SELF-SUSTAINED OSCILLATIONS AND BIFURCATIONS OF MIXED CONVECTION IN A MULTIPLE VENTILATED ENCLOSURE

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A numerical study is made of the self-sustained oscillations and bifurcations of mixed convection in a two-dimensional multiple ventilated enclosure. These two mechanisms of buoyancy [Richardson number (Ri) is a measure] and forcing by the inlet flow [Reynolds number (Re) is a measure] lead to complex interaction. The results are obtained for a range of the Richardson number from 0 to 20 at Pr = 0.701, and the Reynolds number is given in a range of 1000–2500. The results show that depending on the values of Re and Ri, the flow inside the enclosure may be steady, periodic, nonperiodic, or turbulent; and the velocity and temperature fields may show an asymmetric structure for certain combinations of the control parameters, even though the boundary conditions are steady and symmetric. Further, certain features of nonlinear dynamical systems such as bifurcation, self-sustained oscillations are also seen. The simulation results also show that when the inlet flow angle \( \theta \) equals 70° (proved to be the most unstable among four given values of \( \theta \)) the enclosure loses its stability at Re = 1500, Ri = 0, and Re = 1000, Ri = 0.5; transforms into periodic oscillatory flow via the steady symmetry breaking Hopf bifurcation; and becomes nonperiodic-unstable at Re = 2000

KEY WORDS: mixed convection, enclosure, oscillation, bifurcation

1. INTRODUCTION

The self-sustained oscillations and bifurcations of mixed convection in a multiple ventilated enclosure is the focus of the present work. Flow geometry is illustrated in Fig. 1. In the figure, an external airstream enters into the enclosure from the inlets and exits from the outlets. In an enclosure, these two mechanisms of the external forced stream, and the buoyancy-driven flow generated because of the difference in temperatures between the wall and the incoming flow, lead to the possibility of complex flows. Because of the interaction between buoyancy [Richardson number (Ri) is a measure] and forcing by the inlet flow [Reynolds number (Re) is a measure], the flows are substantially unstable and may exhibit self-sustained oscillations and bifurcation phenomena. Thus, the flows will easily undergo transition from laminar flow to chaos and will show very complex nonlinear characteristics, even at small Reynolds and Richardson numbers, and the transition into different flow depends on the magnitudes of the Reynolds number and the Richardson number. Therefore, it is important to understand the oscillations and bifurcations of mixed convection in a multiple ventilated enclosure.

Various researchers investigated the flow instability of mixed convection in an enclosure using experimental and numerical methods. Paolucci and Chenoweth (1989) numerically studied the transition from laminar to chaotic flow in a differentially heated vertical cavity. Angirasa (2000) presented a numerical study of mixed convection flow in an enclosure with an isothermal vertical wall. Forced flow conditions were imposed by providing an inlet at the bottom of the isothermal surface and a vent at the top, facing the inlet. Both positive and negative
temperature potentials were considered. In their study, at higher absolute values of the Grashof number (Gr), the interaction became quite complex and the solution was unstable. In general, heat transfer increases with increasing forced flow for either direction of the buoyancy. Wang and Jaluria (2002) numerically investigated the instability and heat transfer of three-dimensional mixed convection flow in a horizontal rectangular duct, with multiple strip heat sources flush-mounted on the bottom surface at low Reynolds numbers. According to their study, four different flow patterns might exist, and the effect of the numerical results on the cooling of electronic equipment was also discussed. The resonant instabilities and oscillations in twodimensional compressible flow past an open cavity was numerically investigated by Rowley et al. (2002). The compressible Navier-Stokes equations were solved directly (no turbulence model) for cavities with laminar boundary layers upstream.

