldots 9$$

(48)

$$g_i(x_b, t) = g_i^{eq}(x_b, t) + g_i(x_f, t) - g_i^{eq}(x_f, t), \ i = 1, 2, \ldots 5$$

(49)

4. RESULTS AND DISCUSSIONS

The conduction and convection melting problems are solved by LBM with the interfacial tracking method to investigate its applicability and accuracy. The results are compared with the analytical results for the conduction melting problem and the convection melting results in Ref. [8] for validating the model.

4.1. Conduction-Controlled Melting

When natural convection is negligible, the melting front moves at the same velocity at any height. Then this problem can be simplified to a one-dimensional problem governed by conduction. There is an analytical solution for this problem when $Sc$ is equal to 0 [9]. The following nondimensional variables are applied to the governing Eqs. (1–11):

$$\begin{cases} X = \frac{s}{W}, \ S = \frac{s}{W}, \ Fo = \frac{W}{h} \\ \theta = \frac{T - T_m}{T_{m} - T_m}, \ Ste = \frac{c_p(T_{m} - T_m)}{h} \end{cases}$$

(50)
The melting front location can be obtained from

$$S = 2\lambda \sqrt{Fo}$$  \hspace{1cm} (51)

where $\lambda$ is a function of the Stefan number and can be obtained from

$$\lambda \sqrt{\pi} \text{erf}(\lambda) = \frac{Ste}{\sqrt{\pi}}$$  \hspace{1cm} (52)

The temperature distribution in the liquid PCM is

$$\theta(X, Fo) = 1 - \frac{\text{erf}(X/2Fo^{1/2})}{\text{erf}(\lambda)}$$  \hspace{1cm} (53)

The cases for the Stefan number equal to 0.1, 0.5, and 1 are solved for validation. The corresponding $\lambda$ obtained from Eq. (52) are equal 0.220, 0.465, and 0.620, respectively. The melting front moves at the uniform velocity (not a function of $Y$) and the nondimensional computational domain is 1 in the conduction-controlled melting. Meanwhile, the temperature distribution in the computational domain can be obtained in Eq. (53). For the numerical solution, only the energy equation in LBM is solved for the conduction-controlled melting problem and $80 \times 10$ grids are employed.

Figure 5 shows the comparison of melting fronts obtained from the analytical solution and the LBM. The melting front moves faster with increasing Stefan

![Figure 5. Liquid fraction comparison in conduction melting.](image-url)
number. The LBM results agreed with the results from the analytical solutions very well. Figure 6 shows the comparison of the temperature distributions for the three cases when $Fo$ equals 0.3. As discussed above, the case with a higher Stefan number takes a shorter time to reach the same interfacial location. The temperature distribution is closer to a straight line with a lower Stefan number. And it is difficult to distinguish the LBM results from the analytical result for all three cases in Figure 6.

Three conduction melting problem results agree with the analytical results well. The LBM with the interfacial tracking method is valid for the conduction-controlled melting problem.

4.2. Convection-Controlled Melting

The convection-controlled melting problem can be described by Eqs. (13)–(23). No subcooling is considered and the height of the enclosure is equal to its width. Two melting cases are solved by the LBM with the interfacial tracking method. The temperature field, melting front location, and liquid fraction are discussed in two cases to verify the proposed numerical method. The Rayleigh number, the Prandtl number, and the Stefan number are $2.5 \times 10^4$, 0.02, and 0.01, respectively, in Case 1 while these parameters in Case 2 are $2.5 \times 10^5$, 0.02, and 0.01, respectively. After grid number testing, $200 \times 200$ grids are chosen for these two cases.

Figure 6. Temperature comparison in conduction melting.
Bertrand et al. compared the results obtained from different numerical methods for the convection-controlled melting problem. The Fourier number $Fo$ was used in Ref. [8] and it is related to the nondimensional time $\tau$ in LBM by

$$Fo = \tau \cdot Ma/\sqrt{3Ra \cdot Pr}$$

(54)

These results of these two cases are recorded for $Fo$ equaling 4 and 10 in Ref. [8]. The corresponding $\tau$ are 1549 and 3872 for Case 1 and they are 4900 and 12250 for Case 2.

