INVERSE ESTIMATION OF THE FRONT SURFACE TEMPERATURE OF A 3-D FINITE SLAB BASED ON THE BACK SURFACE TEMPERATURE MEASURED AT COARSE GRIDS

Yunpeng Ren, Yuwen Zhang, J. K. Chen, and Z. C. Feng
Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, Missouri, USA

The accuracy of the solution of the inverse heat conduction problem (IHCP) can be improved by using fine grids, but measured temperature data on fine grids are usually not available due to limits imposed by the size and spacing of the temperature sensors. A new method is proposed to recover the front surface temperature of a finite slab using fine grids based on the back surface temperature measured with coarse grids. Effects of heat conduction in the sensor substrate and contact thermal resistance are also taken into account. The proposed method is applied to solve a three-dimensional IHCP with stainless steel and aluminum target slabs. The results show that the front surface temperature can be recovered with satisfactory accuracy. Both the distortion of the temperature distribution and the discrepancy from the exact solutions are reduced by using the new method.

1. INTRODUCTION

High-energy laser (HEL) weapons have been progressively evolving since the 1960s. They can deliver a large amount of stored energy to military targets at the speed of light. During laser irradiation, it is critical to have information about the transient temperature on the target surface to understand the resulting thermomechanical response accurately. However, the heated surface is either too hot or inaccessible for attaching sensors. Similar problems occur during re-entry of space vehicle into the atmosphere and in laser manufacturing processes [1]. In these circumstances, the front...
surface temperature can be obtained by solving an inverse heat conduction problem (IHCP) [2–4] based on the measured transient temperature and/or heat flux on the back surface, where it is accessible to attach sensors.

The inverse heat conduction problem has been widely used to estimate unknown boundary conditions and/or thermophysical properties [5–9]. Taler and Zima [10] used a control-volume method to calculate surface heat flux and temperature from temperature measurements at different spatial nodes in one- and two-dimensional inverse heat conduction problems. Prud’homme and Nguyen [11] analyzed the convergence and regularization properties of the conjugate gradient method (CGM) for estimation of a time-dependent boundary heat flux and proved that the CGM had a sequential filtering mechanism. Huang and Tsai [12] solved a transient inverse two-dimensional problem to predict time-dependent irregular boundary configurations with the CGM. Lin et al. [13] used the conjugate gradient approach with an adjoint problem to estimate the heat flux in unsteady laminar flow.

**NOMENCLATURE**

- $c_p$: specific heat, $J/kg \cdot K$
- $d^i(y, z, t)$: dimensionless direction of descent at iteration $k$
- $f$: frequency of periodic laser
- $i_m$: total number of sensors
- $k$: thermal conductivity, $W/m \cdot K$
- $L$: object thickness in the $x$ direction, $m$
- $M$: object length in the $y$ direction, $m$
- $N$: object width in the $z$ direction, $m$
- $q$: heat flux, $W/m^2$
- $q_{max}$: maximum heat flux at Gaussian laser beam center, $W/m^2$
- $q_{f}(y, z, t)$: observed heat flux on front surface, $W/m^2$
- $q[L, y, z, t; q_{f}]$: observed heat flux on back surface, $W/m^2$
- $S$: objective function
- $\nabla S[q_{f}]$: gradient direction of objective functional at iteration $k$
- $\Delta S[q_{f}]$: objective function variation
- $t$: time, $s$
- $t_f$: final time, $s$
- $\Delta t$: time step, $s$
- $T$: temperature, $K$
- $T_0$: initial temperature, $K$
- $T_{f}(y, z, t)$: front surface temperature, $K$
- $w$: $1/e$ radius of Gaussian laser beam, $m$
- $x, y, z$: spatial coordinate, $m$
- $Y_{TL}(y, z, t)$: measured temperature on back surface, $K$
- $\alpha$: surface absorptivity
- $\beta^i$: dimensionless search step size at iteration level $k$
- $\gamma^k$: dimensionless conjugate coefficient at iteration level $k$
- $\delta$: Dirac delta function
- $\lambda(x, y, z, t)$: dimensionless Lagrange multiplier
- $\phi$: standard deviation of heat flux ($W/m^2$) or temperature ($K$) measurements
- $\chi$: dimensionless tolerance used to stop the CGM iteration procedure
- $\omega$: dimensionless random variable having a normal distribution with zero mean and unitary standard deviation
- $\nu$: dimensionless perturbed variable

**Subscripts**

- $0$: initial
- $f$: final

**Superscripts**

- $k$: iteration level
forced convection. Zhou et al. [14, 15] employed the CGM to estimate the front surface heating conditions based on the measured temperature and heat flux with temperature-dependent thermophysical properties for both 1-D and 3-D problems.