More recently, Mataoui et al. (2003) carried out both numerical and experimental studies on the symmetrical interaction between a turbulent plane jet and a rectangular cavity, and the influence of the geometrical characteristics of the cavity on the self-sustained oscillatory motion. The size and aspect ratio of the cavity were varied and together with the jet width, were compared to that of the cavity. Leong et al. (2005) reported the heat-transfer results for mixed convection from a bottom-heated open cavity subjected to an external flow in their numerical study for a wide range of governing parameters (1 ≤ Re ≤ 2000, 0 ≤ Gr ≤ 106) over cavities with various aspect ratios (A

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= 0.5, 1, 2, and 4). They found that the Reynolds number and Grashof number controlled the flow pattern and the occurrence of recirculating cells, while the aspect ratio had a significant influence on the orientation of these cells. Deshpande and Srinidhi (2005) reported the results regarding mixed thermal convection stability in a rectangular cavity.

Deng et al. (2004) investigated the characteristics of the airflow and heat/contaminant transport structures in the indoor environment by means of a convection transport visualization technique. They performed a numerical study of laminar double-diffusive mixed convection in a two-dimensional displacement ventilated enclosure with discrete heat and contaminant sources. In their study, at higher absolute values of the Grashof number the solution was oscillatory. Later, Beya and Lili (2007) extended the work of Deng et al. (2004) by carrying out a numerical investigation to analyze the unsteady double-diffusive mixed convection in a two-dimensional ventilated room due to heat and contaminant sources.

Most recently, Rahman et al. (2007) numerically investigated the mixed convection on a vented enclosure by the finite element method. An external fluid flow entered the enclosure through an opening in the left vertical wall and exited from another fixed opening in the right vertical wall. Various inlet port configurations were extensively studied with the change of governing parameters. The objective of their study was to analyze the effects of Prandtl (Pr), Richardson, and Reynolds numbers on the heat-transfer characteristics of laminar mixed convection. A related numerical study of mixed convection had been performed by Ghasemi and Aminossadati (2008) They numerically investigated the cooling performance of electronic devices, with an emphasis on the effects of the arrangement and number of electronic components. Their analysis used a twodimensional rectangular enclosure under combined natural and forced convection flow conditions and considered a range of Rayleigh numbers. They found that increasing the Rayleigh number could significantly improve the enclosure heat-transfer process, and they also observed the phenomenon of the value of the average Nusselt number fluctuating in a sinusoidal manner.

Search of the literature has shown that there has been little work related to nonlinear characteristics such as self-sustained oscillations, bifurcations, and chaos in ventilated cavities. A systematic study on the oscillations and bifurcations of mixed-convection flow and the heat transfer is needed. The objective of present work is to analyze the effects of the Richardson number, Reynolds number, and inlet flow angle on the oscillations and bifurcation characteristics of laminar mixed convection. The primary focus of this paper is the influence of the inlet flow angle on the heat-transfer stability for a range of Richardson and Reynolds numbers.

2. PHYSICAL MODEL

A schematic representation of the studied configurations is depicted in Fig. 1. The model considered here is a square cavity (L = H) with a uniform constant temperature $T_h$, applied on both vertical walls. The other sidewalls, including the top and bottom of the cavity, are assumed to be adiabatic. It is assumed that the incoming flow is at a uniform velocity $u_i$ and at the ambient temperature $T_c$. All solid boundaries are assumed to be rigid, no-slip walls.

An external airstream enters into the enclosure from the two openings symmetrically placed on top of the enclosure, with four values of $\theta$ (inlet flow angle, defined as the angle between the inlet flow direction and $x$-direction) = 90°, 70°, 45°, and 20°, and exits from another two openings placed on the bottom of both vertical sidewalls. The right inflow opening was located at a distance $w_i = 0.24$ from the left wall of the enclosure, and the right outflow opening was located at a distance $h_i = 0.02$ from the bottom wall of the enclosure. The size of the exit port is double the size of the inlet port, which is equal to $w = 0.02$ L. For brevity's sake, this case is referred to as TS configuration.

