
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/unhb20

A Hybrid Lattice Boltzmann and Finite-Volume Method for Melting with Convection

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Published online: 25 Aug 2014.


To link to this article: http://dx.doi.org/10.1080/10407790.2014.915678

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A HYBRID LATTICE BOLTZMANN AND FINITE-VOLUME METHOD FOR MELTING WITH CONVECTION

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A hybrid lattice Boltzmann and finite-volume model is proposed to solve the natural-convection-controlled melting problem. The lattice Boltzmann method (LBM) is applied to solve the velocity field, while the temperature field is obtained by the finite-volume method (FVM). The D2Q9 model and finite-difference velocity gradient boundary condition are used in the LBM and the SIMPLE algorithm with QUICK scheme is employed in the FVM. An interfacial tracking model based on energy balance at the interface is applied to obtain the location of the solid–liquid interface. The results from the present hybrid method are validated with experimental results, and good agreement is obtained.

1. INTRODUCTION

Melting and solidification play important roles in many engineering applications such as energy storage, material processing, and electronics cooling. Numerous experimental and numerical investigations have been carried out by researchers around the globe. Benard et al. [1] studied the melting in a rectangular enclosure experimentally and numerically. Dong et al. [2] and Wang et al. [3] investigated the melting with natural convection in a rectangular enclosure experimentally. Jany and Bejan [4] performed scale analysis to analyze the melting problem with natural convection. Zhang and Bejan [5] studied the natural-convection melting with conduction in the solid theoretically. Bertrand et al. [6] compared different numerical methods for the melting driven by natural convection. Huber et al. [7] proposed a lattice Boltzmann method (LBM) for melting with natural convection. Gao and Chen [8] solved the convection-dominated melting problem in porous media using the LBM.

The key point to solve the melting problem numerically is how to obtain the location of the interface, and there are several methods [9, 10] to achieve this goal.

Received 28 December 2013; accepted 8 March 2014.
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Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/unhb.
The enthalpy method [11] is one of the frequently used methods. It can solve the melting problem taking place at a fixed temperature or a range of temperature. Zhang and Chen [12] proposed an interfacial tracking method for the melting under ultrafast laser heating. Chen et al. [13] and Li et al. [14] applied this method to solve natural-convection-controlled melting in a rectangular enclosure under constant wall temperature and discrete heating, respectively. All of the above applications of interfacial tracking method are based on the finite-volume method (FVM) [15], which has high efficiency and whose solution always satisfies the conservation laws.

Use of the lattice Boltzmann method has been growing rapidly during the last three decades. It has been applied to solve various heat transfer problems [16–23] and has showed advantages for fluid flow in complex geometries [24]. Therefore, a hybrid LBM and FVM that uses the LBM to solve the momentum equation and employs the FVM to obtain solution of the energy equation is a promising numerical method, since the main advantage of the LBM lies in obtaining a converged velocity field. Peng et al. [25] coupled the FVM and LBM on unstructured meshes. Mishra and Roy [26] solved the transient conduction and radiation problem using the hybrid method. Ganaoui and Semma [27] reported the results of a phase-change problem obtained by a hybrid method based on an entropy model. The authors employed a hybrid lattice Boltzmann and finite-volume method to solve natural convection

### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( a )</td>
<td>particle acceleration</td>
</tr>
<tr>
<td>( c )</td>
<td>lattice speed</td>
</tr>
<tr>
<td>( c_p )</td>
<td>specific heat, ( \text{J/kg K} )</td>
</tr>
<tr>
<td>( c_s )</td>
<td>sound speed</td>
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<tr>
<td>( e )</td>
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<td>liquid fraction</td>
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<td>( F_0 )</td>
<td>Fourier number</td>
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<tr>
<td>( g )</td>
<td>gravity acceleration, ( m/s^2 )</td>
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<tr>
<td>( G )</td>
<td>effective gravitational acceleration, ( m/s^2 )</td>
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<td>( h_d )</td>
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<td>( k )</td>
<td>thermal conductivity, ( \text{W/m k} )</td>
</tr>
<tr>
<td>( k' )</td>
<td>modified thermal conductivity, ( \text{W/m k} )</td>
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<tr>
<td>( K_n )</td>
<td>coefficient in Chapman-Enskog expansion</td>
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<td>( M_a )</td>
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<td>( p )</td>
<td>pressure, ( \text{Pa} )</td>
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<tr>
<td>( P )</td>
<td>nondimensional pressure</td>
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<td>( P_r )</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>( R_a )</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>( s )</td>
<td>location of melting front, ( \text{m} )</td>
</tr>
<tr>
<td>( S_c )</td>
<td>subcooling</td>
</tr>
<tr>
<td>( S_t )</td>
<td>Stefan number</td>
</tr>
<tr>
<td>( t )</td>
<td>time, ( \text{s} )</td>
</tr>
</tbody>
</table>

### Superscripts

- \( e \): east face of the control volume
- \( l \): liquid \( n \)-octadecane
- \( s \): solid \( n \)-octadecane
- \( w \): west face of the control volume

The enthalpy method [11] is one of the frequently used methods. It can solve the melting problem taking place at a fixed temperature or a range of temperature. Zhang and Chen [12] proposed an interfacial tracking method for the melting under ultrafast laser heating. Chen et al. [13] and Li et al. [14] applied this method to solve natural-convection-controlled melting in a rectangular enclosure under constant wall temperature and discrete heating, respectively. All of the above applications of interfacial tracking method are based on the finite-volume method (FVM) [15], which has high efficiency and whose solution always satisfies the conservation laws.

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in an enclosure [28]. The objective of this article is to apply the hybrid method to solve melting with natural convection based on the interfacial tracking method. The results will be compared with experimental results for validation.

2. PROBLEM STATEMENT

Figure 1 shows an enclosure filled with phase-change material (PCM). The left vertical wall is kept at a constant temperature $T_h$, which is above the melting temperature $T_m$, while the right cold wall is kept at a constant temperature $T_c$, which is below $T_m$. Meanwhile, the top and bottom of the enclosure are adiabatic. Nonslip conditions are applied to all the boundaries. The initial temperature of the system is at $T_c$.

The following assumptions are made:

1. The PCM is pure and homogeneous.
2. The liquid phase of the PCM is Newtonian and incompressible.
3. Boussinesq approximation is applied to the liquid phase.
4. The volume change due to the phase change is negligible.
5. Natural convection of the liquid phase is laminar.

Figure 1. Phase-change model.
The problem can be described by the following governing equations:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  \hspace{1cm} (1)

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  \hspace{1cm} (2)

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g (T - T_m)
\]  \hspace{1cm} (3)

\[
\left( \rho c_p \right) \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  \hspace{1cm} (4)

Equations (1)–(4) are subject to the following boundary and initial conditions:

\[
x = 0, \quad u = 0, \quad v = 0, \quad T = T_h
\]  \hspace{1cm} (5)

\[
x = W, \quad u = 0, \quad v = 0, \quad T = T_c
\]  \hspace{1cm} (6)

\[
y = 0, \quad u = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = 0
\]  \hspace{1cm} (7)

\[
y = H, \quad u = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = 0
\]  \hspace{1cm} (8)

Melting front:

\[
x = s, \quad T = T_m
\]  \hspace{1cm} (9)

\[
x = s, \left[ 1 + \left( \frac{\partial s}{\partial y} \right)^2 \right] \left( k_s \frac{\partial T_s}{\partial x} - k_l \frac{\partial T_l}{\partial x} \right) = \rho h_s \frac{\partial s}{\partial t}
\]  \hspace{1cm} (10)

Initial condition:

\[
t = 0, \quad T = T_c
\]  \hspace{1cm} (11)

The momentum equation is solved by the LBM, while the FVM is applied to the energy equation. For the LBM, Mach number Ma is needed and is defined as

\[
Ma = \frac{u_c}{c_s}
\]  \hspace{1cm} (12)

where \( u_c \) is a characteristic velocity that equals \( \sqrt{g \beta (T_h - T_m) H} \).

