Experimental study on natural convection in a cylindrical envelope with an internal concentric cylinder with slots

Kun Zhang a,b, Mo Yang b,* , Jin Wang b, Yuwen Zhang c

aCollege of Mechanical Engineering, University of Lanzhou Jiaotong, Lanzhou 730010, China
bCollege of Power Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China
cDepartment of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA

ARTICLE INFO

Article history:
Received 4 April 2013
Received in revised form 7 September 2013
Accepted 14 September 2013
Available online 22 October 2013

Keywords:
Natural convection
Rayleigh number
Static bifurcation
Self-sustained

ABSTRACT

Detailed experimental analysis is presented for natural convection in a cylindrical envelope with an internal concentric cylinder with slots. For the case of $\phi = 90^\circ$, where $\phi$ is the angle of slot opening from the vertical axis, and two types of temperature fields were obtained at the same Rayleigh number of $5.49 \times 10^4$. It is demonstrated that the steady state solution was not unique and was dependent on the initial conditions, which is related to static bifurcation. For the two cases of $\phi = 0^\circ$ and $\phi = 45^\circ$, the natural convection turned to unsteady, although the boundary conditions were not time-dependent. The most obvious oscillation regions were above the inner cylinder, and the amplitude increased with increasing Rayleigh numbers. The nonlinear characteristic of these problems led to the multiplicity of convections and self-sustained oscillation.

© 2013 Elsevier Masson SAS. All rights reserved.

1. Introduction

Natural convection is a recurrent phenomenon in the world and most of these flows are unsteady. Studies of unsteady-state natural convections have attracted increasing interest over the last decades due to the desire to improve the phenomenological understanding of natural convection, especially those studies of unsteady flow patterns and temperature fields related to nonlinear characteristics. Natural convection in a cavity is one of the classical models to study nonlinear characteristics. Many helpful results from the researches of natural convection in a cavity indicated that laminar steady flow and heat transfer are observed for lower Rayleigh number, while unsteady convection is obtained at high Rayleigh numbers [1–3]. Debasish and Muralidhar [4] experimentally investigated Rayleigh–Benard convection in a cavity using interferometric tomography. The results showed a sequence of transitions from stable laminar flow to unsteady flow and ultimately to turbulent flow. Mukutmoni and Yang [5,6] simulated Rayleigh–Benard convection in a small cavity with aspect ratio and confirmed that the oscillation and chaos existed. Zhang et al. [7] numerically investigated three different types of static bifurcation at $Ra = 5000$ with different initial conditions. Tae et al. [8] studied mixed convection in the cavity with simulation and experiment. The temperature field fluctuated when buoyancy force was higher. Zhan et al. [9] numerically studied the nonlinear phenomenon of natural convection in a 3-D rectangular cavity. The results showed that the static and dynamic bifurcation appeared with different initial conditions. Experimental studies of this problem were also performed by them [10], and their results verified the conclusions in the numerical studies that the self-sustained phenomenon of flow fields occurred in the steady conditions. Deshpande and Srinidhi [11] numerically studied the mixed convection in a cavity and observed the certain features of dynamic systems like bifurcation, period doubling and chaos. Benouaguef [12] numerically studied the unsteady natural convection in an air-filled square enclosure. They plotted the temporal evolutions of the hot global Nusselt number and the attractors in a space trajectory, and discussed the effect of the Rayleigh number on the route to the chaos.

Many experimental investigations and numerical simulations have been conducted to study unsteady natural convection in horizontal cylindrical annuli in steady conditions. Powe et al. [13,14] visualized the flow patterns with smoke as a tracer and obtained the Rayleigh number at which the flow will change from a steady flow to an unsteady flow. Rao et al. [15] investigated the transient oscillatory phenomena and numerically determined the critical Rayleigh number at which unsteady flow occurs. Cheddadi et al. [16] numerically and experimentally studied the bifurcation...
phenomenon of natural convection at different initial conditions, and showed that the flow pattern was not unique and depended on the initial conditions at high Rayleigh number. However, the static bifurcation phenomenon was not observed in their experiments. Liu et al. [17] investigated the stability of natural convection by measuring the total heat transfer coefficient and the distribution of radiation temperature of water, air and silicone. The results indicated that the critical Rayleigh number dictated the transition from steady to unsteady. Mizushima et al. [18] numerically investigated the bifurcation phenomenon and obtained the critical Rayleigh number when the flow changes into multi-vortex. Yoo [19] investigated numerically the bifurcation sequences to the chaos for the natural convection in horizontal concentric annuli in detail.