Figure 7 shows the temperature field for Case 1 for $Fo = 4$. All the isotherms are close to straight line normal to the bottom. The region with $\theta$ higher than 0 is the liquid PCM region. The temperature is linear to $X$ in the liquid PCM region basically. It shows the melting process is still governed by conduction at $Fo = 4$. Figure 8 compares the numerical melting front with that in Ref. [8]. They are both nearly constant to $Y$ and agree with each other well. It also supports the melting process is still governed by conduction. The temperature field for $Fo$ equaling 10 is shown in Figure 9. The temperature in the liquid PCM zone increases with increasing $Y$. It indicates convection has dominated the melting process. Figure 10 shows the comparison of the melting fronts obtained from the numerical method and those from the reference. The LBM melting front moves faster in the higher location due to convection effect. And these two results agree with each other well. The liquid fraction is the percentage of liquid PCM in the total PCM and it indicates the melting ratio. The Nusselt number

![Figure 7. Case 1: Temperature field ($Fo = 4$).](image-url)
Figure 8. Case 1: Melting front comparison ($Fo = 4$).

Figure 9. Case 1: Temperature field ($Fo = 10$).
where the heat transfer coefficient $h$ can be obtained by

$$h(T_h - T_m) = -k \frac{\partial T}{\partial x} \bigg|_{x=0}$$  \hspace{1cm} (56)

Substituting Eqs. (12) and (56) to Eq. (55), one can obtain

$$Nu = - \frac{\partial \theta}{\partial X} \bigg|_{X=0}$$  \hspace{1cm} (57)

Then the average Nusselt number is

$$Nu_{avg} = \int_0^1 - \frac{\partial \theta}{\partial X} \bigg|_{X=0} dY$$  \hspace{1cm} (58)

The average Nusselt number changing with time is shown in Figure 11a. It turns to be a constant when $Fo$ is greater than 4. This indicates the heat transfer rate
Figure 11. Case 1: Nusselt number and liquid fractions.
Figure 12. Case 2: Streamlines ($Fo = 4$).

Figure 13. Case 2: Temperature field ($Fo = 4$).
Figure 14. Case 2: Melting front comparison ($Fo = 4$).

Figure 15. Case 2: Streamlines ($Fo = 10$).
Figure 16. Case 2 temperature field ($Fo = 10$).

Figure 17. Case 2: Melting front comparison ($Fo = 10$).
Figure 18. Case 2: Nusselt number and liquid fractions.
through the left heat wall reaches a fixed value. Figure 11b shows a comparison of the liquid fraction obtained from different methods. The liquid fraction is linear to Fo when Fo is greater than 4. It means the melting reaches the quasi-steady state. And the numerical and reference results agree with each other very well.

The melting front location and liquid fraction were also reported for Case 2. Meanwhile, Hannoun et al. discussed the fluid field bifurcation for the same case [39]. It was reported that two vortices are in the liquid PCM region when Fo is greater than 2.8 in Case 2. Figure 12 shows the streamlines for Case 2 for Fo = 4. There are two vortices in the liquid PCM zone, which agree well with the results in Ref. [39]. On the other hand, Figure 13 shows the temperature field for the same Fo. There are two curves on most isotherms, led by the velocity results. The LBM melting front agrees with Ref. [8] very well as shown in Figure 14. Basically the melting front moves faster as the height increases even if there are two vortices in the liquid PCM zone. When Fo increases to 10, the streamlines and temperature field are shown in Figure 15 and Figure 16, respectively. Two vortices still appear in the liquid PCM zone, and the upper one appears much stronger than the other one. The LBM melting front agrees with that in Ref. [8] as shown in Figure 17 for Fo = 10. Ref. [40] reported the average Nusselt oscillation phenomenon in a low Prandtl natural convection problem. They reported that the left heat boundary average Nusselt number reached a constant number for the case \( Ra = 1 \times 10^4 \), \( Pr = 0.01 \) while it oscillated around a fixed number for the case \( Ra = 5 \times 10^4 \), \( Pr = 0.01 \). It is reasonable to believe that when \( Ra \) equals to \( 1 \times 10^5 \), the average Nusselt number would also oscillate because the higher \( Ra \) gives a nonlinear effect to the governing equations. Figure 18a shows the variation of the Case 2 average Nusselt number with time. After \( Fo = 4 \), the average Nusselt number oscillates around 5. Figure 18b shows the variation of the liquid fraction for the entire enclosure with time. The liquid fraction’s resulting tendency is not linear to the \( Fo \) at the very beginning (conduction-controlled stage). It then turns to be linear to the time when \( Fo \) is greater than 2. It means that the melting process reaches a quasi-steady state. And it has a good agreement with the reference result.

These two convection results agree well with the reference ones. Using the LBM with the interfacial tracking method is valid for the convection-controlled melting problem.

5. CONCLUSIONS

LBM with the interfacial tracking method is applied to simulate the melting problem. Both conduction- and convection-controlled melting problems are solved for validation. The numerical results agreed with the analytical results for conduction-controlled melting. For convection-controlled melting, the agreement between the results from the present method agreed with those in the reference very well. Therefore, the proposed numerical method is valid for the melting problem simulation.

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