3. MATHEMATICAL MODEL

The flow is considered to be two-dimensional, unsteady, and laminar, and the physical properties are assumed to be constant, except for the density in the body force term of the $y$ momentum equation, which is treated by using the Boussinesq approximation. The viscous dissipation and Joule heating in the energy equation is neglected. The working fluid is assumed to be air (Pr = 0.701). Taking into account the aforementioned assumptions, the governing equations can be written in nondimensional form as follows:

Continuity equation–

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$
\( \frac{\partial U}{\partial X} + U \frac{\partial U}{\partial Y} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\text{Re}} \) \tag{2}

\( \frac{\partial V}{\partial X} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \) \tag{3}

Energy equation–

\( \frac{\partial \Theta}{\partial X} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{1}{\text{Re Pr}} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right) \) \tag{4}

The mixed convection parameter seen in the above equation, \( \text{Ri} \), defined as \( \text{Gr}/\text{Re}^2 \), is a characteristic number for the mixed-convection process that indicates the relative dominance of the natural- and forced-convection effect, and \( \text{Re} \) is based on the rectangular cavity length and the velocity of jet at the inlet.

The nondimensional numbers seen in the equations above, \( \text{Re}, \text{Gr}, \text{and Pr} \), are Reynolds number, Grashof number, and Prandtl number, respectively, and they are defined as:

\[ \text{Gr} = \frac{g \beta (T_c - T_h) H^3}{\gamma^2}, \quad \text{Re} = \frac{u_i H}{\nu}, \]

\[ \text{Ri} = \frac{\text{Gr}}{\text{Re}^2}, \quad \text{and} \quad \text{Pr} = \frac{\nu}{\alpha}. \]

The dimensionless parameters in the equations above are defined as follows:

\[ U = \frac{u}{u_i}, \quad F = \frac{u_i \tau}{H}, \quad X = x/H, \quad Y = y/H, \]

\[ \Theta = (T - T_h)/(T_c - T_h), \quad P = \frac{p}{\rho u_i^2}. \]

The Nusselt number (\( \text{Nu} \)) at the sidewall is a measure of the convective heat-transfer coefficient at the surface. The average Nusselt numbers at two sidewalls are obtained using the expression

\[ X = 0, \quad \text{Nu}_x = -\frac{1}{H} \int_0^H \frac{\partial \Theta}{\partial X} \big|_{X=0} dY \]

\[ X = L, \quad \text{Nu}_y = -\frac{1}{H} \int_0^H \frac{\partial \Theta}{\partial X} \big|_{X=L} dY. \]

The boundary conditions for TS configuration are

\[ X = 0, \quad \Theta = 0, \quad U = V = 0 \]

\[ X = 1, \quad \Theta = 0, \quad U = V = 0 \]

\[ Y = 0, \quad \frac{\partial \Theta}{\partial Y} = 0, \quad U = V = 0 \]

\[ Y = 1, \quad \frac{\partial \Theta}{\partial Y} = 0, \quad U = V = 0. \]

At the inlet: \( 0.24 \leq X \leq 0.26, 0.74 \leq X \leq 0.76 \):

\[ U = 0, \quad V = -1, \quad \Theta = 1. \]

At the outlet: \( 0.02 \leq Y \leq 0.06 \):

\[ \frac{\partial V}{\partial X} = 0, \quad \frac{\partial \Theta}{\partial X} = 0. \]

4. NUMERICAL PROCEDURE AND CODE VALIDATION

The semi-implicit method for pressure-linked equations (SIMPLE) method on the staggered grids is adopted here, and the governing differential equations are discretized by the quadratic upwind interpolation of convective kinematics (QUICK) scheme. The computational procedure is similar to that described by Patankar (1981). The computational domain consists of \( 80 \times 80 \) bi-quadratic elements, and the grid sizes have been optimized. The computational time step is \( \Delta F = 0.001 \) The results presented here are independent of grid sizes and time step.