In the mathematical formulation of the IHCP, either temperature or heat flux should be measured to provide information to solve the ill-posed problem. Generally, temperature on the back surface is preferred, since it can be measured with less uncertainty than the heat flux [16–20]. However, in actual applications, the area density of the sensors is often limited by the size and spacing of the sensors, which places a limit on the detailed information for recovering the front surface temperature.

In this article, the CGM is incorporated into a three-dimensional model to estimate the front surface temperature by using the measured temperature on the back surface. All the thermophysical properties are treated as temperature-dependent. A new method is proposed to overcome the limit of the number of sensors by interpolating the measured temperature of the back surface to provide the data needed to solve the IHCP at fine grids. Effects of heat conduction in the sensor substrate and the contact thermal resistance between the sensor substrate and finite slab are also considered.

2. PROBLEM DESCRIPTION

Consider a 3-D object with dimensions of \( L \times M \times N \) as shown in Figure 1. Initially, the finite slab has a uniform temperature \( T_0 \) and then is exposed to a high-intensity, Gaussian laser beam \( q_1(y, z, t) \) on the front surface \( (x = 0) \) from \( t = 0 \). The purpose of the inverse algorithm is to recover the temperature distribution \( T_1(y, z, t) \) on the front surface based on the temperature measurement data \( Y_{TL}(y_i, z_i, t) \) on the back surface \( (x = L) \). In this study, the front surface temperature \( T_1(y, z, t) \) is estimated in an indirect way, i.e., the net heat flux \( q_1(y, z, t) \) on the front surface is

Figure 1. Physical model.
recovered first using the inverse algorithm and then the temperature $T_1(y, z, t)$ on the front surface is obtained as a by-product by employing Fourier’s law. The measured temperatures on the back surface are employed in the objective function.

2.1. Direct Problem

The direct problem can be stated as follows:

\[
\rho c_p(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[ k(T) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ k(T) \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[ k(T) \frac{\partial T}{\partial z} \right]
\]

for $0 < x < L, 0 < y < M, 0 < z < N, t > 0$

\[T = T_0 \quad \text{for } 0 \leq x \leq L, 0 \leq y \leq M, 0 \leq z \leq N, t = 0,\]

\[-k(T) \frac{\partial T}{\partial x} = q_1(y, z, t) \quad \text{for } x = 0, t > 0\]

\[-k(T) \frac{\partial T}{\partial x} = 0 \quad \text{for } x = L, t > 0\]

\[-k(T) \frac{\partial T}{\partial y} = 0 \quad \text{for } y = 0, M, t > 0\]

\[-k(T) \frac{\partial T}{\partial z} = 0 \quad \text{for } z = 0, N, t > 0\]

In the above direct problem, the heat flux on the front surface $q_1(y, z, t)$, is considered to be known and the back surface is considered to be adiabatic. The objective of the direct problem here is to determine the transient temperature in the target.

2.2. Inverse Problem

For the inverse problem, the heat flux at $x = 0$ is unknown and needs to be recovered, but everything else in the direct problem is known. The extra known information for recovering the heat flux on front surface is the temperature measurements on the back surface ($x = L$), $Y_{TL}(y, z, t)$. The inverse problem can thus be stated as follows: The temperature measurements on the back surface, $Y_{TL}(y, z, t)$, are utilized to recover the front surface heat flux. The front surface temperature is computed according to the temperature–heat flux relation defined by Fourier’s law.

The solution of the inverse problem is obtained by minimizing the following objective function:

\[
S(q_1) = \sum_{i=1}^{m} \int_{0}^{t_f} \left( Y_{TL}(y_i, z_i, t) - T[L, y_i, z_i, t; q_1] \right)^2 dt
\]
is the number of temperature sensors (here \(i_m = 20 \times 20 = 400\), which means that 400 sensors are used in all) on the back surface.