Applying the following nondimensional variables,

\[
\begin{align*}
X &= \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{\sqrt{g\beta}}, \quad V = \frac{v}{\sqrt{g\beta}}, \quad Pr = \frac{v}{u_c}, \quad Ra = \frac{g \beta (T_h - T_m) H^3}{\nu^2}, \\
\tau &= \frac{t \sqrt{\nu c_p}}{H}, \quad \theta = \frac{T - T_m}{T_h - T_m}, \quad P = \frac{\rho}{\rho_c^2}, \quad Ste = \frac{\rho h_s (T_h - T_s)}{h_s}, \quad Sc = \frac{T_m - T_m}{T_h - T_m}
\end{align*}
\]  \hspace{1cm} (13)
to Eqs. (1)–(4), the dimensionless governing equations can be obtained as

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

(14)

\[
\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + Ma \sqrt{\frac{Pr}{3 Ra}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]

(15)

\[
\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + Ma \sqrt{\frac{Pr}{3 Ra}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Ma^2 \theta}{3}
\]

(16)

\[
\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = Ma \sqrt{\frac{1}{3 Ra Pr}} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

(17)

\[
X = S, \quad \theta = 0
\]

(18)

\[
X = S, \quad \frac{Ma \cdot Ste}{\sqrt{3 Ra Pr}} \left[ 1 + \left( \frac{\partial S}{\partial Y} \right)^2 \left( \frac{k_s \partial T_s}{k_f \partial x} - \frac{\partial T_f}{\partial x} \right) \right] = \frac{\partial S}{\partial \tau}
\]

(19)

\[
X = 0, \quad U = 0, \quad V = 0, \quad \theta = 1
\]

(20)

\[
X = W/H, \quad U = 0, \quad V = 0, \quad \theta = -Sc
\]

(21)

\[
Y = 0, \quad U = 0, \quad V = 0, \quad \partial \theta/\partial Y = 0
\]

(22)

\[
Y = 1, \quad U = 0, \quad U = 0, \quad \partial \theta/\partial Y = 0
\]

(23)

Initial condition:

\[
\tau = 0, \quad \theta = -Sc
\]

(24)

Meanwhile, the Fourier number Fo, which is defined as

\[
Fo = \frac{kt}{\rho c_p H^2}
\]

(25)

plays an important role in the melting problem discussion. Fourier number has a relation to the nondimensional time as

\[
Fo = \frac{\tau \cdot Ma}{\sqrt{3 Ra \cdot Pr}}
\]

(26)
3. HYBRID LBM AND FVM METHOD FOR MELTING PROBLEM

3.1. Lattice Boltzmann Method for Velocity Fields

The lattice Boltzmann method is applied to solve the velocity field in the hybrid method for the melting with convection. The Boltzmann equation can be used to describe the statistical behavior of a fluid [29]:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \mathbf{a} \cdot \nabla f = \Omega_{\text{collision}}$$

(27)

where \( f \) is the density distribution, \( \mathbf{a} \) is the particle acceleration, \( \mathbf{v} \) is the particle velocity, and \( \Omega_{\text{collision}} \) is the collision operator. Equation (27) can be applied to any direction \( i \) in the phase space. The particle velocity and collision operator in the \( i \)th direction are \( \mathbf{e}_i \) and \( \Omega_i \), respectively. The collision operator \( \Omega_{\text{collision}} \) makes solution of Eq. (27) very challenging. To satisfy the conservations of mass and momentum, \( \Omega_i \) has to meet the following conditions:

$$\sum_i \Omega_i = 0$$

(28)

$$\sum_i \mathbf{e}_i \Omega_i = 0$$

(29)

