Effects of Film Evaporation and Condensation on Oscillatory Flow and Heat Transfer in an Oscillating Heat Pipe

An advanced theoretical model of a U-shaped minichannel, a building block of a closed-end oscillating heat pipe, has been developed. Thin film evaporation in the evaporator and thin film condensation in the condenser, axial variation of surface temperature, and pressure loss at the bend are incorporated in this model. The sensible heat transfer coefficients between the liquid slug and the wall are obtained by analytical solution for laminar liquid flow and by empirical correlations for turbulent liquid flow. The effects of the inner diameter, evaporator temperature on the thermally induced oscillatory flow and heat transfer performance, and the mechanism of film condensation and evaporation are investigated. [DOI: 10.1115/1.4002780]

Keywords: film evaporation, film condensation, oscillating heat pipe

1 Introduction

The increased demand on high performance required by the information technology industry has raised thermal design challenges due to both increased heat dissipation from the CPU and higher heat density. While conventional heat sinks or spreaders become severely inadequate at these high levels of heat fluxes, oscillating (or pulsating) heat pipe (OHP or PHP) shows a promise to meet the next generation CPU thermal requirements with a low profile heat sink. The OHP is a type of two-phase heat transfer devices, invented and patented in the 1990s by Akachi [1]. Due to its potential heat transport capability, fast thermal response, simple structure, and low cost of construction, the OHP will play a key role in the electronics cooling.

Typically, an OHP consists of a plain meandering long tube of capillary dimensions with many U-turns and joined end to end and the evaporator and condenser sections are located at these turns. Compared with the traditional heat pipes, the unique feature of the OHP is that the vapor and liquid flow in the same direction, and therefore, there is no friction between the liquid and vapor phases. The inner diameter must be sufficiently small ranging from 0.1 mm to 5 mm for the capillary force to create liquid slugs and vapor plugs [2]. There is no wick structure to return the condensate liquid from the condenser to evaporator sections; heat is transported from the evaporator section to the condenser section by the oscillation of the working fluid in the axial direction. When the evaporator section is heated, the vapor pressure at this end increases due to liquid film evaporation. At the condenser section, when the vapor plugs are cooled, the condensing liquid film is formed inside the channel and the pressure in the condenser section is lowered. This increased pressure in the evaporator and reduced pressure in the condenser result in a pressure difference. This pressure difference pushes the liquid slug from the evaporator section to the condenser section and, conversely, the cold vapor and liquid from the condenser to the evaporator. The oscillatory flow in an OHP can be sustained by the alternative change of the pressure at two ends of the liquid slugs.

In the past decade, extensive experimental and theoretical works have been conducted to understand the mechanism of the OHP. Dobson and Harms [3] developed a mathematical model to study the behavior of an OHP with an open-end. They assumed the heat transfer coefficients between the heat pipe wall and the vapor and neglected the effect of surface tension and heat transfer between the liquid and its surroundings. Hosoda et al. [4] investigated the formation of vapor plugs in a meandering closed loop heat transport device (MCL-HTD) with a simplified numerical model neglecting the liquid film between the tube wall and the vapor plug and the effect of surface tension. Wong et al. [5] presented a theoretical model of the OHP operation based on the Lagrangian approach of slug flow in a serpentine tube. By using a pressure pulse generated by the local heat input into the vapor bubble, the pressure and velocity variations in the OHP were obtained. The liquid film between the vapor plug and the wall and the effect of surface tension were not included.

Zhang and Faghi [6] analyzed the thin film evaporation and condensation in the evaporator and condenser sections of an OHP with an open-end. The results showed that the heat transfer in an OHP is mainly due to the exchange of sensible heat and phase-change heat transfer is the driving force of the oscillation. Shafii et al. [7], based on thin film evaporation and condensation, established a theoretical model of an OHP and concluded that the surface tension has little effect on the frequency and amplitude of the oscillation motion. Liang and Ma [2] presented a mathematical model describing the oscillation characteristics of slug flow in a capillary tube. It was demonstrated that the internal diameter, vapor plug size, and unit cell numbers determine the oscillation and capillary force, gravitational force, and initial pressure distribution of the working fluid that significantly affects the frequency and amplitude of the oscillation motion in the capillary tube. Zhang and Faghi [8] investigated the liquid-vapor pulsating flow in an OHP with arbitrary number of turns. The results showed that for an OHP with fewer than six turns, the amplitude and frequency of oscillation are independent of the number of turns. Ma et al. [9] developed a mathematical model predicting the fluid motion and temperature drop in an OHP. The numerical results indicate that the oscillating motions occurring in the OHP significantly en-
hances the heat transfer in the OHP. Experimental results indicated that there exists an onset temperature difference for the excitation of oscillating motions in an OHP. Despite the extensive study about the OHP, the mechanisms of oscillating motions occurring in the OHP, including the detailed relationship between the oscillating frequency and dimensions, temperature difference between the evaporating and condensing sections, filling ratio, working fluids, and operating temperature, have not been fully understood. The authors [10] extended the modeling capability by including the effects of initial temperature, axial variation of surface temperature, and pressure drop at the bend. It was demonstrated that the initial temperature has a very profound effect in the oscillating motion and heat transfer performance. The authors also investigated the capillary and gravitational effects on the oscillatory flow and heat transfer of an OHP by comparing the displacement of the liquid slug and temperature and pressure of the vapor plug’s latent and sensible heat transfers [11]. The results confirmed that for all cases under the same inner diameter, the gravitational force has more effect on the frequency of the oscillation than the amplitude. As the inner diameter of the OHP increases, the heat transfer capability is improved. The comparison of the effect with and without surface tension indicated that the effect of surface tension on the oscillation can be neglected. However, the heat transfer coefficients in the evaporator and condenser sections were assumed to be constants in Refs. [10,11].

