Nonlinear dynamics study based on uncertainty analysis in electro-thermal excited MEMS resonant sensor

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ABSTRACT

Nonlinear vibration model of double-clamped resonant beam driven by electro-thermal excitation in a MEMS resonant pressure sensor is established. Inherent heat elevation of electro-thermal excitation is taken into account besides the nonlinear geometric effect. An approximate solution for this model is obtained via Galérkin procedure and multiple scales method. The sample-based stochastic model is established to investigate the influence of inherent heat elevation on vibrating nonlinearity, including linear natural frequency \( f_{ln} \), nonlinear frequency offset \( F_{off} \), resonator amplitude \( A_{max} \), and the non-linear factor \( F_{nl} \), considering uncertainty distributions of structure size and excitation voltage due to fabricating or control errors. The results reveal that the dc bias of the excitation signal has significant effect on vibrating nonlinearity, which is verified by experiments. The results can be used as reference for sensor design and operation with respect to proposed nonlinear effects.

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1. Introduction

With the development of MEMS manufacturing technology, the resonant sensors are being rapidly miniaturized. As one of the main representative resonant sensors, MEMS resonant pressure sensor becomes a better alternative to its conventional macro counterpart [1,2], which is used in avionics, industry, and instrumentation, for their outstanding performances like small volume, high accuracy, stability and negligible hysteresis [3,4].

However, when it is vibrating with large amplitude, nonlinear vibration of the resonator occurs due to the small size of the resonator, which is highly undesirable in most of the resonant sensors [4,5]. After experimentally observed nonlinear behavior in a
MEMS silicon resonant pressure sensor was reported first [6], non-linear behavior of MEMS resonators was reported in the literatures [7–13]. The nonlinear dynamics of resonator driven by electrostatic and piezoelectric methods have been well examined but works on the electrothermally excited resonators are scarcely found. It is believed that the electro-thermal excitation method have certain advantages of simply fabrication [14] since it was been used for microcantilever resonance first [15]. However, the inherent heat elevation of electro-thermal excitation leads nonlinear dynamics of resonator to be more complicated [4].

Meanwhile, the influence of size and excitation parameters on nonlinear dynamics of resonator is not analyzed for the cases that the size and excitation parameters are not exactly the design values but value in a certain distribution instead due to fabricating or control errors; the size and excitation parameters are difficult to be measured but are just estimated from the performances [16]. Therefore, a systematic methodology incorporating the parametric uncertainty distribution to analyze the effects of the size and excitation parameters is needed. Peng et al. applied a sample-based stochastic model to investigate the influences of different uncertain parameters in the solid–liquid–vapor phase change processes and identified the key parameters that had dominant effects [17]. The effects of uncertainty were also investigated with a sampling-based stochastic model in the optical fiber drawing process [18, 19] and resin transfer modeling [20]. More applications of the stochastic model in the optical fiber drawing process [18, 19] and resin transfer modeling [20] were verified by detection experiments.

### 2. Vibration modeling and analysis

#### 2.1. Working mechanism

Fig. 1 shows the structure diagram and photograph of the resonant beam in a MEMS silicon resonant pressure sensor. The resonator is a clamped-clamped resonant beam that is fabricated on a square diaphragm. The square diaphragm together with the clamped-clamped resonant beam is the main sensing structure. The measured pressure makes the square diaphragm deformation which changes the stress in the resonant beam. Thus, the natural frequency of the resonant beam changes with measured pressure. The resonant beam is electrothermally excited by the excitation thermal resistor located at the center of the beam. Piezoresistive detection resistor located at the root area detects alternating stress when the resonant beam is vibrating. The frequency of the changing resistance caused by alternating stress is considered to be the natural frequency when the beam is in the resonant state. By measuring the frequency of the changing resistance, the natural frequency is obtained and the measured pressure could be obtained.