The LBGK model simplifies the collision operator with the function

$$\Omega_i = -\frac{1}{\tau_v} (f_i - f_i^{\text{eq}})$$

(30)

where \( \tau_v \) is the relaxation time and \( f_i^{\text{eq}} \) is the equilibrium density distribution. The equilibrium density distribution \( f_i^{\text{eq}} \) can be obtained from the Maxwell-Boltzmann distribution,

$$f_i^{\text{eq}}(\rho, \mathbf{u}) = \left( 1 + \frac{1}{c_s^2} \mathbf{e}_i \cdot \mathbf{u} + \frac{1}{2c_s^4} \mathbf{Q}_i : \mathbf{u}\mathbf{u} \right)$$

(31)

$$\mathbf{Q}_i = \mathbf{e}_i \mathbf{e}_i - c_i^2 \mathbf{I}$$

(32)

where \( c_s \) is the speed of sound and \( \mathbf{I} \) is the identity tensor.

The D2Q9 model shown in Figure 2 can be used for a 2-D problem. The velocity in every direction is

$$\mathbf{e}_i = \begin{cases} (0, 0) & i = 1 \\ \mathbf{c}(-\cos \frac{2i\pi}{9}, -\sin \frac{2i\pi}{9}) & i = 2, 3, 4, 5 \\ \sqrt{2c} \left[ -\cos \left( \frac{(2i+1)\pi}{4} \right), -\sin \left( \frac{(2i+1)\pi}{4} \right) \right] & i = 6, 7, 8, 9 \end{cases}$$

(33)
where \( c \) is the lattice speed. Equation (27) can be simplified as

\[
f_i(r + e_i \Delta t, t + \Delta t) - f_i(r, t) = \frac{1}{\tau_i} \left[ f_i^{eq}(r, t) - f_i(r, t) \right] + F_i \quad i = 1, 2, \ldots, 9
\]  
(34)

where \( \Delta t \) is the time step.

The body force \( F_i \) in the system is

\[
F_i = \Delta t \cdot G \cdot \frac{e_i \cdot u_i}{p} f_i^{eq}
\]  
(35)

When the velocity is low, Eq. (31) can be simplified as

\[
f_i^{eq} = \rho \omega_i \left[ 1 + \frac{e_i \cdot u_i}{R_s T} + \frac{(e_i \cdot u)^2}{2R_s^2T^2} - \frac{u_i \cdot u_i}{R_s T} \right]
\]  
(36)

where

\[
\omega_i = (2\pi R_s T)^{-m/2} \exp \left( -\frac{e_i \cdot e_i}{2R_s T} \right)
\]  
(37)

Equation (36) can be further simplified in terms of the speed of sound in the lattice unit \( c_s \):

\[
f_i^{eq} = \rho \omega_i \left[ 1 + \frac{e_i \cdot u_i}{c_s^2} + \frac{(e_i \cdot u)^2}{2c_s^4} - \frac{u_i \cdot u_i}{2c_s^2} \right]
\]  
(38)
where

\[
    \omega_i = \begin{cases} 
        \frac{4}{9} & i = 1 \\
        \frac{4}{9} & i = 2, 3, 4, 5 \\
        \frac{1}{36} & i = 6, 7, 8, 9 
    \end{cases} \tag{39}
\]

And the macroscopic properties can be obtained by the following equations:

\[
    \rho = \sum_{i=1}^{9} f_i \tag{40}
\]

\[
    \rho u = \sum_{i=1}^{9} e_i f_i \tag{41}
\]

\[
    p = \rho c_s^2 \tag{42}
\]

The Navier-Stokes equation can be reached through the Chapman-Enskog expansion [24] when the kinematic viscosity \( \nu \) is related with relaxation time \( \tau_v \) in Eq. (34) by

\[
    \nu = c_s^2 \left( \tau_v - \frac{1}{2} \right) \Delta t \tag{43}
\]

The nonslip boundary condition is applied to the LBM at the fixed boundaries as well as the solid–liquid interface. The finite-difference velocity gradient method [30] is used in this article to replace all the density distributions on the boundary:

\[
    f_i = f_i^{\text{eq}} - \frac{\tau \omega_i}{c_s^2} Q_i : \rho \nabla u \tag{44}
\]