In order to fully discover the mechanisms of the OHP operation and performance, the film evaporation and film condensation need detailed investigation. Khurstev and Faghri [12] developed a physical and mathematical model of the evaporating thick liquid film attached to the liquid-vapor meniscus in a circular microtube. The numerical results obtained for water demonstration that the formation of extended thick liquid films in microtubes can take place due to high-velocity vapor flow under high rates of vaporization. Begg et al. [13] established a model of forced-convective film condensation in a circular miniature tube. The model predicted the shape of the liquid-vapor interface along the condenser and the length of the two-phase flow region. Observations from a flow visualization experiment of water vapor condensing in a horizontal glass tube confirm the existence and qualitative features of annular film condensation, leading to the complete condensation phenomenon in small diameter (d<3.5 mm) circular tubes.

Since the evaporation and condensation over the liquid film in an OHP are the driving forces of the oscillatory flow, precise understanding of phase-change heat transfer is desirable. However, the evaporation and condensation heat transfer coefficients are simply assumed in most works. The objective of this study is to develop an analytical model for predicting the oscillation motion and heat transfer in an OHP. The model considers the film evaporation and condensation, the effect of axial variation of the surface temperature, as well as the pressure loss at the bend.

2 Theoretical Model

The physical model is a vertically placed U-shaped minichannel with its two ends sealed (see Fig. 1(a)). The U-shaped minichannel is treated as a straight tube (see Fig. 1(b)) with the pressure loss in the bend considered. The length of each evaporator section, which is located at the two ends of the pipe, is \( L_e \) and evaporator temperature is maintained at \( T_e \). The condenser section maintained at \( T_c \) is located between two evaporator sections and its length is \( L_c \). The length of the liquid slug is \( L_p \), which depends on the filling ratio, and \( x_p \) is used to represent the displacement of the liquid slug. When the liquid slug is exactly in the middle of the U-shaped miniature channel, \( x_p \) is zero. When the liquid slug shifts to the right side, \( x_p \) is positive; when it moves to the left side, \( x_p \) is negative.

If the initial value of displacement \( x_p(0) \) is positive (see Fig. 1(a)), part of the vapor plug in the left side is in contact with the condenser section, and condensation in the left part will cause the pressure of the left vapor plug \( p_{l1} \) to decrease. The pressure difference between the two vapor plugs causes the liquid slug to move to the left direction. The evaporation of the liquid film left behind by the liquid slug in the right part of the channel causes the pressure of the right vapor plug \( p_{r2} \) to increase and the pressure difference between the two ends of the liquid slug further increases. When the displacement \( x_p \) becomes zero, there is neither evaporation nor condensation in two vapor plugs but the liquid slug will keep moving due to its inertia. When the liquid moves to the left side (\( x_p < 0 \)), the pressure difference changes its sign. The oscillation of the liquid slug can be sustained by alternative evaporation and condensation in the two vapor plugs.

The following assumptions are made in order to model heat transfer and fluid flow in the heat pipe.

1. The liquid is incompressible and the vapor behaves as an ideal gas under reversible and adiabatic processes with no temperature and pressure gradients in the vapor regions.
2. In every time-step, liquid and vapor interface is quasi steady state so that equilibrium liquid films can be established.
3. Heat transport in the thin film is due only to the conduction in the radial direction.
4. Shear stress at the liquid-vapor interface is negligible.
5. Heat conduction in the liquid slug is assumed to be one-

![Fig. 1 Physical model](http://heattransfer.asmedigitalcollection.asme.org/on/01/16/2016 Terms of Use: http://www.asme.org/about-asme/terms-of-use)
dimensional in the axial direction and heat exchange between the liquid and the wall is considered by a convective heat transfer coefficient.

(6) The U-shaped minichannel is assumed to be a straight pipe and the effect of pressure loss at the bend is considered using an empirical correlation.

(7) The evaporator and condenser sections are maintained at constant but different temperatures.

2.1 Governing Equations for Oscillatory Flow. The momentum equation for the liquid slug is

\[\frac{d^2x_p}{dt^2} = (p_{v1} - p_{v2})A - \Delta p_bA - 2\rho_gAx_p - \pi dL_p\tau_p\]  

where \(\tau_p = c_i\rho_0 v^2/2\) is the shear stress, the friction coefficient \(c_i\) is [14]

\[c_i = \begin{cases} 
16/Re, & Re \leq 2200 \\
0.078 Re^{-0.2}, & Re > 2200 
\end{cases} \]  

and \(\Delta p_b\) is the pressure loss at the bend [15]

\[\Delta p_b = \begin{cases} 
\zeta \rho \frac{v_p^2}{2}, & v_p > 0 \\
-\zeta \rho \frac{v_p^2}{2}, & v_p < 0 
\end{cases} \]  

where \(\zeta\) is the pressure loss coefficient. The effect of gravity on the motion of the liquid slug has been included in the momentum equation.