Comparatively, little works have been reported on unsteady natural convection heat transfer in more complex domain, such as a cylindrical envelope with an internal concentric cylinder with slots. Some early studies for those complex models have been focused on the convective heat transfer enhancement. Kuleek assumed that the heat transfer enhancement of a cylindrical envelope with an internal slotted cylinder and found the convective heat transfer coefficient with slots could be enhanced by as much as 50%. They indicated that the unsteady oscillated phenomenon on steady conditions will be investigated in detail.

2. Experimental systems

2.1. Experimental model

The problem under consideration, as shown in Fig. 1, is the natural convection heat transfer in a horizontal cylindrical envelope with an internal concentric cylinder with slots. The inner and outer cylinders are kept at uniform but different temperatures \( T_i \) and \( T_o \), respectively, with \( T_i > T_o \). As a result of the temperature difference between the two circles, density gradient occurs and leads to natural convection. It is assumed that the air in the enclosure is of Boussinesq type. The included angle \( \phi \) is defined for the angle between central line of internal slotted cylinder and vertical axis. Three different inclined angles (\( \phi = 0^\circ \), \( \phi = 45^\circ \) and \( \phi = 90^\circ \)) are selected for those models in our experiments.

2.2. Experimental setup

The experimental setup consists of anti-seismic table, experimental section and optical system. A schematic diagram of the total experimental setup is presented in Fig. 2. The anti-seismic table was manufactured by University of Shanghai for Science and Technology (see Fig. 3) and the flatness is less than 0.10 mm/m². The experimental study was carried out using laser holographic interferometry technique [25,26]. The effect of initial conditions on the final temperature fields and the multiplicity of solutions related to static bifurcation will be investigated. Numerical simulation of the natural convection at the same parameters will also be performed to compare with the experimental results. The self-sustained oscillated phenomenon on steady conditions will be investigated in detail.
interferometry technique. The optical system consisted of a light source, a half pass lens, six mirrors, two beam expanders and two collimating lenses. They were fixed on the anti-seismic table. The dotted lines in Fig. 2 depict the light path in this experiment. The light source is 30 mW helium–neon continuum laser. The microscope, the Fourier lens and the dielectric mirror were used for beam expander, collimating lens and steering mirrors, respectively. The imaging system consisted of the holograph plate using silver halide and the film rack. To ensure the precision of replacement, the holograph plate top was fixed on the rack. The container with liquid was moved up and down for developing, fixation and bleaching. The interferogram were formed on the holograph plate when the inner and outer cylinders were kept at different temperatures. The interference fringes correspond to isotherms and all interferograms were recorded by a camera.

Fig. 4 shows the schematic of experimental test section. To provide the uniform surface temperature in the axial direction, test cylinders were made from copper of \( L = 280 \) mm long and the wall thickness was 1 mm. The inner radius of cylindrical envelope is \( r_o = 50.5 \) mm. Two plastic water pipes were placed around the outer cylindrical envelope to form flow channels for cooling water in the form of a double helix. The two plastic water pipes were covered with cotton insulation. The internal concentric cylinder with slots was made of two slotted copper cylinders and enveloped by copper bars. The internal space between the slotted copper cylinders was used as flow channels for hot water. The hot water can flow in and out from one side to another side by four copper tubes. Those copper tubes were embedded into the two extremities of the internal cylinder with slots and were used for supporting the internal concentric cylinder with slots. The outer and inner radii of internal concentric cylinder with slots are \( r_j = 21 \) mm and \( r_i = 16.3 \) mm, respectively. The slotted angle is \( \alpha = 42.4^\circ \). The front and rear of the test section were enveloped by two 19-mm-thick tempered glasses to satisfy adiabatic condition.