### 2.3. Conjugated Gradient Method for Minimization

The iterative process based on the CGM \([3, 4]\) is now derived for estimation of the unknown heat flux \(q_1(y, z, t)\) by minimizing the objective function \(S\) given by Eq. (7). The front surface heat flux \(q_1(y, z, t)\) at iteration \(k + 1\) is advanced by

\[
q_1^{k+1}(y, z, t) = q_1^k(y, z, t) - \beta_k^k d_k^k(y, z, t)
\]

where \(\beta_k^k\) is the search step size from iteration \(k\) to \(k + 1\), which will be addressed in the next section, and \(d_k^k(y, z, t)\) is the direction of descent (i.e., search direction), given by

\[
d_k^k(y, z, t) = \nabla S[q_k^k] + \gamma^k d_k^{k-1}(y, z, t)
\]

which is a conjugation of the gradient direction \(\nabla S[q_k^k]\) at iteration \(k\) and the direction of descent \(d_k^{k-1}(y, z, t)\) at iteration \(k - 1\). The conjugate coefficient \(\gamma^k\) is determined by

\[
\gamma^k = \frac{\sum_{i=1}^{i_m} \int_0^T \nabla S[q_k^k]\{\nabla S[q_k^k] - \nabla S[q_k^{k-1}]\}dt}{\sum_{i=1}^{i_m} \int_0^T \{\nabla S[q_k^{k-1}]\}^2dt}
\]

with \(\gamma^0 = 0\). To perform the iterations according to Eq. (8), the step size \(\beta_k^k\) and the gradient of the objective functional \(\nabla S[q_k^k]\) need to be determined. To do so, a sensitivity problem and an adjoint problem are constructed in the following.

### 2.4. Sensitivity Problem and Search Step Size

The limiting approach is employed for the governing equations, boundary, and initial conditions of the direct problem \([4]\). After some manipulations, the sensitivity problem can be stated as follows:

\[
\frac{\partial (pc_p \Delta T)}{\partial t} = \frac{\partial^2 (k \Delta T)}{\partial x^2} + \frac{\partial^2 (k \Delta T)}{\partial y^2} + \frac{\partial^2 (k \Delta T)}{\partial z^2}
\]

for \(0 < x < L, 0 < y < M, 0 < z < N, t > 0\)

\[\Delta T(x, y, z, 0) = 0 \quad \text{for } 0 \leq x \leq L, 0 \leq y \leq M, 0 \leq z \leq N, t = 0\]

\[-\frac{\partial (k \Delta T)}{\partial x} = \Delta q_1(y, z, t) \quad \text{for } x = 0, t > 0\]
\[
\frac{-\partial(k \Delta T)}{\partial x} = 0 \quad \text{for } x = L, t > 0 \tag{14}
\]
\[
\frac{-\partial(k \Delta T)}{\partial y} = 0 \quad \text{for } y = 0, M, t > 0 \tag{15}
\]
\[
\frac{-\partial(k \Delta T)}{\partial z} = 0 \quad \text{for } z = 0, N, t > 0 \tag{16}
\]

The above equations are used to determine the temperature variation \(\Delta T(x, y, z, t)\) caused by the perturbation \(\Delta q_1(x, y, z, t)\).

The objective function at iteration \(k + 1\) can be obtained by replacing \(q_1\) in Eq. (7) with \(q_1^{k+1}\) given by Eq. (8), i.e.,
\[
S[q_1^{k+1}] = \sum_{i=1}^{I_0} \int_0^{T_f} \left\{ Y_{TL}(y_i, z_i, t) - T[L, y_i, z_i, t; (q_1^k - \beta^k d^k)] \right\}^2 dt \tag{17}
\]

Linearizing \(T[L, y_i, z_i, t; (q_1^k - \beta^k d^k)]\) by the Taylor series expansion, Eq. (17) becomes
\[
S[q_1^{k+1}] = \sum_{i=1}^{I_0} \int_0^{T_f} \left\{ Y_{TL}(y_i, z_i, t) - T[L, y_i, z_i, t; q_1^k] - \beta^k \Delta q(d^k) \right\}^2 dt \tag{18}
\]