The energy equations of the two vapor plugs are

\[\frac{dm_{v1}}{dt} = \frac{T_{v1}}{c_v} + \frac{\pi L}{4} \frac{dx_p}{dt} \]  

\[\frac{dm_{v2}}{dt} = \frac{T_{v2}}{c_v} - \frac{\pi L}{4} \frac{dx_p}{dt} \]  

The vapor plugs behave as an ideal gas, i.e.,

\[p_{v1}(L_e + x_p) \frac{\pi L}{4} dx = m_{v1}R_g T_{v1} \]  

\[p_{v2}(L_e - x_p) \frac{\pi L}{4} dx = m_{v2}R_g T_{v2} \]  

Combining the above equations, the masses and temperatures of two vapor plugs are obtained as the following [16]:

\[m_{v1} = \frac{\pi d^2 p_0}{4R T_0} \left(\frac{p_{v1}}{p_0}\right)^{\gamma/\gamma} (L_e + x_p) \]  

\[m_{v2} = \frac{\pi d^2 p_0}{4R T_0} \left(\frac{p_{v2}}{p_0}\right)^{\gamma/\gamma} (L_e - x_p) \]  

\[T_{v1} = T_0 \left(\frac{p_{v1}}{p_0}\right)^{(\gamma-1)/\gamma} \]  

\[T_{v2} = T_0 \left(\frac{p_{v2}}{p_0}\right)^{(\gamma-1)/\gamma} \]  

where \(\gamma\) is the ratio of the specific heats and \(\gamma=1.3\) for steam. The effects of evaporation and condensation on the mass variation of the vapor plugs can be calculated as the following:

\[\frac{dm_{v1}}{dt} = \begin{cases} 
-Q_{vap} h_{v0,v}^*, & x_p > 0 \\
Q_{cond} h_{v0,v}^* + Q_{vap} h_{v0,v}^*, & x_p < 0 
\end{cases} \]  

where \(h_{v0,v}^* = h_{v0,v} + 0.68 c_p(T_{v1} - T_e)\) and \(h_{v0,v}^* = h_{v0,v} + 0.68 c_p(T_{v2} - T_e)\), as defined in Ref. [17].

The initial condition of the U-shaped minichannel in this study is chosen to be identical to the reference state of this system.

\[x_p = x_{p0}, \quad t = 0 \]  

\[p_{v1} = p_{v2} = p_0, \quad t = 0 \]  

\[T_{v1} = T_{v2} = T_0, \quad t = 0 \]  

\[m_{v1} = \frac{\pi d^2 p_0}{4R T_0} (L_e + x_{p0}) \]  

\[m_{v2} = \frac{\pi d^2 p_0}{4R T_0} (L_e - x_{p0}) \]  

2.2 Heat Transfer. The heat transfer in an OHP is defined as the total heat transferred from the heating section to the cooling section that consists of two parts: the latent heat transfer due to evaporation and condensation and the sensible heat transfer due to the heat transfer between the minichannel wall and the liquid plugs in the form of a single-phase heat transfer.

2.2.1 Latent Heat Transfer. Figures 1(c)–1(e) show the physical models of film evaporation and condensation heat transfer. With the complication of the two-phase heat transfer in the evaporator and condenser sections, the modeling is divided into two parts: evaporator section and condenser section. In the evaporator section, the liquid film is divided into three regions: nonevaporating region, thin film region, and intrinsic meniscus region. In the condenser, the liquid film is divided into two regions: thin film region and intrinsic meniscus region.

In Fig. 1(b), if the displacement \(x_p\) is positive, a relatively flat liquid film is formed in the condenser section with an intrinsic meniscus attached and condensation occurs (see Fig. 1(c)). In the evaporator, when \(v_p\) is positive, the liquid slug moves forward and a thin film is formed and evaporation occurs on the thin film and meniscus (see Fig. 1(d)); when \(v_p\) is negative, the liquid slug moves backward and a thick film is formed (see Fig. 1(e)). The difference between the thick film and the thin film is that the thick film’s surface is convex, as shown in Fig. 1(e) [12]. The length of the flat thin film in the condenser section \(L_f\) is calculated from the left end of the condenser to the left end of the liquid the slug accounting for the condensation of the left vapor slug (see Fig. 1(c)); the length of the thin film in the evaporator \(L_f\) is calculated from the transient point to the nonevaporating point (see Figs. 1(d) and 1(e)). The shape of the meniscus depends on the contact angle, as shown in Fig. 1. It is assumed that the contact angle depends on the direction of the liquid slug movement only and its value will not change unless the liquid slug changes its direction. The assumption is valid for most of the time except when the liquid slug changes its direction.