#### 2.2. Nonlinear vibration modeling of double-clamped resonant beam

Inherent heat elevation of electro-thermal excitation causes the natural frequency offset, and nonlinear vibration affects this shift. Taking the inherent heat elevation and nonlinear geometric effect into account, the equation of motion of the resonant beam is obtained in line with Euler–Bernoulli beam modeling when the external forces and moments are considered. The lateral deflection $w(x, t)$ is given by

$$
EI \left( \frac{d^4 w(x,t)}{dx^4} + \frac{c}{\rho} \frac{d^2 w(x,t)}{dt^2} + \rho B \frac{d^2 w(x,t)}{dt^2} \right) - (N + \frac{EA}{2L} \int_0^L \left( \frac{d^2 w(x,t)}{dx^2} \right)^2 dx \frac{d^2 w}{dx^2} \right) = [\delta(x - 0.5L - L_0) - \delta(x - 0.5L + L_0)]\left[M_{\omega_{00}}(t) + M_{\omega_{0\omega}}(t)\right]
$$

The boundary conditions of this clamped-clamped resonant beam are

$$
w(0, t) = w(L, t) = 0
$$

where $x$ is the distance along the resonant beam from one clamped end, and $t$ is time. The $\delta$ denotes dimensional quantities. Damping coefficient $c$ is a dependent of the vibration frequency when the resonator vibrating in liquid or air [26,27]. However, the resonant beam in this pressure sensor is sealed in a vacuum chamber with low pressure. Damping force is generated by internal friction of the beam material, which is not fully understood. The damping coefficient is simplified to be frequency-independent in the neighborhood of resonance and the value is determined by the measured quality factor [4]. $M_{\omega_{00}}(t)$ and $M_{\omega_{0\omega}}(t)$ are driving moments developed by the dynamic thermal components of the thermal power of excitation resistor. For the driving moments are located under the thermal resistor, unit step function is used to describe the position of the driving moments. $\delta$ is the second order derivative of the unit step function.
where $U_{ac}$ is amplitude of the ac excitation signal, and $\omega$ are frequency of the ac exciting signal. According to Ohm's law, there are static thermal power $P_{st}$ and two dynamic thermal powers $P_{dy}(\omega)$ and $P_{dy}(2\omega)$.

$$P_{st} = \frac{(U_{dc}^2 + 0.5U_{dc}^2)R}{2}$$

(4)

$$P_{dy}(\omega) = \frac{(2U_{dc}U_{ac}(\omega)\sin(\omega t)\cos(\omega t))}{R}$$

(5)

$$P_{dy}(2\omega) = \frac{(0.5U_{dc}^2\cos(2\omega t))}{R}$$

(6)

Because the thermal diffusion length is smaller than the thickness of beam, heat generated by the dynamic thermal power only conducts in the direction of beam thickness under resistor area. The temperature field cause thermal moments which are used to drive the resonant beam into vibration. Thermal moments mainly include two frequency parts $M_{d0}(t)$ and $M_{d20}(t)$ [4]:

$$M_{d0}(t) = \frac{24U_{dc}U_{ac}EaLm(\omega)}{2\pi RBl} \left[ \cos \omega t - \phi_d(\omega) \right]$$

$$M_{d20}(t) = \frac{6U_{dc}U_{ac}(2\omega)}{2\pi RBl} \left[ \cos 2\omega t - \phi_d(2\omega) \right]$$

(7)

where

$$M_{d0}(\omega) = \frac{1}{2\pi^2} \left[ \frac{1}{\gamma} + \frac{1}{\gamma(1+r)} + \frac{2}{\gamma(1+\exp(-\gamma(1+r)))} \right]$$

$$\phi_d(\omega) = -\arctan \left[ \frac{1 - \gamma - (1 + \gamma)e^{-\gamma r} - 2e^{-\gamma} - \gamma\cos(\gamma) - \sin(\gamma)}{1 - e^{-\gamma r} - 2e^{-\gamma} - \sin(\gamma)} \right]$$

$$\gamma = H(\rho C_W/2\lambda)^{1/2}$$

and $j=1,2$.

It can be found from Eq. (6) that thermal moment of frequency $\omega$ is 4 times of the thermal moment of frequency $2\omega$ when $U_{dc} = U_{ac}$. So the thermal moment of $\omega$ is selected as the incentive in order to facilitate the subsequent signal processing.