In Eq. (18), the temperature is obtained from the direct problem [Eqs. (1)–(6)] with the estimated \(q_1^k\) at \(x = 0\). The sensitivity function \(\Delta q(d^k)\) is the heat flux variation at \(x = L\) and time \(t\), which is calculated using Fourier’s law based on temperature variation \(\Delta T\) obtained by Eqs. (11)–(16) by letting \(\Delta q_1(y, z, t) = d^k(y, z, t)\). The calculation of heat flux variation \(\Delta q(d^k)\) will be described in detail later. The search step size \(\beta^k\) can be determined by minimizing the objective function, Eq. (18), with respect to \(\beta^k\):
\[
\beta^k = \frac{\int_{i=1}^{I_0} \int_0^{T_f} \{ T[L, y_i, z_i, t; q_1] - Y_{TL}(y_i, z_i, t) \} \cdot \Delta T[L, y_i, z_i, t; d^k] \cdot dt}{\sum_{i=0}^{I_0} \int_0^{T_f} \{ \Delta T[L, y_i, z_i, t; d^k] \}^2 \cdot dt} \tag{19}
\]

### 2.5. Adjoint Problem and Gradient Equation

The adjoint problem is also obtained by using a limiting approach:

\[
\rho c_p \frac{\partial \lambda}{\partial t} + k \frac{\partial^2 \lambda}{\partial x^2} + k \frac{\partial^2 \lambda}{\partial y^2} + k \frac{\partial^2 \lambda}{\partial z^2} + \sum_{i=1}^{I_0} 2 \{ T[x, y_i, z_i; q_1] - Y_{TL}(y_i, z_i, t) \} \delta(x - L) \delta(y - y_i) \delta(z - z_i) = 0 \tag{20}
\]

for \(0 < x < L, 0 < y < M, 0 < z < N, t > 0\)
\[ \lambda(x, y, z, t_f) = 0 \quad \text{for} \ 0 \leq x \leq L, 0 \leq y \leq M, 0 \leq z \leq N, \ t = t_f \]  

\[ -k \frac{\partial \lambda}{\partial x} = 0 \quad \text{for} \ x = 0, L, \ t > 0 \]  

\[ -k \frac{\partial \lambda}{\partial y} = 0 \quad \text{for} \ y = 0, M, \ t > 0 \]  

\[ -k \frac{\partial \lambda}{\partial z} = 0 \quad \text{for} \ z = 0, N, \ t > 0 \]  

where \( \lambda(x, y, z, t) \) is the Lagrange multiplier and \( \delta(\cdot) \) is the Dirac delta function.

The gradient of the least-square norm is

\[ \nabla S[\mathbf{q}_1(y, z, t)] = \lambda(0, y, z, t) \]  

### 2.6. Stopping Criterion

The discrepancy principle is used as the stopping criterion [3, 4]:

\[ S[\mathbf{q}_1^k] < \chi \]  

where \( \chi \) denotes the tolerance and \( k \) is the total iteration number required to satisfy Eq. (26).

It is assumed that the absolute value of temperature residuals can be approximated by

\[ |Y_{TL}(y_i, z_i, t) - T[L, y_i, z_i, t; \mathbf{q}_1]| \approx \phi \]  

where \( \phi \) is the standard deviation of the measurement. Substituting Eq. (27) into Eq. (7), the tolerance of the stopping criterion is obtained:

\[ \chi = i_m \phi^2 t_f \]  

### 3. MESHING AND INTERPOLATIONS

The 3-D problem defined above is solved numerically. The meshes in the \( y \) and \( z \) directions are uniform (see Figure 2). In order to consider heat conduction in the sensor substrate, which has different thermophysical properties, a modified uniform grid system (see Figure 3) is employed. In Figure 3a, the grids from 1 to \( L_3 \) represent the target material and the grid points \( L_2 \) and \( L_1 \) represent the sensor substrate. In order to use a uniform grid system to solve heat conduction in both the target and the sensor substrate, a modified grid system in the \( x \) direction as shown as Figure 3b is used. It can be seen that the size of the last grid is increased from \( \Delta x_1 \) to \( \Delta x_2 \), which is the size of the grids before \( L_2 \). In doing this, the properties such as thermal
conductivity and specific heat should be modified using the following method:

\[
\hat{k} = \frac{\Delta x_2}{\Delta x_1} k \quad \hat{c}_p = \frac{\Delta x_1}{\Delta x_2} c_p
\]  

(29)

where \(k\) and \(c_p\) are the real thermal conductivity and the specific heat of the sensors, which are made of Kovar. The density of the last grid is not modified and remains the same as that of the sensor substrate. After this, a uniform grid system containing both target material and sensor substrate is constructed.