The outer cylindrical envelope is heated and the internal concentric cylinder with slots is cooled. The constant-temperature water is provided by two water tanks and flows into the circle flume. The temperatures of both cylinders change according to the water temperature. The four thermocouples were embedded into the two extremities of cylindrical copper for detecting temperature. The differences in temperature readings for the two extremities were about 0.1 °C. All the temperatures were monitored continuously using a data acquisition instrument.

3. Experimental processes

3.1. Experimental procedure

a) According to the design of experimental setup in Fig. 2, all experimental equipments’ positions are checked and fixed. The light path is adjusted.
b) The experimental equipments are placed in the laboratory for 12 h so that the temperature of experimental system and environment are virtually the same. The surface of holograph plate in the darkroom faces light and the exposure time is
15 s. The container with liquid is moved up and down for developing, fixation, and bleaching.

c) The holograph plate is washed by 100% alcohol. The two-dimensional film of experimental equipment can be obtained when the holograph plate is dry.

d) The constant temperature water in the tank is filled into the circle volume. The temperature of both outer cylindrical copper and inner cylindrical copper with slots are varied by changing the water temperature. Four thermocouples are embedded into the cylindrical coppers for the measurement of temperature. The interference stripes can be observed when the temperature difference exists.

e) The temperature fields at different conditions are recorded by camera.

3.2. Disposal of temperature field

The actual temperature field can be calculated in terms of holographic interferometry principle. The fringes can be considered to represent isotherms since the index of refraction is a function of temperature [27]. The temperature can be obtained from the following correlation [28]:

\[
T = \frac{1}{T_e + \frac{j m K L M P_o}{10^6}}
\]

where \(T_e\) is the environmental temperature (K), \(\lambda = 0.6328 \times 10^{-6}\) is the wavelength of the light (m), \(m\) is ordinal of stripes, \(R = 8.314\) is the universal gas constant (J/mol K), \(L\) is the longitudinal effective length of the cylindrical copper (m), \(K = 0.2256 \times 10^{-6}\) is the Gladstone–Dale constant (m³/g), \(M = 29\) is molecular mass of air (g/mol), and \(P_o = 1.01325 \times 10^5\) is the atmospheric pressure (Pa).

For the case of \(\varphi = 0^\circ\), the environmental temperature \((T_e = 291.4\ K)\), the temperatures of outer cylindrical copper \((T_o = 294.8\ K)\) and inner cylindrical copper with slots \((T_i = 321.7\ K)\) were measured by thermocouples. The known temperature \(T_i\) of inner cylindrical is used for computing the value \(m (-11.4)\) of high temperature wall by Eq. (1). There exist nine interference stripes in Fig. 5. Because the difference between the value \(m\) of adjacent interference stripes is 1, the value \(m\) of every interference stripe can be obtained. All of the temperatures corresponding to interference stripes can be calculated by Eq. (1), and the values from inside to outside are\(318.8\ K, 315.9\ K, 313.1\ K, 310.3\ K, 307.6\ K, 305.0\ K, 302.4, 299.8\ K,\) and \(297.3\ K,\) respectively.

The Rayleigh number, \(Ra\), and dimensionless temperature, \(\Theta\), are defined as:

\[
Ra = \frac{\beta g D^3 (T_i - T_o)}{\alpha H \nu}, \quad \Theta = \frac{T - T_o}{T_i - T_o}
\]

where \(\beta\), \(\alpha\) and \(\nu\) are thermal expansion coefficient, thermal diffusivity and kinematic viscosity of the fluid, respectively. \(D = r_o - r_i\) is the gap width of annulus, and \(g\) is the gravitational acceleration. For the above case, the value of \(Ra\) is equal to \(5.64 \times 10^4\) and the values of \(\Theta\) from inside to outside are obtained as shown in Fig. 5. The results show that the experimental precision can support the following study.