For the static thermal power, temperature gradient in the thickness direction of the beam is sufficiently small to be neglected, because thermal diffusion length is much larger than the thickness of the beam. The heat generated by static thermal power only conducts along the axial direction of the beam to the clamped boundary that is considered to be ideal heat sink. The conducting heat causes temperature distribution along the axial direction of the beam, which is known as inherent heat elevation of electro-thermal excitation. Thermal axial force caused by this temperature distribution changes the natural frequency of resonant beam.

Unfortunately, the inherent heat elevation of electro-thermal excitation is inevitable, and it affects the sensor performances and may even make sensitive element failure when it is serious. The thermal axial force $N_t$ is

$$N_t = -EaL\left[ \frac{U_{dc}^2 + 0.5U_{ac}^2}{2\pi R B} \right] x_0 \left[ \frac{x_0^2}{L^2} \right]$$

(8)

The axial load $N_p$ in the beam is caused by diaphragm displacement that is formed due to the measured pressure $P$. This measured pressure $P$ changes the axial stress in resonant beam, and the axial load in resonant beam is given by [16].

$$N_p = BH\left[ \frac{49(1-u^2)}{96} \frac{L_{ad}^2}{H_{ad}^{2}} \right] \left[ 1 - \left( \frac{L}{L_{ad}} \right)^2 \right] P$$

(9)

$N_t$ together with $N_p$ is the axial load $N$ of the resonant beam. Compared to $N_t$, $N_p$ is a large value and mainly changes the natural frequency of the beam, while $N_t$ causes natural frequency offset of the resonant beam which leads to measurement errors. In order to study the effects of $N_t$, $N_p$ is set as constant values in this paper.

It will facilitate the identification of the order of magnitude of some variables to introduce dimensionless quantities and will be convenient for the multiple scales method. So let

$$w(x, t) = \frac{w(\hat{x}, \hat{t})}{r}, x = \frac{\hat{x}}{r}, \hat{t} = \frac{\hat{t}}{r}, t = \frac{\hat{t}}{\sqrt{r/BH}}$$

(10)
where \( \omega_0 \) is the fundamental natural frequency of the beam under linear vibration. In non-dimensional form, Eq. (1) becomes
\[
\frac{\partial^2 W}{\partial x^2} + 24c_0^2 \phi = 2(q + \phi) \omega_0^2 \sin(\omega_0 t),
\]
and the linear natural frequency is
\[
f_m = \frac{\omega_0}{2\pi}.
\]

After setting the coefficients, Eq. (14) becomes
\[
q + \phi = -2\epsilon_\mu \mu - \epsilon q^3 + \epsilon K(\phi) \cos(\omega t - \phi_0(\alpha)),
\]
where \( \phi(\omega) \) is the phase given by Eq. (7). The coefficient \( k_1, k_2, k_3 \) and amplitude of excitation force \( F(\omega) \) are:
\[
\begin{align*}
k_1 &= c \int_0^1 W^2(x) \omega_0^2 dx, \\
k_2 &= \frac{\rho BH_0 L^4}{2} \int_0^1 W^2(x) dx, \\
k_3 &= -\frac{2\rho BH_0 L^4}{3} \int_0^1 W^2(x) dx.
\end{align*}
\]

Typically, \( k_3 \) is small and \( k_2 = \epsilon \) is set to be used as the small perturbation parameter in the subsequent perturbation analysis in multiple scales method. And
\[
k_1 = 2c \mu \omega_0 L^4.
\]

where
\[
\mu = \frac{C_{00} L^4}{E B H^2} \int_0^1 W^2(x) dx.
\]