The accuracy of the IHCP solution can be improved by using fine grids, but measured temperature data on fine grid are usually not available due to limits on the size and spacing of the temperature sensors. A new method is proposed here to recover the front surface temperature of a finite slab using fine grids based on the measured back surface temperature in coarse grids. The back surface temperature

![Figure 2. Illustration of meshing on the back surface.]

Figure 3. Modified grid system along \(x\) direction (color figure available online).
is measured by a square array of $20 \times 20$. The sensor array is centered on the back surface, and the sensors are evenly spaced, as shown in Figure 2, where a uniform mesh is shown based on the arrangement of the sensors. For simplicity of illustration, only a $10 \times 10$ sensor array is plotted in Figure 2. The solid lines represent the faces of control volumes. The black solid circular dots represent the locations of the temperature sensors, which are located exactly at the centers of the control volumes.

In this study, a new method, as shown in Figure 4, is proposed to overcome the limit on the number of data points per unit area. The black points represent sensor locations while the open circles represent the grid points used in the IHCP analysis. The temperatures at the open diamond points are obtained by averaging the two closest black points and the star points are obtained by averaging the four surrounding black points. Finally, the temperatures at open circles, which will be used in the simulations, are obtained by averaging four surrounding points: two diamond points, one black point, and one star point. However, the values at the open circles on the perimeter are set to the value at the black point in the same grid, since the temperature gradient is zero at the adiabatic boundaries. By doing this, a doubled array with size of $40 \times 40$ is obtained to recover the front surface temperature.

4. RESULTS AND DISCUSSION

4.1. Generation of Simulated Measurement Data

Instead of conducting actual experiments, measurement data for temperature are generated numerically by solving the direct problem described by the governing Eq. (1) with initial conditions and boundary conditions given by Eqs. (2)–(6). The heat flux on the front surface in the direct problem for generating measured data
The back surface is dynamic with the following form:

\[
q_1(y, z, t) = q_{\text{max}} \cdot \alpha \cdot \exp\left\{-\frac{(y - 0.5 M)^2 + (z - 0.5 N)^2}{w^2}\right\} \cdot \left[1 + \sin(2\pi ft)\right]
\]

where \( \alpha \) is surface absorptivity, \( q_{\text{max}} \) is the maximum heat flux at the center of the heating flux spot, \( w \) is the \( \frac{1}{e} \) radius of the Gaussian laser beam, and \( f \) is frequency. Their values are set to be \( \alpha = 0.05 \), \( q_{\text{max}} = 5,000 \, \text{W/cm}^2 \), \( w = 10.0 \, \text{mm} \), and \( f = 2.0 \, \text{Hz} \).

The dimensions of the 3-D object considered are \( L \times M \times N = 2.15 \, \text{mm} \times 100 \, \text{mm} \times 100 \, \text{mm} \). The density of the 3-D object is considered to be constant and uniform. However, the thermal conductivity and specific heat are temperature-dependent. Two different target materials, i.e., stainless steel 304 (SS 304) and aluminum alloy 2024-T6 (Al 2024-T6), were used in the simulations. For the SS 304, its density is a constant, \( \rho = 7,900 \, \text{kg/m}^3 \), and thermophysical properties are expressed as (see Figure 5)

\[
k(T) = 10.06453 + 0.01719 \cdot T - 1.85055 \times 10^{-6} T^2
\]

\[
c_p(T) = 447.1588 + 0.18516 \cdot T - 1.95486 \times 10^{-5} \cdot T^2
\]

For the Al 2024-T6, its density is \( \rho = 2,770 \, \text{kg/m}^3 \) and the thermal properties are (see Figure 6)

\[
k(T) = 171.8571 + 0.02571 \cdot T
\]

\[
c_p(T) = 704.3571 + 0.5607 \cdot T
\]

Figure 5. Thermal physical properties of SS 304 (color figure available online).
The substrate of the sensors is made of Kovar with density 8,360 kg/m³, thermal conductivity 17.3 W/m K, and specific heat $c_p = 368.73 + 0.443 \cdot T$, as shown in Figure 7.