4. Results and discussion

The experiments were performed for three different inclined angles \(\varphi\) and the parameters are listed in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Average high temperature (°C)</th>
<th>Average low temperature (°C)</th>
<th>(\alpha \times 10^{-6}) (m²/s)</th>
<th>(\nu \times 10^{-6}) (m²/s)</th>
<th>(\varphi) (degree)</th>
<th>Ra</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.83</td>
<td>20.22</td>
<td>22.15</td>
<td>15.53</td>
<td>0</td>
<td>23,600</td>
</tr>
<tr>
<td>48.74</td>
<td>22.53</td>
<td>23.60</td>
<td>16.48</td>
<td>0</td>
<td>54,900</td>
</tr>
<tr>
<td>67.06</td>
<td>23.91</td>
<td>24.3</td>
<td>16.96</td>
<td>45</td>
<td>68,200</td>
</tr>
<tr>
<td>57.44</td>
<td>22.48</td>
<td>22.15</td>
<td>15.53</td>
<td>90</td>
<td>22,300</td>
</tr>
<tr>
<td>30.00</td>
<td>20.88</td>
<td>22.15</td>
<td>17.45</td>
<td>0</td>
<td>29,988</td>
</tr>
<tr>
<td>48.95</td>
<td>21.82</td>
<td>23.60</td>
<td>16.48</td>
<td>90</td>
<td>57,000</td>
</tr>
<tr>
<td>68.30</td>
<td>22.97</td>
<td>25.00</td>
<td>17.45</td>
<td>90</td>
<td>82,000</td>
</tr>
</tbody>
</table>

Fig. 4. Schematic diagram of the test section: 1, cotton insulation; 2, cold water; 3, plastic pipe; 4, outer cylindrical copper; 5, inner cylindrical copper with slots; 6, hot water; 7, air; 8, copper sleeve; 9, tempered sight glass.

Fig. 5. Interference stripes at \(Ra = 5.64 \times 10^4\).
4.1. Static bifurcation

For the case of $\varphi = 90^\circ$, the experiment revealed that two types of steady temperature fields can be obtained for the same Rayleigh number. When the almost initial zero velocity and uniform temperature fields are used for initial conditions, both the first and second types of temperature fields at $\text{Ra} = 2.36 \times 10^4$ are obtained randomly after many experiments (see Figs. 6(a) and 7(a)). The second type of developed temperature field in Fig. 7(a) is easier to obtain than the first type of temperature field in Fig. 6(a). It is worth noting that the initial conditions seem to be the same at every experiment, in which there exist some subtle and inevitable perturbations.

The first type of temperature field for the case of $\text{Ra} = 2.36 \times 10^4$ is shown in Fig. 6(a). Because in this experiment the size of holograph plate was smaller than that of the outer cylindrical envelope, the interference stripes near the right cannot be shown in Fig. 6. When the first type of velocity and temperature field is used as initial fields, the final temperature fields at $\text{Ra} = 5.49 \times 10^4$ and $\text{Ra} = 7.81 \times 10^4$ are steady and kept the same shapes of the first type after a short time, as shown in Fig. 6(b) and (c). The general shapes of interference stripes remained unchanged with increasing Rayleigh number. The interference stripes spacing near the outer cylinder surface is smallest at the top. This indicates that the largest temperature gradient and heat transfer occur at the top of outer cylindrical envelope. The heat transfer rate decreases from top to bottom. The largest thickness and smallest temperature gradient occur at the bottom. When the Rayleigh number is lower, the stripes are wider and the numbers are fewer. When the Rayleigh number increases, the stripes become thinner and the numbers increase. The above phenomenon in the annulus space of a horizontal cylindrical envelope with an internal concentric cylinder with slots of $\varphi = 90^\circ$ is very similar to those of natural convection in the annulus between horizontal concentric cylinders. There is a discrepancy of natural convection heat transfer between the two structures at the center of cylindrical envelope. Flow and heat transfer also occur at the internal domain of the slotted cylinder. The curved interference stripes at slotted place can be observed in Fig. 6. Because of the changed internal cylinder with slotted, heat transfer increased due to the larger heat transfer surface area between the air and the high temperature wall.