Grid refinement tests were performed for the case \( \theta = 90^\circ, \text{Re} = 2250, \text{and Ri} = 0 \) using three uniform grids \( 50 \times 50, 80 \times 80, \) and \( 100 \times 100 \). The results show that when we change the mesh size from a grid of \( 80 \times 80 \) to a grid of \( 100 \times 100 \), the average Nusselt number (\( \text{Nu}_x \)) undergoes an increase of only 0.31%; then, because of the calculation cost, the \( 80 \times 80 \) grid is retained.

The three different sizes of time step considered for the time -step refinement tests for the case \( \theta = 90^\circ, \text{Re} = 3000, \) and \( \text{Ri} = 0 \), are 0.002, 0.001, and 0.0005. Results show that when we change the time step size from 0.001 to 0.0005, the average Nusselt number (\( \text{Nu}_x \)) undergoes an increase of only 0.23%; then, because of calculation cost, \( \Delta F \) is considered as 0.001.

The test for validation of the computer code is performed by comparing with the solutions which are available in the literature by De Vahl Davis (1983) and is shown in Table 1.
TABLE 1: Comparison of the results for validation with de Vahl Davis (1983)

<table>
<thead>
<tr>
<th></th>
<th>( Ra = 10^3 )</th>
<th></th>
<th>( Ra = 10^4 )</th>
<th></th>
<th>( Ra = 10^5 )</th>
<th></th>
<th>( Ra = 10^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present</td>
<td>de Vahl Davis (1983)</td>
<td>( \Delta% )</td>
<td>Present</td>
<td>de Vahl Davis (1983)</td>
<td>( \Delta% )</td>
<td>Present</td>
</tr>
<tr>
<td>( \overline{Nu} )</td>
<td>1.108</td>
<td>1.117</td>
<td>0.81</td>
<td>2.201</td>
<td>2.238</td>
<td>1.66</td>
<td></td>
</tr>
<tr>
<td>( Nu_{\text{max}} )</td>
<td>1.489</td>
<td>1.505</td>
<td>1.06</td>
<td>3.459</td>
<td>3.528</td>
<td>1.96</td>
<td></td>
</tr>
<tr>
<td>( Nu_{\text{min}} )</td>
<td>0.682</td>
<td>0.692</td>
<td>1.45</td>
<td>0.589</td>
<td>0.586</td>
<td>0.51</td>
<td></td>
</tr>
</tbody>
</table>

The present results have a very good agreement with the results by De Vahl Davis (1983). From the comparison it can be decided that the current code can be used to predict the heat-transfer characteristics for the present problem.

5. RESULTS AND DISCUSSIONS

5.1 Oscillations and Bifurcations of Mixed Convection with TS Configuration

5.1.1 Effect of Reynolds Number and Richardson Number

(1) \( Re = 1000, \) \( Ri \) varying from 0 to 10, and \( \theta \) fixed at 90°—

Results are obtained for \( Re = 1000, \) \( Pr = 0.701, \) and \( Ri = 0, 0.5, 1.0, 1.5, 2.0, 2.5, \) and 10. Variation of the average Nusselt number at the left sidewall \( (Nu_x) \) with dimensionless time \( (F) \) for different Richardson numbers are shown in Figs. 2(a)–2(f), respectively. Numerical results show that depending on the increasing \( Ri, \) the flow inside the cavity may be observed as follows.

(a) Steady solution \( (Ri = 0, 0.5) \)

At very low values of the Richardson number, such as 0 and 0.5, the inertia force of the fluid is dominant compared to the buoyancy force, the flow is in the regime of forced convection and the disturbance induced by the buoyancy force cannot destroy the original system balance, then the solution of \( Nu_x \) is steady, as shown in Figs. 2(a) and 2(b).