When the resonant beam is under linear vibration, \( k_2 = \omega_0^2 \), and \( k_2 = 1 \) for the case of non-dimensional process. \( \omega_0 \) is the fundamental natural frequency of the beam under linear vibration affected by the inherent heat elevation of electro-thermal excitation. So \( \omega_0 \) should contain the offset value caused by the inherent heat elevation to ensure that \( k_2 = 1 \). When the inherent heat elevation is taken into account, \( \omega_0 \) is
\[
\omega_0 = \left( \frac{\rho BH^4 L^4}{24} \int_0^1 W^2(x) dx \right)^{0.5} - \frac{N}{\rho BH^2 L^2} \int_0^1 W^2(x) dx.
\]
Inherent heat elevation of electro-thermal excitation is caused by heat of the excitation resistor. It is affected by excitation voltage and the structure size, including the resistance of excitation resistor $R$. The uncertain input parameters investigated here include the length of the resonant beam $L$, width of the resonant beam $B$, thickness of the resonant beam $H$, resistance of excitation resistor $R$, excitation dc voltage $U_{dc}$, and the excitation ac voltage $U_{ac}$. It is not possible to obtain the exact distribution of uncertain input parameters, but a general distribution of the input parameters can be estimated from the performances of different sensor samples. It is reasonable to assume that all of these input parameters obey the Gaussian distribution [17,18]. In Gaussian distribution, design value of the input parameter is set as the mean value $\mu$ while the standard deviation $\sigma$ shows the uncertainty of the input parameter. The coefficient of variance (COV), $\sigma/\mu$, is defined to represent the degree of uncertainty of the input parameter. After obtaining the distribution of the input parameter, Monte Carlo sampling (MCS) method is utilized to randomly select every input parameter from its prescribed Gaussian distribution and combining them together as one sample.

The variability of output parameter is highly dependent on the number of samples chosen [17]. In order to ensure that the number of input parameter samples is proper and the samples selected are representative, a stochastic convergence analysis is conducted. The standard deviation and mean value of the input and output parameter with different numbers of samples are also analyzed. After sufficient samples are selected, the deterministic nonlinear vibration model is used to calculate the output parameters for every input sample. Effect of the input parameter variability on the output parameter uncertainty is assessed by obtaining the output parameter combinations through this deterministic model. The probability distribution is generated from the resulting combinations of output parameter. The output parameters of interest in this study include: the linear natural frequency $f_{nr}$, nonlinear frequency offset $F_{off}$, amplitude $A_{mp}$, and the nonlinear factor $F_{non}$. The output parameters are uncertain due to the uncertain input parameters. In order to quantify the uncertainty of the output parameter, the interquartile range (IQR) is defined as the difference between the 25th percentile (P25) and the 75th percentile (P75) [17,18].

$$IQR = P75 - P25$$ (29)

### 3.2. Experiment method

In order to verify the effect of inherent heat elevation of electro-thermal excitation and influence of input parameters on vibrating nonlinearity obtained by the sample-based stochastic model, experiments on two real MEMS resonant sensor samples that have the same designed sizes are conducted. Verification of the size effects on the vibrating nonlinearity is difficulty to achieve due to the difficulty to obtain the real sizes of sensor samples and the limited number of sensor samples. The experiment is focused on the verification of the effects of heat elevation of electro-thermal excitation caused by the voltage parameters on the vibrating nonlinearity of the resonant beam: the linear natural frequency $f_{nr}$.
nonlinear frequency offset $F_{\text{off}}$, amplitude $A_{\text{mp}}$, and the non-linear factor $F_{\text{nl}}$.

Natural frequency can be detected directly by detecting the output vibration signal of the sensor. This natural frequency contains the linear natural frequency $f_{\text{rn}}$ and the nonlinear frequency offset $F_{\text{off}}$. In order to obtain the linear natural frequency $f_{\text{rn}}$ and the non-linear frequency offset $F_{\text{off}}$ separately, natural frequency is detected when the excitation voltage is low to ensure that the resonant beam works under linear vibration. From the results it can be found that the square of natural frequency of the resonant beam is in good linear relationship with static power when $U_{\text{dc}} = U_{\text{ac}}$. According to this linear relationship, linear natural frequency $f_{\text{rn}}$ is calculated in a large excitation voltage range. When the nonlinearity appears, nonlinear frequency offset $F_{\text{off}}$ appears, which can be obtained by detected natural frequency subtracting linear natural frequency $f_{\text{rn}}$. And the nonlinear factor $F_{\text{nl}}$ is calculated according to the linear natural frequency $f_{\text{rn}}$ and the nonlinear frequency offset $F_{\text{off}}$.