If the boundary condition is steady and symmetric, one may expect that the final temperature field should also be symmetric. However, this is sometimes not the case as evidenced from Fig. 7. For the steady and symmetric boundary conditions, the second type of temperature fields is asymmetric. The second type of temperature field at $\text{Ra} = 2.36 \times 10^4$ from almost zero initial velocity and uniform temperature fields is shown in Fig. 7(a). When the second type of asymmetric velocity and temperature field in Fig. 7(a) is used as initial fields, the final temperature fields is asymmetric in Fig. 7(b) and (c) about a vertical axis although the boundary condition is symmetric. It is obvious in this figure that the asymmetric phenomenon exists inside and above the slotted cylinder. The largest temperature gradient and heat transfer occur at the right top of outer cylindrical envelope. The interference stripes spacing on the right side in the annulus space are smaller than the stripe spacing on the left side, so the overall heat transfer rate on the right side is larger than that on the left side. The heat transfer rate above the slotted cylinder is higher than that below the slotted cylinder. This is because when natural convection heat transfer occurred above the slotted cylinder, the bottom wall was heated and the top wall was cooled; it was beneficial to the strength of convection and heat transfer. In the region below the slotted cylinder, the bottom wall was cooled and the top wall was heated, and the strength of convection was restrained and the heat transfer rate was decreased.
The second type of temperature field verified that the asymmetric solutions can be obtained for the steady and symmetric boundary conditions due to the nonlinear characteristics of this system. The experimental results indicated that for the Rayleigh numbers from $2.36 \times 10^4$ to $7.81 \times 10^4$, the multiplicity of solutions are obtained and are related to static bifurcations of this system. There exist two domains for the Rayleigh numbers based on whether the temperature field is independent or dependent of the initial conditions. Because our experiments were limited to relatively high Rayleigh numbers, the data for the case of $Ra < 10^4$ was not obtained due to difficulties in forming distinct interference stripes, and the critical Rayleigh number related to the static bifurcation point was not detected by experimental method.

Numerical solutions of this problem were carried out to further investigate the bifurcation phenomenon. The governing equations were discretized by the finite volume method, and the central difference scheme was adopted to discretize the diffusion terms for the numerical viscosity. The SIMPLE algorithm with Quick scheme was used for handling the pressure velocity coupling. The time step is selected as 0.01 and 100 $\times$ 50 $\times$ 50 grid points are adequate to yield accurate results for the present computations. Because the numerical results are only used to be compared with experimental results, the numerical method in more detail is not depicted and is available elsewhere [29].

When the Rayleigh number was lower than $1.6 \times 10^3$, the natural convection was inconspicuous and the heat transfer rate was low; the symmetric solutions can be reached from arbitrary initial fields and it was independent of the initial conditions. When the Rayleigh number was greater than $1.9 \times 10^3$, the steady state solution was not unique and depends on the initial conditions. The critical Rayleigh number related to static bifurcation was between $1.6 \times 10^3$ and $1.9 \times 10^3$. For a Rayleigh number range of $2.36 \times 10^4$ to $7.81 \times 10^4$, the temperature field depends on the initial conditions. Almost zero initial velocity and uniform temperature was used as initial conditions, and the two types of temperature fields at $Ra = 2.36 \times 10^4$ were obtained randomly after many numerical calculations. There also existed some small and inevitable perturbations in the initial conditions at every numerical calculation. Those perturbations induced by rounding error play an important role in affecting the final solutions. When the two different types of velocity and temperature fields at $Ra = 2.36 \times 10^4$ are respectively used as initial fields, the velocity vector fields for $Ra = 5.49 \times 10^4$ at the center of the outer cylinder are obtained by numerical method, as is seen in Fig. 8. There are two large vortices in the annulus between the inner and outer cylinders. The differences of the two types of velocity vector fields occur obviously above and in the internal cylinder with slots. For the second type of velocity vector field, the left vortex is larger than the right vortex in the outer cylindrical envelope. The temperature field is related to the flow pattern. The first and second of temperature fields obtained from numerical solution corresponding to the experimental solutions are shown in Fig. 8. It can be seen that the numerical results are in good agreement with the experimental results and the static

![Fig. 8. Two types of numerical results at Ra = 5.49 × 10^4 for φ = 90° (a) first type of temperature field, (b) second type of temperature field.](image-url)
bifurcation of this system can be verified by both numerical and experimental methods. The reason for the first and second temperature fields is the static bifurcation phenomenon of this system, and the initial conditions play a determining role in problems involving instability. The final bifurcation results are regarded to be sensitive to the initial conditions, and the initial conditions are expected to determine which of the branches the solutions will finally be led to. When the different types of temperature and velocity vector fields (including the zero initial fields with different perturbations) are used for the initial fields, the different types of temperature and velocity vector field under the same conditions can be obtained for a narrow band of parameters.