2.2.1.1 Film condensation. The physical model of the film condensation and the cylindrical coordinate system used in the condenser section are shown in Fig. 1(c). Both vapor plugs and liquid slug oscillate along the z-coordinate. The mass and energy balances for the liquid film shown in Fig. 1(c) yield the following:

\[\int_{R_0}^{R_t} rw_f(r)dr = \frac{1}{2\pi h_0} (\bar{m}_{\text{in}} - \frac{Q}{h_0}) \]  

where \(r\) is the radial coordinate, \(\delta\) is the thickness of the liquid film, \(w_f\) is the liquid velocity along the radial direction, and \(\bar{m}_{\text{in}}\) is the liquid mass flow rate in the inner meniscus region at the transient point, which is supplied by the condensation for \(z<0\).
is the heat flow rate through a given cross section due to condensation from the film for \( z \geq 0 \) and defined as follows:

\[
Q(z) = 2\pi R \int_0^z q_w(z)dz
\]  
(20)

where \( q_w(z) \) is the heat flux at the solid-liquid interface due to heat conduction through a cylindrical film with a thickness of \( \delta \):

\[
q_w = k_f \frac{T_w - T_\delta}{R \ln[R/(R - \delta)]}
\]  
(21)

where \( T_\delta \) is the local temperature of the liquid-vapor interface, which differs from the saturated bulk vapor temperature due to the interfacial resistance and effects of curvature and disjoining pressure on saturation pressure over evaporating films. Combining Eqs. (20) and (21), the following equation is obtained:

\[
dQ = 2\pi k_f \frac{T_w - T_\delta}{R \ln[R/(R - \delta)]} dz
\]  
(22)

The heat flux across the thin film is defined as [18]

\[
q_\delta = \left( \frac{2\alpha}{2 - \alpha} \right) \frac{h_w}{\sqrt{2\pi R_g T_w}} \left[ \frac{p_v}{\sqrt{T_v}} - \frac{(p_{sat})_\delta}{\sqrt{T_\delta}} \right]
\]  
(23)

where \( \alpha \) is the accommodation coefficient and \( p_v \) and \( (p_{sat})_\delta \) are the saturation pressures corresponding to \( T_v \) and at the liquid-vapor interface, respectively. The following two algebraic equations given by the extended Kelvin equation in Refs. [17,18] should be determined to solve \( T_\delta \):

\[
(p_{sat})_\delta = p_{sat}(T_\delta) \exp\left[ \left( \frac{p_{sat}(T_\delta) + p_d - \sigma K}{\rho R_g T_\delta} \right) \delta \right]
\]  
(24)

where \( K \) is the local curvature of the liquid-vapor interface. Under steady state conditions, \( q_w(R - \delta)/R = q_w \), it follows from Eqs. (21) and (23) that

\[
T_\delta = T_w + \frac{(R - \delta)}{k_f} \ln\left[ \frac{R}{R - \delta} \right] \left( \frac{2\alpha}{2 - \alpha} \right) \frac{h_w}{\sqrt{2\pi R_g T_w}} \left[ \frac{p_v}{\sqrt{T_v}} - \frac{(p_{sat})_\delta}{\sqrt{T_\delta}} \right]
\]  
(25)

Equations (24) and (25) determine the interfacial temperature \( T_\delta \) and pressure \( (p_{sat})_\delta \) for a given vapor pressure \( p_v = p_{sat}(T_v) \), the wall temperature \( T_w \), and the liquid film thickness \( \delta \). The disjoining pressure for water can be expressed as

\[
p_d = \rho R_g T_\delta \ln\left[ \frac{\delta}{(3.3)^6} \right]
\]  
(26)

where \( \alpha = 1.5336 \) and \( b = 0.0243 \).

The momentum equation for viscous flow in a liquid film is written in the Stokes approximation.

\[
\frac{1}{r} \frac{\partial}{\partial r}\left( r \frac{\partial w_l}{\partial r} \right) = \frac{1}{\mu_l} \frac{dp_l}{dz}
\]  
(27)

which is subject to the following boundary conditions:

\[
w_l = 0 \quad \text{at} \quad r = 0
\]  
(28)

\[
\frac{\partial w_l}{\partial r} = 0 \quad \text{at} \quad r = R - \delta
\]  
(29)

Solving Eq. (27) with boundary conditions specified by Eqs. (28) and (29), the velocity profile is obtained

\[
w_l = \frac{1}{\mu_l} \frac{dp_l}{dz} \frac{1}{4} \left( R^2 - r^2 \right) + \frac{(R - \delta)^2}{2} \ln \frac{R}{R - \delta}
\]  
(30)

Substituting Eq. (30) into Eq. (19), the following equation for the liquid pressure is:

\[
dp_l = \frac{\mu_l}{2\pi R_p \phi} \left( \frac{Q}{h_w} - \bar{m}_{1,ln} \right)
\]  
(31)

where

\[
F = \frac{R^4 + (R - \delta)^2 \left[ \ln \frac{R}{R - \delta} + \frac{1}{2} \right] - R^2 + (R - \delta)^2 - R^2}{16 + 2 \left( \ln \frac{R}{R - \delta} + \frac{1}{2} \right) - 4 + \frac{(R - \delta)^2 - R^2}{8 - 4}}
\]  
(32)

The pressure difference between the vapor and liquid phases is due to capillary and disjoining pressure effects [17].

\[
p_v = p_d = \sigma K - p_d
\]  
(33)

Since the variation of the liquid film thickness in the condenser section is very small, the curvature of the liquid film is [17]

\[
K = \frac{d^2 \delta}{dz^2} + \frac{1}{R - \delta}
\]  
(34)

Substituting Eqs. (26) and (34) into Eq. (33) and differentiating Eq. (33) with respect to \( z \), one obtains

\[
\frac{d}{dz} \left( p_v - p_d \right) = \frac{d}{dz} \left[ \left( \frac{d^2 \delta}{dz^2} + \frac{1}{R - \delta} \right) - \rho R_g T_\delta \ln\left( \frac{\delta}{(3.3)^6} \right) \right]
\]  
(35)