(b) Periodic solution \( (Ri = 1.0, 1.5) \)

As the Richardson number increases to \( Ri = 1.0, \) the inertia and buoyancy forces balance each other, which then results in a mixed convection. These two mechanisms of buoyancy \( (Ri \) is a measure) and forcing by the inlet flow \( (Re \) is a measure) lead to complex interaction. The flow inside the cavity loses its stability at \( Re = 1000, \) \( Ri = 1.0, \) then transforms into periodic oscillatory flow via the steady symmetry breaking Hopf bifurcation as shown in Figs. 2(c) and 2(d). The dimensionless time period is 5, a time period of streamline plots and temperature contours are displayed in Fig. 3. When the Richardson number further increases to \( Ri = 1.5, \) the flow retains periodic oscillation but the dimensionless time period decreases to 4.4.

(c) Trend for steady-state solution \( (Ri \geq 2.0) \)

When the Richardson number further increases to \( Ri \geq 2.0, \) the buoyancy force gradually becomes the dominant mechanism to drive the convection of the fluid and the flow is in the regime of natural convection. The trend for steady-state solution is observed as shown in Figs. 2(e)–2(g). One can observe that the average Nusselt number may not change with dimensionless time \( F \) when the Richardson number increases to \( Ri = 2.5, \) which indicates that the solutions attain steady state again.

(2) \( Re = 2000, \) \( Ri \) varying from 0 to 20, and \( \theta \) fixed at 90°—

When the Reynolds number is increased to \( Re = 2000 \) for \( Ri \) varying from 0 to 20, variation of the average Nusselt number at the left sidewall \( (Nu_x) \) with dimensionless time \( (F) \) for different Richardson numbers is shown in Figs. 4(a)–4(f), respectively. Numerical results show that depending on the increasing \( Ri, \) the flow inside the cavity may be observed as follows: with increasing \( Ri \) for a given \( Re, \) the solution may exhibit a change from steady state to periodic oscillation, and then to nonperiodic oscillatory state. As \( Ri \) increases further, the flow inside the cavity becomes stable again [shown in Figs. 4(a)–4(f)]. According to that, three kinds of heat-transfer regimes exist for the magnitude of \( Ri: \) a forced-convection-dominated
FIG. 2: Variation of $\text{Nu}_x$ with $F$ at $\theta = 90^\circ$ $\text{Re} = 1000$

FIG. 3: Period of streamline plots and temperature contours at $\theta = 90^\circ$ $\text{Re} = 1000$, $\text{Ri} = 1.0$

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regime, a mixed-convection regime, and a buoyancy-dominated (or natural-convection) regime.

At low values of the Richardson number, the forced convection due to the driven force dominates the flow mechanism, and the solution retains a steady state. As the Richardson number increases to $R_i = 0.5$, the system can not retain balance because of the enhanced disturbance of the buoyancy forces, which then results in oscillatory solution. When the Richardson number further increases to $R_i = 1.0$, the interaction between buoyancy and forcing by the inlet flow becomes more intense, and this behavior results from the onset of nonperiodic oscillatory solution. When the Richardson number further increases to $R_i \geq 2.0$, the buoyancy force gradually becomes the dominant mechanism to drive the convection of the fluid, and the flow is in the regime of natural convection. The trend for steady state solution can be observed. One can find that, with the increasing $R_i$ from $R_i = 2.0$, the amplitude of oscillation becomes smaller and smaller, and the Nusselt numbers do not change with dimensionless time $F$ when $R_i$ reaches $R_i = 20$, which indicates that the solution attains a steady state again.

5.1.2 Effect of Inlet Flow Angle ($\theta$)

Numerical results are obtained when $\theta = 20^\circ$, $45^\circ$, $70^\circ$, and $90^\circ$ for different $R_i$ and $Re$. $R_i$ is fixed at 0 (natural convection effect is not considered) while $Re$ increases. Figure 5 represents a period of the streamline plots and temperature contours when $Re = 1500$, $R_i = 0$, $\theta = 70^\circ$. Results show that the velocity and temperature fields lose their symmetry and transform to asymmetric structure because of periodic oscillation, while the solutions remain in a stable state for $\theta = 20^\circ$, $45^\circ$, and $90^\circ$.