The vibration amplitude of the resonant beam cannot be detected directly because the beam is sealed in a vacuum chamber. The normalized amplitude of output vibrating signal is used as a substitute to the normalized vibration amplitude of the beam.

The two test samples are fabricated by silicon micromachining technology, and the fabrication processes is shown in Fig. 3. The sensing structure was fabricated on a silicon wafer $<100>$ $p$-type with resistivity of 7–10 $\Omega$cm. The SiO$_2$ layers with thickness of 1 $\mu$m was grown on both wafer surfaces by thermal oxide, and a Si$_3$N$_4$ layer with 1300–1450 Å thickness was deposited by LPCVD, as shown in Fig. 3(a). Anisotropically etching was used to form a cavity from the wafer back-side, by which the diaphragm was processed as shown in Fig. 3(b), and the predefined diaphragm thickness was decided by the timed etch stop. A shallow cavity was etched under the resonant beam by deep reactive ion etching (RIE) process from the wafer front-side, as shown in Fig. 3(c). The etched substrate wafer was fusion bonded to a $<100>$ $n$-type structural wafer to form the resonant beam as shown in Fig. 3(d). The bonded wafers were immediately annealed at 1150 °C for 3 h, and followed by thinning, grinding, and polishing process to obtain a 15 $\mu$m thickness film. The thermal excitation resistor and Piezoresistive detection resistor were made by boron diffusion with nominal resistance of 200 $\Omega$ for each resistor, as shown in Fig. 3(c). The contact holes, the connecting wires, and electrode pads were made by Al sputtering and wet etching. The resonant beam was released by deep RIE process, as shown in Fig. 3(f).

The output vibration signals of the sensor samples are detected by a homemade integrated testing instrument. The excitation signal is generated by DDS signal source in integrated testing instrument of resonant silicon microstructure transducer. The DDS signal source can realize controllable sweep function with minimum frequency sweep interval of 0.1 Hz. With this frequency sweep interval, the frequency sweep is carried out for the measurement under normal atmospheric temperature and pressure. The results are recorded, processed, and plotted with a program code using Matlab. Fig. 4 shows a typical detected result of the output vibration signal. When the amplitude of the output vibration signal reaches the maximum value in amplitude–frequency characteristic curve, resonant beam is in resonant state, and the frequency of excitation signal is treated as the natural frequency of the resonant beam.
4. Results and discussions

4.1. Analysis of the sample-based stochastic model

The properties of the resonant silicon beam in this paper are: the Young's modulus is $E = 133$ GPa, the Poisson's ratio is $v = 0.278$, the material density is $\rho = 2329$ Kg $m^{-3}$, the thermal conductivity of the material is $k = 150 Wm^{-1}K^{-1}$, the thermal expansion coefficient of the material is $\alpha = 2.4 \times 10^{-6}K^{-1}$, the specific heat of the material is $C = 733$ J $Kg^{-1}K^{-1}$, and the drive resistor length is $l_R = 100 \mu m$.

The output parameter distributions are obtained according to the uncertain input parameter distributions of $L$, $H$, $B$, $U_{ac}$, $U_{dc}$ and $R$, using the sample-based stochastic model. In order to get the real distribution of the output parameters, large numbers of input samples are needed, which may lead to massive computation and inefficiency. So a minimum quantity of input parameters which can represent the input sample distribution and guarantee steady output distribution should be obtained by convergence analysis.

In the process to obtain number of input samples $N_s$, the classical design values of the input parameters are set as their nominal mean values: $L = 1500 \mu m$, $H = 15 \mu m$, $B = 90 \mu m$, $U_{ac} = 2 V$, $U_{dc} = 2 V$, and $R = 200 \Omega$. However, there is a certain size bias for fabricated sensor sample due to processing errors, especially for the smaller size such as beam thickness. Meanwhile, control parameters of excitation signal also fluctuate when driving the resonant beam due to the control errors. Additionally, there is difficulty for a real sensor sample to determine the exact input parameters but the general distributions of the input parameters can be estimated to be a reference to set the COVs of input parameters. According to the general distribution of input parameters, the COVs of all input parameters are set as 0.04.