Since the vapor pressure is constant in the vapor region, the liquid pressure gradient is obtained below

\[
\frac{dp_l}{dz} = \frac{d}{dz} \left[ \sigma \left( \frac{d^2 \delta}{dz^2} + \frac{1}{R - \delta} \right) - \rho R_g T_\delta \frac{b \delta}{dz} \right]
\]  
(36)

The third-order derivative of film thickness \( d^3 \delta/dz^3 \) is solved with Eqs. (31) and (36), i.e.,

\[
\frac{d^3 \delta}{dz^3} = \frac{1}{\sigma} \left[ \rho R_g T_\delta \frac{b \delta}{dz} - \frac{\mu_l}{2\pi R_p \phi} \left( \frac{Q}{h_w} - \bar{m}_{1,ln} \right) \right] - \frac{1}{(R - \delta)^2} \frac{d\delta}{dz}
\]  
(37)

which can be together with Eq. (22) with the following boundary conditions:

\[
\delta_{z=0} = \delta_i
\]  
(38)

\[
\frac{d\delta}{dz} \bigg|_{z=0} = 0
\]  
(39)

\[
\frac{d^2 \delta}{dz^2} \bigg|_{z=0} = 0
\]  
(40)

\[
\bar{Q}_{z=0} = 0
\]  
(41)

\[
\frac{d\delta}{dz} = 0, \quad \frac{d^2 \delta}{dz^2} = \frac{1}{R - \delta}, \quad z = L_1
\]  
(42)

where \( \delta_i \) is unknown and can be determined by solving the above equations iteratively until \( (d\delta/dz)|_{z=0} = 0 \) is satisfied. At the same time, the transition film thickness \( \delta_i \) and the total heat transfer amount for the thin film \( \bar{Q}_{t,ln} \) are obtained.

Considering the moving contact angle, the thickness beyond the transition point is calculated by [17]

\[
R' = (R - \delta_i) \cos \theta
\]  
(43)

\[
\delta_{me} = R - \sqrt{R'^2 - ((R - \delta_i) \tan \theta + s)^2}
\]  
(44)

The heat transfer in the meniscus region is then

\[
Q_{c,ln} = 2\pi R_k (T_{c,ln} - T_e) \int_0^{R_k} \frac{1}{\delta_{me}} ds
\]  
(45)

The total condensation heat transfer amount for the condenser is
\[ Q_{\text{cond}} = Q_{e1} + Q_{e2} \]  

### 2.2.1.2 Thin film evaporation

The simplified physical model of thin film evaporation in the evaporator section is shown in Fig. 1(d). The liquid film thickness in the evaporator section satisfies [6]

\[ \frac{d}{dz}(\sigma K - p_d) = 3 \mu_l \left( \frac{m_{l,1n} - Q}{h_{lv}} \right) \]  

which was obtained by analyzing the mass and momentum balance of an evaporating film. Since the film thickness is thinner than the case of condensation, the Cartesian coordinate system is used. The curvature of the liquid film \( K \) is calculated by

\[ K = \frac{d^2 \delta}{dz^2} \left[ 1 + \left( \frac{d \delta}{dz} \right)^2 \right]^{-3/2} \]  

and the disjoining pressure \( p_d \) is the same as in the condensation model. \( Q \) in Eq. (47) is the heat flow rate through a given cross section due to liquid film evaporation when \( z \geq 0 \) and defined as follows:

\[ Q = 2 \pi R \int_0^z k(T_w - T_d) \delta ds \]  

Instead of considering the third-order ordinary differential Eq. (47), the following four first-order equations, including four unknown variables: \( \delta, \Delta, p_{cap} \), and \( Q \), should be considered using the standard Runge–Kutta procedure with their respective boundary conditions

\[ \frac{d \delta}{dz} = \Delta \]  

\[ \frac{d \Delta}{dz} = [1 + (\Delta^2)]^{3/2} \left( \frac{p_{cap} + p_d}{\sigma} \right) \]  

\[ \frac{dp_{cap}}{dz} = -\frac{3 \mu_l}{\rho h_{lv} \sigma} Q(z) \]  

\[ \frac{dQ}{dz} = k(T_w - T_d) \delta \]  

\[ \delta_{l|z=0} = \delta_0 \]  

\[ \Delta_{l|z=0} = 0 \]  

\[ p_{cap}_{|z=0} = -\frac{\sigma}{R - \delta_0} p_d \]  

\[ Q_{|z=0} = 0 \]  

where \( \delta_0 \) is the nonevaporating film thickness calculated by the following equation:

\[ \delta_0 = 3.3 \left\{ 1 - \exp \left[ \frac{p_{sat}(T_w) - p_e + \sqrt{T_w/T_e} + \sigma K}{p_R T_w} \right] \right\}^{1/3} + \ln \left( \frac{p_e}{p_{sat}(T_w)} \right) \left( \frac{T_w}{T_e} \right) \]  

Following the approach of Khrustalev and Faghri [19] for the prediction of evaporation from a hemispherical liquid–vapor meniscus with a radius of \( R - \delta_0 \) and considering the effect of the moving contact angle, the vapor velocity at \( z = 0 \), \( \bar{\nu}_{v,lm} \) can be defined as

\[ \bar{\nu}_{v,lm} = \frac{2}{h_{lv} \rho_l (R - \delta_0)} \int_0^{L_d} \frac{T_w - T_d}{\delta k_l} \times \sin \left[ \arccos \frac{s}{R - \delta} \right] ds \]  

where \( R_d = R - (R - \delta_0) \tan \theta \).