In the inverse heat transfer analysis, the simulated back surface temperature is employed in the objective function as shown in Eq. (7). The recovered front surface temperatures will be compared with those calculated from the above direct problem to examine the accuracy of the present inverse heat conduction algorithm.

Figure 6. Thermal physical properties of Al 2024-T6 (color figure available online).

Figure 7. Thermal physical properties of sensor substrate (color figure available online).
Figure 8. Comparison of front surface temperature contours for SS 304: exact (left), and IHCP results for 20 \times 20 mesh (middle) and 40 \times 40 mesh (right) (color figure available online).
The simulated measurement data computed from Eqs. (1)–(6) provide the exact (errorless) measurements. To account for the measurement error in the back temperature, we let

\[ Y_{TL}(y, z, t) = Y_{TLE\text{exact}}(y, z, t) + \omega \phi \]  

(35)

where \( Y_{TLE\text{exact}}(y, z, t) \) is the data simulated from the direct problem, \( \omega \) is the standard normal random variable, and \( \phi \) represents the standard deviation of the measurement noise and is set to 2 K in our simulation.

4.2. Results of IHCP

The finite-difference method is used to solve the above direct problem, sensitivity problem, and adjoint problem. The final time is chosen as \( t_f = 2.0 \) s, and the time step is \( \Delta t = 0.05 \) s.

Figure 8 shows the exact and recovered solutions of temperature on the front surface of the SS 304 obtained by using original measured data on a 20 \( \times \) 20 mesh and interpolated back surface data with 40 \( \times \) 40 mesh. The left column is the exact solution of temperatures on the front surface, the middle column is obtained by employing original measured data (20 \( \times \) 20) on the back surface, and the right column is the recovered temperature on the front surface by interpolating the back surface data with a 40 \( \times \) 40 mesh. It can be seen that the contours around the spot edge appear more “ragged” for the 20 \( \times \) 20 array. By doubling the grid number in the \( y \) and \( z \) directions, the front surface temperature can be recovered with higher overall accuracy. The distortion between exact solutions and recovered solutions arising from measurement errors is also reduced. The root mean square (RMS) defined as follows is shown in Figure 9.

![Figure 9](http://example.com/figure9.png)

Figure 9. RMS for SS 304 at different times (color figure available online).
Figure 10. Comparison of front surface temperature contours for Al 2024-T6: exact (left), and IHCP results for 20 × 20 mesh (middle) and 40 × 40 mesh (right) (color figure available online).
\[ \text{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (A_i - B_i)^2} \]  

where \( A \) and \( B \) are the recovered and exact temperatures to be compared, and \( n \) is the total number of sample points at one time instant.

The RMS errors can demonstrate the overall quality of the inverse algorithm since they are calculated over the entire front surface. It can be observed from Figure 9 that at the beginning and the end, the RMS shows higher values than those in the middle range of time, which is the limit of the CGM itself. By employing interpolated measured data, the RMS is reduced from about 9 K to about 7 K.

Figure 10 shows the results for Al 2024-T6 obtained by using original measured data on a 20 × 20 mesh and interpolated back surface temperatures with a 40 × 40 mesh. Compared to the results for SS 304, due to the higher thermal conductivity and specific heat, the Al 2024-T6 target is heated to a lower temperature and the sensors on the back surface can obtain enough information to recover front surface temperatures over a longer duration. This is confirmed by the results that from \( t = 0.2 \text{ s} \) to \( t = 1.8 \text{ s} \). The front surface temperature can be well recovered in both magnitudes and shapes. The RMS shown in Figure 11 has a lower value of around 2.3 K. Similarly, both discrepancy and distortion between the exact solutions and recovered solutions are reduced by using the interpolated measured data, which is shown in the last column of Figure 11. The RMS is also reduced from about 2.3 K to about 1.5 K.

5. CONCLUSIONS

The conjugated gradient method is employed to recover front surface temperature of a 3-D object with temperature-dependent thermophysical properties.
Two different materials, i.e., stainless steel 304 and aluminum alloy 2024-T6, are used as target materials. A new method by doubling the grid numbers in the $y$ and $z$ directions and interpolating the measured temperature data on the back surface is proposed to overcome the limit on the number of sensors. The results show that the front surface temperature can be recovered with higher accuracy and less distortion compared with that recovered by original measured data on coarse grids.

REFERENCES