So the density distribution can be related to strain rate tensor \( S \) due to the symmetry of \( Q_i \):

\[
    f_i = f_i^{\text{eq}} - \frac{\tau \omega_i}{c_s^2} Q_i : S \tag{45}
\]

where

\[
    S = \frac{1}{2} \left[ \nabla u + (\nabla u)^T \right] \tag{46}
\]

The velocity field in the phase-change problem can be obtained by the LBM discussed above.

### 3.2. Finite-Volume Method for Energy Equation

The finite-volume method is employed to solve the energy balance in Eq. (4) for the temperature field. Equation (4) is integrated over each control volume shown in Figure 3 for one time step. The QUICK scheme [13] is applied to this process. It is worth noting that the locations of the computing nodes in the LBM and FVM are

\[
    \frac{1}{2} \left[ \nabla u + (\nabla u)^T \right]
\]
different: they are respectively located at the center or the corner of the control volume for the FVM or LBM shown in Figure 3. Therefore, central difference is needed to obtain the velocity on the face of the control volume from the LBM results. And the time integration is performed implicitly by adding the information in the last time step to the source term [27].

3.3. Interfacial Tracking Method

The phase-change interface location is calculated by the interfacial tracking method, and the porous media assumption is applied to the phase-change zone [31]. For this 2-D melting problem, the location of the melting front in the control volume is where melting takes place and changes every time step. Figure 4 shows three possible locations of the melting fronts in the control volume. Assuming that the size of the control volume in the $x$ direction is $(\Delta X)_p$, the liquid fraction $f_p$, can be obtained by

$$f_p = \frac{S - X_p + (\Delta X)_p/2}{(\Delta X)_p}$$

where $X_p$ is the center of the control volume in the $x$ direction.

![Figure 3. Control volume in FVM.](image-url)
In this interfacial tracking model, the temperature at the center of the control volume $P$ in which melting happens is assumed to be at the melting point. This assumption will cause an error when the melting front is not exactly at $P$ as shown in Figures 4b and 4c. Since this mainly affects the energy balance, a correction to the conductivity can be applied to fix this error. The interfacial tracking model uses the following corrections to $k_s$ and $k_l$ in the melting control volume:

$$k_s = \frac{(\delta X)_c}{(\delta X)_c + (0.5 - f_p)(\Delta X)_p} k_s$$  \hspace{1cm} (48)

$$k_l = \frac{(\delta X)_w}{(\delta X)_w - (0.5 - f_p)(\Delta X)_p} k_l$$  \hspace{1cm} (49)

Applying Eqs. (48) and (49) to Eqs. (18) and (19), we can reach

$$X = S, \quad 0_p = 0$$  \hspace{1cm} (50)
With these settings, the interfacial tracking model can be used in simulating the melting process. Then the interface location can be obtained by

\[
S = S^0 + \frac{\partial S}{\partial \tau} \Delta \tau
\]

(52)

where \(S^0\) is the interface location in the last time step and \(\Delta \tau\) is the time step.

### 3.4. Porous Media Model

Melting of pure phase-change material takes place at a fixed temperature, and the solid and liquid phases are separated by a clear interface. In order to obtain a reasonable velocity field near the solid–liquid interface, it is assumed that there is a phase-change zone between the solid and the liquid zones as shown in Figure 1. Both liquid and solid exist in the one control volume in the phase-change zone, and the fractions of liquid and solid depend on the degree of melting (solid–liquid interface within the control volume). Semma et al. [31] suggested that the phase change zone can be considered as a porous medium and solved the phase-change problem with pure LBM. There are several methods for the fluid flow in porous media using the LBM. Spaid and Phelan [32] proposed a model for microscale flow in fibrous media by solving the Brinkman equation directly, while Dardis and McCloskey [33] avoided solving Brinkman equation by adding a porous media effect term in the collision term in Eq. (34). Guo and Zhao [34] fulfilled the fluid flow simulation by introducing another force term depending on the parameters of the porous medium.