The total amount of heat transfer through the meniscus can be obtained using

\[ Q_{e2} = \rho_l A \bar{\nu}_{v,lm} h_{lv} \]  

where \( s \) is the coordinate along the solid-liquid interface, shown in Fig. 1(d), and the liquid film thickness \( \delta \) for \( s = 0 \) is calculated by Eqs. (43) and (44).

As the thin film thickness increases, the thin film is considered to transform into meniscus when the disjoining pressure drops to 1/1000 of its value at the nonevaporating film. Meanwhile, with the transient film thickness, the total evaporative heat transfer amount for the entire thin film \( Q_{e1} \) is obtained and \( Q_{e2} \) is solved by Eq. (63). The total evaporative heat transfer for the evaporator section is obtained.

### 2.2.1.3 Thick film evaporation

The physical model of the thick film evaporation and the cylindrical coordinate system used in the evaporator section are shown in Fig. 1(e). The modeling of the thick film evaporation is similar to the thin film condensation so that the general equations can be used to describe the evaporation.

The mass and energy balances for the liquid film shown in Fig. 1(e) can be calculated by Eq. (19). Here, \( m_{l,1n} \) is the liquid mass flow rate in the inner meniscus region at \( z = 0 \), which supplies the evaporating film with liquid for \( z \geq 0 \). \( Q(z) \) is the heat flow rate through a given cross section due to liquid film evaporation for \( z \geq 0 \) and defined as in Eqs. (20) and (21). Following the same procedure as in the condenser section, we obtain the following equation:

\[ \frac{dp_l}{dz} = \frac{\mu_l}{2 \rho h_{lv} \sigma} \left\{ \frac{Q}{h_{lv}} \left[ \frac{R^4}{16} + \frac{(R - \delta_0)^2}{2 \left( F + \frac{(R - \delta_0)^2 - R^2}{4} \right)} \right] \right\}^{-1} \]  

where

\[ F = \frac{(R - \delta_0)^2}{2} \left( \frac{\ln R - R - \delta_0}{2} \right) \frac{R^2}{4} \]  

The pressure difference in the vapor and liquid phases is due to capillary and disjoining pressure effects [17].

\[ p_e - p_l = \sigma K - p_d \]  

Introducing the slope of thick film

\[ d \delta dz = \Delta \]  

which can be obtained by substituting Eq. (48) into Eq. (63)

\[ \frac{d \Delta}{dz} = [1 + (\Delta^2)]^{3/2} \left( \frac{p_e - p_l + p_d}{\sigma} \right) \frac{\cos(\arctan \Delta)}{R - \delta} \]  

The four first-order differential equations, Eqs. (40), (64), (65), and (67), include four unknown variables: \( \delta, \Delta, p_l, \) and \( Q \). Therefore, four boundary conditions are set forth at \( z = 0 \)

\[ \delta = \delta_{fr} \]  

\[ \Delta = 0 \]  

\[ p_l = p_{e,fr} = \frac{\sigma}{R - \delta_{fr}} p_d \]  

\[ Q = 0 \]  

The boundary conditions specified in Eqs. (67) and (68) were obtained from the assumptions that the pressure variation in the
meniscus liquid is negligible compared with the capillary pressure and that the meniscus smoothly transforms into a liquid film. The boundary condition (69) directly follows from Eq. (20). There are also three parameters, $\delta_p$, $m_{in}$, and $w_{vap}$, and an additional variable $T_g$ involved in this problem. They will be considered using additional algebraic equations and some constitutive boundary conditions including those at the end of the liquid film ($z=L_2$).

The liquid film inevitably ends with a microfilm (Fig. 1(d)) so that its thickness at $z=L_2$ is very small and that the disjoining pressure is very important in the microfilm region, allowing the local interface temperature $T_g$ to approach $T_w$ suppressing evaporation. On the contrary, $\delta_p$ is expected to be thick enough for the disjoining pressure to be zero. At the end of the microfilm ($z=L_2$), the liquid film is nonevaporating ($T_g=T_w$) with the length of $\delta_p$. The parameter $m_{in}$ should be found using a constitutive boundary condition at the end of the microfilm.

\[ m_{in} = \frac{Q}{h_{li}(z=L_2)} \]  
(70)

which states that the liquid mass flow rate $z=L_2$ is zero and is consistent with Eq. (19).

The above equations and the boundary conditions have been solved using the standard Runge–Kutta procedure and the shooting method on parameter $m_{in}$ to satisfy Eq. (70). All the unknown variables were found with the accuracy of 0.0005%. During the numerical procedure, the step size of $dz$ was $10^{-7}$ m, and the thermophysical properties of the saturated vapor and liquid were recalculated for each of the points at the corresponding vapor temperature $T_v$. $L_2$ was determined by checking if $\delta_p$ was thick enough for the film evaporation and condensation models with meniscus; if the liquid slug is moving backward, solve for $Q_{evp}$ and $Q_{cond}$ from thick film evaporation and thin film condensation models with meniscus.

Obtain the new masses of the two vapor plugs $m_{i1}$ and $m_{i2}$ by accounting for the change of vapor masses from Eqs. (12) and (13).