The stochastic convergence analysis of the mean values of input parameters is conducted and the results are shown in Fig. 5. It can be seen that the mean values of the input parameters frequently fluctuate when the number of samples is less than 300. The mean values of the input parameters still oscillate but the changing amplitude is less than 1% at $N_s = 300$. Thus, it is sufficient to ensure that the nominal mean values of input parameters are steady when $N_s = 300$.

The stochastic convergence analysis of the standard deviations of the input parameters is also conducted and the results are in Fig. 6. It can be seen that the standard deviations still oscillate significantly even with more than 300 input samples. The reason for this is that the deviation is a higher order moment and converges much more slowly compared with the mean values [17]. When the sample number increases to 600, the standard deviation of $L$, $H$, $B$, $U_{ac}$, $U_{dc}$ and $R$ converges within 1.67%, 1.04%, 1.41%, 1.70%, 0.57%, and 1.92%, respectively.

Stochastic convergence analysis is conducted to the output mean values and the results are shown in Fig. 7. It can be found that the mean values of the output parameters converge very fast. The mean values of all output parameters are within 1% when the number of samples is greater than 500. The stochastic convergence analysis of the standard deviations of the output parameters is shown in Fig. 8. The standard deviation converges to within 4.05% for $f_{rn}$, 6.42% for $F_{off}$, 4.14% for $A_{mp}$, and 2.97% for $F_{onl}$ when numbers of input samples is 300, and it is 1.05% for $f_{rn}$, 0.17% for $F_{off}$, 0.43% for $A_{mp}$, and 0.43% for $F_{onl}$ within 2% for all output parameters when 600 samples are used. According to the above discussion, the
Fig. 7. Stochastic convergence analysis of the mean value of the output parameters: (a) $f_{rn}$, (b) $F_{off}$, (c) $A_{mp}$, (d) $F_{nol}$.

Fig. 8. Stochastic convergence analysis of the standard deviation of the output parameters: (a) $f_{rn}$, (b) $F_{off}$, (c) $A_{mp}$, (d) $F_{nol}$.

minimum number of samples $N_s = 600$ and this number of samples will be used to conduct following analyses.

Fig. 9 shows the IQRs of $f_{rn}$, $F_{off}$, $A_{mp}$, and $F_{nol}$ as functions of the COVs of the input parameters $L$, $H$, $B$, $U_{ac}$, $U_{dc}$ and $R$. When the COV of one input parameter increases from 0.01 to 0.1, the COVs of other parameters are kept unchanged at 0.01. In the IQR analysis of the linear natural frequency of the resonant beam $f_{rn}$, the IQR of $f_{rn}$ significantly increases from 1.569 KHz to 14.008 KHz when the COV of $L$ increases from 0.01 to 0.1. This indicates that linear natural frequency of the resonant beam $f_{rn}$ greatly depends on beam length $L$. Besides, the IQR of $f_{rn}$ increases from 1.618 KHz to 8.371 KHz when the COV of $H$ increases from 0.01 to 0.1, which shows that the beam thickness $H$ can also affect the resonant frequency significantly. On the contrary, the COVs of other input parameters such as width of the beam $B$ and resistance of the resistor $R$ are relatively less important to $f_{rn}$. And the dc bias $U_{dc}$ and the amplitude $U_{ac}$ of ac excitation signal also have little effects on $f_{rn}$.

In the IQR analysis of the nonlinear frequency offset $F_{off}$, all input parameters have large influences on the IQR of $F_{off}$ while the beam thickness $H$ and the resistance of excitation resistor $R$ almost have the same influences to the IQR of $F_{off}$. Both effects of $U_{dc}$ and $U_{ac}$ on $F_{off}$ are significant. And $U_{dc}$ is more important to $F_{off}$ than $U_{ac}$, which means that large $U_{dc}$ can lead to large change of $F_{off}$.