In this work, the velocity field in the phase-change zone is obtained using the porous media model [33]. It is assumed that the density distribution in each node is uniform throughout the volume of each node, and each particle moves only 1 or \(\sqrt{2}\) lattice spacing in one step in the nine directions as shown in Figure 1. The real situation of the computing node \(N_i\) in the phase-change zone is shown in Figure 5a. The interface location is not right on the node \(N_{i+1}\) and the liquid fraction on node \(N_i\) is \(n_s\). Then the volume of node \(N_i\) is assumed to be filled with a porous medium with permeability \(1 - n_s\) as shown in Figure 5b. The density distribution in the phase-change zone can be expressed as

\[
f_i(r + e_i \Delta t, t + \Delta t) - f_i(r, t) = \frac{1}{c_v} \left[ f_i^{eq}(r, t) - f_i(r, t) \right] + F_i + \Delta^{PCM},
\]

(53)

where \(\Delta^{PCM}\) is the porous media effect term. When \(n_s\) equals 1, it means that the node \(N_i\) is pure free fluid node so that \(\Delta^{PCM}\) has no effect on the simulation process. When \(n_s\) equals 1, it means the node \(N_i\) is solid boundary node and Eq. (53) is reduced to a standard bounce-back boundary condition [31].
4. RESULTS AND DISCUSSION

The hybrid method is validated by comparing the results with the experimental results in [35]. In the experimental system, a 15 mm × 15 mm enclosure is filled with n-octadecane as PCM. The melting point of the n-octadecane is 300.7 K, and its latent heat is 244 kJ/kg. The viscosity is 0.0039 N s/m², and the density is 774 kg/m³. The thermal conductivity, specific heat, and thermal expansion coefficient are 0.152 W/m K, 2,180 J/kg K, and 0.00085 K⁻¹, respectively. The Prandtl number equals 56.2 and the subcooling is zero. Regarding the definition of Rayleigh number and Stefan number, they only change with the temperature difference \( T_h - T_m \) linearly when the other parameters are all fixed in the system.

The results of numerical solution for Stefan number \( \text{Ste} = 0.0451 \) and \( \text{Ste} = 0.0959 \) are compared with the results in [35]. The corresponding temperature differences are 5.3 K and 11.2 K, respectively. The Rayleigh numbers for these two cases are \( 3.27 \times 10^5 \) and \( 6.95 \times 10^5 \), respectively.

For the case that \( \text{Ste} = 0.0451 \), the melting process is governed by the conduction at the beginning. The melting interface is almost a vertical straight line during that period. After the nondimensional time \( \tau \) becomes greater than 7,500, the melting velocity varies depending on the \( Y \) location in the enclosure. The PCM closer to the top of the enclosure melts faster than that closer to the bottom. And this difference becomes bigger as time grows. The temperature filed of this case when \( \tau = 52,700 \) is shown in Figure 6. Generally, the higher the location is, the higher temperature is due to the natural convection. This temperature field governed by the natural convection leads to the sloping interface. Figure 7 shows the comparison of melting fronts between the numerical results and the experimental results when \( \tau = 52,700 \) for the case of \( \text{Ste} = 0.0451 \). It can be seen that the numerical results agree with the experimental results very well.