4.1 Solve the liquid temperature distribution from Eq. (72) and calculate $Q_w$ and $Q_v$ with Eqs. (76) and (77).

After the time-step independent test, it was found that the time-step independent solution of the problem can be obtained when the time-step is $\Delta t=1 \times 10^{-3}$ s, which is used in all numerical simulations.

3 Numerical Procedure

The governing equations in the physical model above can be solved numerically. An explicit finite difference scheme is employed to solve the governing equations of the vapor plugs and the liquid slug. An implicit scheme with uniform grid [20] is employed to solve the transient heat transfer in the liquid slug. The results of each time-step can be obtained by the following numerical procedure outlined below.

1. Assume the two vapor plug temperatures $T_{v1}$ and $T_{v2}$.
2. Check the moving direction of the liquid slug. If the liquid slug is moving forward, solve for $Q_{evp}$ and $Q_{cond}$ from thin film evaporation and condensation models with meniscus; if the liquid slug is moving backward, solve for $Q_{evp}$ and $Q_{cond}$ from thick film evaporation and thin film condensation models with meniscus.
3. Obtain the new masses of the two vapor plugs $m_{i1}$ and $m_{i2}$ by accounting for the change of vapor masses from Eqs. (12) and (13).
4. Calculate the pressure of the two vapor plugs $p_{v1}$ and $p_{v2}$ from Eqs. (8) and (9).
5. Solve for $T_{v1}$ and $T_{v2}$ from Eqs. (10) and (11).
6. Solve for $x_p$ from Eq. (1).
7. Compare $T_{v1}$ and $T_{v2}$ obtained in step 6 with assumed values in step 1. If the differences meet the small tolerance, then go to step 8; otherwise, the above procedure is repeated until a converged solution is obtained.
8. Solve for the liquid temperature distribution from Eq. (72) and calculate $Q_w$ and $Q_v$ with Eqs. (76) and (77).

4 Results and Discussion

The present model is used to simulate a vertically U-shaped minichannel with the following parameters: $L_c=0.1$ m, $L_v=0.2$ m, $d=3.34$ mm, $T_v=100°C$, $T_c=60°C$, $p_0=47,359$ Pa, $T_f=80°C$, $\theta_f=33$ deg, and $\theta_c=85$ deg. Unless stated, the above conditions remain the same for each case.

Figure 2 shows the variation of liquid slug displacement $x_p$, the vapor pressure $p_v$, and the vapor temperature $T_v$ with time when $T_v=90°C$ and $T_c=70°C$. The period of oscillation of the liquid slug is 0.081 s and the amplitude of the displacement of the liquid slug is 0.009 m. The highest and lowest temperatures of vapor plugs are 89.99°C and 73.08°C, respectively; the largest temperature drop between wall and vapor is only 17°C. The corresponding highest and lowest saturated vapor pressures are 53,550 Pa and 43,552 Pa.

Figure 3(a) shows the latent heat transfer into and out of the left vapor plug. It can be seen that when the left end of the liquid slug moves into the evaporator section, the evaporative heat transfer of vapor 1 starts at a small rate, which is due to the thin film and meniscus heat transfer mode. When the liquid slug reaches the leftmost point and then moves backward, the rate of evaporation increases sharply due to the thick film heat transfer mode as the liquid slug moves back and vapor temperature drops. As vapor 1 expands, its temperature drops and the rate of evaporation in-
creases until the left end of the liquid slug moves into the condenser section, where condensation occurs. The thin film condensation of vapor 1 starts and the condensation rate gradually increases since the length of the condensation film increases when the liquid slug moves to the right. Meanwhile, due to condensation and expansion, the vapor temperature decreases incessantly and hinders condensation. Therefore, the rate of condensation first reaches its maximum value and then decreases to its minimum value after the liquid slug approaches the rightmost point. When the liquid slug moves back, the temperature of vapor 1 decreases slightly and again raises due to compression; as a result, the condensation rate increases again. However, as the liquid slug returns to the evaporator section, the length of the condensation film decreases to zero, condensation stops, and evaporation starts again. As can be seen, the length of the condensation film and temperature difference determine the condensation rate; the temperature difference and the film thickness determine the evaporation rate because the thickness of the evaporation film is small compared with that of the thin film condensation.

Figure 3(b) shows the sensible heat transfer into and out of the liquid slug. It can be seen that the contribution of sensible heat on the total heat transfer amount is larger than that of latent heat. The average heat transferred due to evaporation and condensation for one period of the heat pipe operation is 0.46 W. The average sensible heat transfer for the same time period is 2.01 W, which represents an overall contribution of 81.8%. It can be concluded that the heat transfer in an OHP is mainly due to the exchange of sensible heat. The role of thin film evaporation and condensation on the operation of OHPs is mainly on sustaining the oscillation of the liquid slug so that heat transfer is highly enhanced, although the contribution of latent heat on the overall heat transfer is not significant.