The IQR analysis of $A_{mp}$ indicates a strong relationship between the IQRs and the COVs of beam thickness $H$. The influence of beam length $L$ is relatively minor and the influence of beam width $B$ is also insignificant. The effects of $U_{dc}$ and $U_{ac}$ on $F_{off}$ are as strong as the effect of $R$. And $U_{dc}$ is more important to $A_{mp}$ than $U_{ac}$.
Fig. 9. The IQR of the output parameters with different COVs of the input parameters.

(a) IQR of $f_n$ with different input parameters COV

(b) IQR of $F_{\text{off}}$ with different input parameters COV

(c) IQR of $A_{\text{mp}}$ with different input parameters COV

(d) IQR of $F_{\text{set}}$ with different input parameters COV

Fig. 10. The IQR of the output parameters with different values and COVs of $U_{\text{ac}}$ and $U_{\text{dc}}$.

(a) IQR of $f_n$ with different values and COV of $U_{\text{ac}}$ and $U_{\text{dc}}$

(b) IQR of $F_{\text{off}}$ with different values and COV of $U_{\text{ac}}$ and $U_{\text{dc}}$

(c) IQR of $A_{\text{mp}}$ with different values and COV of $U_{\text{ac}}$ and $U_{\text{dc}}$

(d) IQR of $F_{\text{set}}$ with different values and COV of $U_{\text{ac}}$ and $U_{\text{dc}}$
The IQR analysis of the nonlinear factor $F_{\text{nl}}$ shows that the nonlinear factor $F_{\text{nl}}$ greatly depends on $U_{dc}$ and $U_{ac}$, while the resistance of excitation resistor $R$ also has strong influence to the IQR of $F_{\text{nl}}$. Other size parameters have little influence to $F_{\text{nl}}$. At the same time, the effect of $U_{dc}$ are greater than the effect of $U_{ac}$.

The excitation voltages have strong impact on the vibration nonlinearity of the resonant beam when the size parameters are fixed in the specific sensor sample. The relationship between the IQRs and different values $U_{ac}$ and $U_{dc}$ are investigated. Fig. 10 shows the IQRs of $f_{\text{rn}}$, $F_{\text{off}}$, $A_{\text{mp}}$, and $F_{\text{nl}}$ at different values and different COVs of $U_{ac}$ and $U_{dc}$. When the COVs of $U_{ac}$ or $U_{dc}$ increase from 0.01 to 0.1, the COVs of other parameters keep unchanged. It can be found from Fig. 10 that the COV of $U_{dc}$ causes a more significant increase of the IQR of the output parameter than the COV of $U_{ac}$, which indicates a relatively high dependence of $f_{\text{rn}}$, $A_{\text{mp}}$, and $F_{\text{nl}}$ on $U_{dc}$ and the dependence is greater for large $U_{dc}$. As $U_{dc}$ increases from 1 V to 2 V and then to 3 V, the IQRs of $f_{\text{rn}}$, $F_{\text{off}}$, $A_{\text{mp}}$, and $F_{\text{nl}}$ increase, which indicates that larger excitation voltage lead to large $F_{\text{off}}$, $A_{\text{mp}}$, and $F_{\text{nl}}$. However, for $f_{\text{rn}}$, larger excitation voltage causes more frequency offset of $f_{\text{rn}}$, so the $f_{\text{rn}}$ becomes low.

### 4.2. Analysis of experiment

From the analysis of sample-based stochastic, it can be found that $U_{dc}$ has significant impact on the vibration nonlinearity of the resonant beam especially when the size parameters are fixed. The natural frequency, which contain the linear natural frequency $f_{\text{rn}}$ and nonlinear frequency offset $F_{\text{off}}$, is detected in a large range of $U_{dc}$ when $U_{ac} = 2$ V. The detected results of two sensor samples together with the theoretical results are shown in Fig. 11. It can be seen that the frequency offsets of linear natural frequency $f_{\text{rn}}$ caused by inherent heat elevation of both samples are high, and it is more serious for sample 2, $f_{\text{rn}}$ obtained by theoretical calculation and detections are consistent while theoretical natural frequency and detected natural frequencies have a relatively large differences. The differences are because nonlinear frequency offsets $F_{\text{off}}$ of both samples are less than theoretical results, which may be caused by undesirable boundary and simplification of the beam into single crystal silicon beam in the vibration model.