Figure 5. Porous media assumption: (a) real interface location, (b) assumption.
The case that Ste equals 0.0959 is also studied. The temperature difference is larger than the previous case. Therefore the natural-convection effect is more pounced. The liquid PCM zone begins to be governed by the natural convection when $\tau$ is around 5,700. The temperature field is shown in Figure 8 when $\tau$ equals 37,250. Similar to the previous case, basically the higher temperature locates in the

![Figure 6. Temperature field, Ste = 0.0451, $\tau = 52,700$.](image1)

![Figure 7. Melting-front comparison, Ste = 0.0451, $\tau = 52,700$.](image2)
higher location due to the natural convection. It can also be seen that the nondimensional temperature gradient near the interface is larger than in the previous case. Since this case has a larger temperature difference, it has a larger temperature gradient, which can lead to higher melting rate. Figure 9 shows the comparison of melting
fronts between the numerical and the experimental results when \( \tau \) is 37,250 for the case that \( \text{Ste} = 0.0959 \). The numerical results agree with the experimental results very well. Several facts may cause the slight difference between the numerical and experimental results. The discrepancies may be caused by the facts that the volume change due to the phase change is assumed to be negligible in the numerical method, and keeping the heated wall at a constant temperature during experiments is also challenging.

More numerical solutions are carried out for the cases that Stefan numbers are 0.0658, 0.0309, and 0.0212, respectively. The corresponding Rayleigh numbers are \( 4.77 \times 10^5 \), \( 2.24 \times 10^5 \), and \( 1.54 \times 10^5 \), respectively. Figure 10 shows the comparisons of Nusselt numbers for different Stefan numbers. The Nusselt number \( \text{Nu} \) that represents the heat transfer rate from the heated wall to the liquid PCM is defined as

\[
\text{Nu} = \frac{hH}{k_l} \tag{54}
\]

where the heat transfer coefficient \( h \) can be obtained by

\[
h(T_h - T_m) = -k \left. \frac{\partial T}{\partial x} \right|_{x=0} \tag{55}
\]

Substituting Eqs. (13) and (55) to Eq. (54), one can obtain that

\[
\text{Nu} = -\left. \frac{\partial \theta}{\partial X} \right|_{X=0} \tag{56}
\]

Figure 10. Nusselt number comparison.
Then the average Nusselt number is

\[ \text{Nu}_{\text{avg}} = \int_{0}^{1} \left. \frac{\partial \theta}{\partial X} \right|_{X=0} dY \] (57)

The parameter \( \text{Fo} \times \text{Ste} \) is an important time measure in the melting problem. For the conduction governed melting problem we can reach [7]

\[ \text{Nu}_{\text{avg}} \sim (\text{Fo} \times \text{Ste})^{-1/2} \] (58)

For all the five cases considered, the conduction dominates the melting process at the beginning. When \( \text{Fo} \times \text{Ste} \) is less than 0.005, the Nusselt numbers are almost the same in the five cases; this means that the melting processes for all five cases are governed by the conduction during that period. Then the Nusselt numbers become different and they are higher for higher Stefan numbers. Natural convection is more pronounced with larger temperature difference, which leads to higher Stefan number. After some time the Nusselt numbers approach with different constant numbers, which means that the melting processes reach quasi-steady state. After reaching this state, the liquid fraction becomes linear with time, which agrees well with Figure 11. And the liquid fractions have different slops to \( \text{Fo} \times \text{Ste} \), since the higher Stefan number case has higher Nusselt number.

The melting front comparison when \( \text{Fo} \times \text{Ste} \) equals 0.033 is shown in Figure 12. As discussed above, the natural convection has governed the melting process at that instance. We can see that the melting rate close to the top of the enclosure is higher for the case with higher Ste, and the melting rate close to the bottom of the enclosure is

![Figure 11. Comparison of liquid fractions.](image-url)
higher for the case with lower Ste. This indicates that the natural-convection effect is more important for the higher Ste case, which agrees with the discussion above.

5. CONCLUSIONS

A hybrid lattice Boltzmann and finite-volume method is developed for melting problems. The interfacial tracking method is applied to obtain the location of the interface. Natural-convection-governed melting problems with different Stefan numbers are solved for validation. The numerical results agree with the experimental results very well. Therefore the proposed numerical method is valid for melting problem simulation.

FUNDING

Support for this work by the U.S. National Science Foundation under Grant CBET-1066917, and Chinese National Natural Science Foundation under Grants 51129602 and 51276118 is gratefully acknowledged.

REFERENCES