4.2. Self-sustained oscillation

For the case of $\varphi = 0^\circ$, the evolution of the temperature field from the zero initial velocity and uniform temperature fields was observed at $Ra = 6.46 \times 10^4$. The number of interference stripes increased gradually from the initial time until to reach a fixed value. When the number remained the fixed value, the developed temperature field on the surface seemed to reach steady state. However, the interference stripes started to oscillate with small amplitudes soon afterward. The amplitudes increased to a certain scope and the oscillation of temperature field become more and more obvious with time.

When time was sufficiently long, the natural convection remained oscillatory. The different temperature fields in Fig. 9 are obtained at different times for $Ra = 6.46 \times 10^4$, although the boundary conditions are symmetric and stable. The oscillatory phenomenon occurs obviously at the top of the outer cylindrical envelope, and the amplitude of the temperature decreases gradually from top to bottom. Considering the outer cylindrical envelope, the isotherms are closely packed at the top. The local heat transfer in this region is highest while that at the bottom is lowest. The
unstable temperature fields can be obtained for a Rayleigh number range between $2.4 \times 10^4$ and $8.93 \times 10^4$. When the Rayleigh number was lower, the numbers of stripes were fewer and amplitudes were smaller. When the Rayleigh number increased, the numbers were more and amplitudes increased. Compared with that for the case $\theta = 90^\circ$, the temperature fields were unsteady and more complex due to the different slot location. When the slot was at the top of inner cylinder, it could lead to a different non-linear behavior.

The unsteady temperature fields can also be obtained in steady conditions. Numerical temperature fields at different times for $\theta = 0^\circ$, $Ra = 6.46 \times 10^4$ are shown in Fig. 10. The most obvious oscillation occurs also above the inner cylinder. These behaviors are in good agreement with these observed in our experiment. The
temperature fields are similar to the experimental results. However, the perturbations in initial conditions are different and unpredictable at every experiment or every numerical solution. The problem with nonlinear characteristic is very sensitive to some conditions, so it is very difficult to predict oscillation phenomenon of temperature field exactly.

For the case of $\varphi = 45^\circ$, the temperature fields for $Ra = 6.82 \times 10^4$ at different times are shown in Fig. 11. The largest heat transfer still occurs above the inner cylinder. The asymmetric phenomenon of temperature field is more obvious than the above due to the asymmetric geometry. The temperature field is unsteady under the steady boundary condition. However, the unsteady numerical results have not been obtained by numerical method, even many different initial conditions were used. There exists large discrepancy between the numerical predictions and experimental results. The authors believe that the reason is the multiplicity of solutions related to static bifurcation phenomenon. The unsteady temperature field in experiment is one of these results and the steady numerical result is another one. Because the final bifurcation results are sensitive to the initial fields, the numerical results that can be in good agreement with experimental results depend on the appropriative initial fields. However, the appropriate initial fields have not been identified and the subtle perturbations in the initial fields cannot be controlled.

5. Conclusions

Experimental investigations of temperature fields in a cylindrical envelope with an internal concentric cylinder with slots are carried out. Nonlinear characteristics of natural convection for different types of geometries with different slot locations are discussed and analyzed. For the case of $\varphi = 90^\circ$, two different types of temperature fields were obtained at the same Rayleigh number of $5.49 \times 10^4$. The numerical results for cases with Rayleigh numbers
ranged from $2.36 \times 10^4$ to $7.81 \times 10^4$ were in good agreement with experimental results. It is demonstrated by experiments and numerical solutions that at high Rayleigh numbers, the steady state convection was not unique and depends on the initial conditions. For two cases of $\phi = 0^\circ$ and $\phi = 45^\circ$, the natural convection turned to be unsteady after a short time, although the boundary conditions were steady. The most obvious oscillation regions were above the inner cylinder, where the amplitude of temperature field increased with increasing Rayleigh number. When time was sufficiently long, the natural convection remained oscillatory. The nonlinear characteristics of these problems lead to the multiplicity of convection and self-sustained oscillation.

Acknowledgment

The financial supports from the Chinese National Science Foundation under Grants No. 51276118, 51206072 and 51129602 are gratefully acknowledged.

References