When the Reynolds number further increases to $Re = 2500$, the velocity and temperature fields of $\theta = 45^\circ$ become oscillatory (shown in Fig. 6) and lose symmetric structure, while the solutions remain in a steady state for $\theta = 20^\circ$ and $90^\circ$ [shown in Figs. 7(a) and 7(b)].

The effect of inlet flow angle on the oscillations and bifurcations of mixed convection with TS configuration is evident. The TS configuration with $\theta = 70^\circ$ is proved to be the most unstable in the four values of $\theta$, in consideration of the pure forced convection effect.

The results obtained by taking into consideration the natural convection effect ($R_i = 0.5$) are shown in Fig. 8. Figure 8 shows that the TS configuration with $\theta = 70^\circ$ loses its stability at $Re = 1000$, while $\theta = 90^\circ$ at $Re = 1500$, and $\theta = 45^\circ$ at $Re = 1750$, but the solutions remain stable for $\theta = 20^\circ$. The TS configuration with $\theta = 70^\circ$ is also proved to be the most unstable in the four values of $\theta$, in consideration of both forced and natural convection effect.
FIG. 5: Period of streamline plots and temperature contours at $Re = 1500$, $Ri = 0$, $\theta = 70^\circ$

(a) a period of streamline plots at $Re=1500$, $Ri=0$, $\theta=70^\circ$

(b) a period of temperature contours at $Re=1500$, $Ri=0$, $\theta=70^\circ$

FIG. 6: Oscillation at $Re = 2500$, $Ri = 0$, $\theta = 45^\circ$

(a) Variation of $Nu_x$ with $F$

(b) streamline plot and temperature contour

FIG. 7: Variation of $Nu_x$ with $F$ for $\theta = 20^\circ$ and $90^\circ$

(a) $Re=2500$, $\theta=20^\circ$

(b) $Re=2500$, $\theta=90^\circ$

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5.2 Phase Portraits

A phase portrait is a two-dimensional projection of the phase space (Giannakopoulos, 2002). It represents each of the state variables’ instantaneous state to each other. For the pendulum example, the angular velocity position graph is a phase portrait. Chaotic and other motions can be distinguished visually from each other according to the phase portrait. A fixed point solution is a point in a phase portrait. A periodic solution is a closed curve in a phase portrait, and chaotic solutions are distinct curves in a phase portrait. Figures 9(a)–9(f) show the change of phase portraits for the case $\text{Ri} = 0.5$, $\theta = 70^\circ$, and $\text{Re}$ given in the range of 500–2000.

Figure 9 shows that as $\text{Re}$ increases, the solution may exhibit a change from a steady state to periodic oscillation, and then to a nonperiodic oscillatory state. According to Fig. 9, the phase portraits show the evolution of the attractor from a stable fixed point, to a limit cycle, and finally, to chaos.

6. CONCLUSION

The present work is focused on the self-sustained oscillations and bifurcations of mixed convection in a two-dimensional multiple ventilated enclosures. The conclusions are drawn as follows:
(1) As Ri increases, the solution may exhibit a change from a steady state to periodic oscillation, and then to a nonperiodic oscillatory state. As Ri increases further, the flow inside the cavity becomes stable again.

(2) The velocity and temperature fields may show an asymmetric structure for certain combinations of the control parameters, even though the boundary conditions are steady and symmetric.

(3) The TS configuration with $\theta = 70^\circ$ (proved to be the most unstable in the seven values of $\theta$) loses its stability at Re = 1500, Ri = 0, and Re = 1000. Ri = 0.5 transforms into periodic oscillatory flow via the steady symmetry breaking Hopf bifurcation and becomes nonperiodic-unstable at Re = 2000.

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