In order to verify the effect of $U_{dc}$, different values of $U_{dc}$ are used to drive the resonant beams in the two test samples when $U_{ac}$ are fixed. The amount of change is from $-0.5$ V to $0.5$ V when $U_{dc}$ are 1 V, 2 V, and 3 V, respectively. The experimental results together with the theoretical results of $f_{\text{rn}}$, $F_{\text{off}}$, $A_{\text{mp}}$, and $F_{\text{nl}}$ are shown in Fig. 12. It can be seen that the linear natural frequency $f_{\text{rn}}$ and amplitude $A_{\text{mp}}$ obtained by the theoretical analysis and detections are in good agreement. However, for both sample, the difference between two nonlinear frequency offsets $F_{\text{off}}$ and the difference between two nonlinear factors $F_{\text{nl}}$ obtained by theoretical analysis and experiment detection are large. This is caused by unsatisfactory clamped boundary and assuming beam to be a single silicon beam. From the Fig. 12(a) and (b), it can be seen that linear natural frequency $f_{\text{rn}}$ of sample 2 is larger than that of sample 1, which is caused by the difference of real size parameters. There are a large differences between nonlinear frequency offsets $F_{\text{off}}$ obtained by theoretical analysis and experiment detections. In order to com-
Fig. 12. Experimental and theoretical results of $U_{dc}$ effects.
pare the experimental and theoretical effects of $U_{dc}$ on $F_{off}$, all the $F_{off}$ are normalized by their respective maximums. From Fig. 12(c) and (d), it can be seen that both experimental $F_{off}$ increase when the $U_{dc}$ increases, which are in the same trend with the theoretical $F_{off}$. The amplitude of output vibrating signal is used as a substitute to the vibration amplitude of the beam, which is normalized by the maximum amplitude. From Fig. 12(e) and (f), it can be seen that the amplitudes $A_{mp}$ increase when the $U_{dc}$ increases, and detected results are consistent with the theoretical results, especially for Sample 2. The nonlinear factor $F_{nol}$ depends on the nonlinear frequency offset $F_{off}$ and linear natural frequency $f_{rn}$. So there are large differences between theoretical and experimental results due to the differences between the theoretical $F_{off}$ and experimental $F_{off}$. All the $F_{off}$ are normalized by their respective maximums. From Fig. 12(g) and (h), it can be seen that the nonlinear frequency offsets $F_{nol}$ increase when the $U_{dc}$ increases.

At the same time, large $U_{dc}$ leads to more decrease of $f_{rn}$ and more increase of $F_{off}$, $A_{mp}$ and $F_{nol}$, which agree with the results of sample-based stochastic modeling. The detected results of two samples are not exactly the same due to the different sizes caused by fabrication error.

5. Conclusions

In this paper, a nonlinear vibration model of double-clamped resonant beam driven by electro-thermal excitation is established and solved. From the analysis of sample-based stochastic model and detection experiment, it can be concluded that:

(1) The frequency offset of linear natural frequency $f_{rn}$ caused by inherent heat elevation is large when $U_{dc}$ is high, and it is larger than the nonlinear frequency offset $F_{off}$ that is low in experimental detection.

(2) The dc bias $U_{dc}$ have significant effect on the $f_{rn}$, $F_{off}$, $A_{mp}$ and $F_{nol}$ while the $U_{dc}$ has relatively minor effect on these performances; large $U_{dc}$ leads to more decrease of $f_{rn}$ and more increase of $F_{off}$, $A_{mp}$ and $F_{nol}$, which agree with the results of sample-based stochastic modeling.

(3) Size parameters have different influence on $f_{rn}$, $F_{off}$, $A_{mp}$ and $F_{nol}$ in different ways. Beam length $L$ has stronger influence on $f_{rn}$ and $F_{off}$. Beam thickness $H$ has stronger influence on $A_{mp}$. Resistance of resistor $R$ has strong influence on $F_{nol}$. However, beam width $B$ nearly has no effect on $f_{rn}$, $F_{off}$, $A_{mp}$ and $F_{nol}$.

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References


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