Figure 4 shows the length of condensation and evaporation film thickness profiles versus $z$-coordinates and the length variation of thick film with time. In Fig. 4(a), a flat thin condensation film is formed before the meniscus and the film starts with a thickness that satisfies the length of the liquid film as well as the energy and mass balances. The averaged thickness of the flat thin film is around 100 $\mu$m through which heat flux is 1000 W/m$^2$ and ac-
cumulative condensation rate is 0.46 W. In Fig. 4(b), the thin film region of the liquid film is clearly demonstrated but the thickness of the thin film is very small (~1 μm) and short (~36 μm) compared with the condensing film. This short and thin film transfers 0.46 W while the long and thick condensing film transfers the same amount of heat. Figure 4(c) indicates that the length of the thick film is longer with smaller temperature difference between vapor plug and evaporator section and shorter with larger temperature difference because when the temperature difference is high, the heat flux through the thick film is so high that most of the liquid supplied evaporates and cannot maintain the same length of the thick film. On the contrary, with small temperature difference, the heat flux through the thick film is lower so that sufficient liquid flows into the thick film and maintains a longer thick film. Although the length of the thick film with lower temperature difference is longer, the actual cumulative evaporation rate is small compared with that of the shorter thick film length and larger temperature difference. Since the thickness of the thick film varies around 1 μm, high temperature difference results in extremely high heat flux, which is about $10^5$ W/m². Temperature difference is necessary for high heat transfer through such a thick film but too large temperature difference restrains the formation of the thick film as it dries out the liquid film.

The influences of the evaporator temperature and the inner diameter on the oscillatory flow and heat transfer are then studied and the results are shown in Figs. 5–8. Figure 5(a) shows the effect of the evaporator temperature on the displacement of the liquid slugs. The increase of evaporator temperature has significant effects on the amplitudes and frequencies of liquid slug oscillation. When the temperature difference between evaporator and condenser is high, the vapor pressure varies in a larger range according to the temperature range so that larger pressure difference can be provided. As a result, the amplitude of oscillation is larger when the evaporator temperature is higher. The maximum displacement of the liquid slug oscillation with evaporator temperature of 120°C is 0.0217 m, which is larger than 0.0163 m and 0.0194 m, the maximum displacements when the evaporator temperatures are 100°C and 110°C, respectively. The period of liquid slug oscillation is 0.080 s when the evaporator temperature is 100°C, which is longer than the periods of 0.077 s and 0.079 s when the initial temperatures are 120°C and 110°C, respectively. Based on the period and amplitude of the liquid slug, it can be calculated that the average liquid slug velocities are 0.804 m/s, 0.982 m/s, and 1.127 m/s for 100°C, 110°C, and 120°C, respectively. The increase of the evaporator temperature accelerated the oscillation and enhanced the forced convection heat transfer.

Figures 5(b) and 5(c) show the influence of the evaporator temperature on the latent heat transfer of vapor 1. The latent heat transfer out of the vapor experiences a sharp increase in its magnitude with the increase of the evaporator temperature. Similarly, the latent heat transferred into the vapor shown in Fig. 5(c) experiences an increase in its magnitude. Figure 6 shows the influence of the evaporator temperature on the latent heat transfer of vapor 1 and the sensible heat transfer of the liquid slug. The sensible heat transferred into and out of the system also exhibits the similar trend as above, i.e., the sensible heat transfer rate increases with an increasing evaporator temperature. The system with an evaporator temperature of 120°C has higher average heat transfer rate of 52.59 W compared with 20.95 W and 33.54 W for the system with evaporator temperatures of 100°C and 110°C, respectively.

Figure 7(a) shows the influence of the inner diameter on liquid slug displacement. To study the effect of the diameter on the OHP performance, three different inner diameters of the tube are investigated: 2 mm, 3.34 mm, and 4 mm. Figure 7(a) shows that the amplitude of oscillation of the liquid slug decreases and the frequency of the oscillation of the liquid slug increases when the diameter of the tube decreases. The amplitude of liquid oscillation decreases with an increasing diameter: for the three diameters studied, the amplitudes are 0.0171 m, 0.0163 m, and 0.0158 m, respectively. The periods of the oscillation for the three diameters are 0.082 s, 0.081 s, and 0.080 s, respectively, i.e., the difference between these three cases is not significant. The decrease of the diameter has lesser effect on the frequency than on the amplitude of the liquid slug displacement. The variation of the diameter of the minichannel has great effects on both latent and sensible heat transfers. Figures 7(b) and 7(c) show that the average latent heat transfer rate for the diameter of 2 mm is 0.586 W, which is a 55.3% decrease from 1.31 W for the diameter of 4 mm and a
50.7% decrease from 1.06 W for the diameter of 3.34 mm. Figures 8(a) and 8(b) show the effect of the diameter of the minichannel on the sensible heat transferred into and out of the liquid slug. The average sensible heat transfer rate for the diameter of 2 mm is 11.6 W, which is a 48.1% decrease from 22.35 W for the diameter of 4 mm and a 41.4% decrease from 19.8 W for the diameter of 3.34 mm. The decrease of the total heat transfer rate is mainly due to the decrease of the heat transfer area of the miniature tube.

5 Conclusions

Heat transfer in the evaporator and condenser sections of an oscillating heat pipe are analyzed in this study. The evaporation and condensation heat transfer coefficients are obtained by solving the thin film in the evaporator and thin film in the condenser. The results confirmed that for all cases, the heat transfer in an OHP is mainly due to the exchange of sensible heat. The effect of evaporator temperature on the oscillation and heat transfer is very significant. The decrease of the evaporator temperature decreases the total heat transfer rate and oscillating frequency and dampens the amplitude of the oscillation. The variation of the diameter of the minichannel has a large impact on the heat transfer within the OHP while the oscillation is almost unaffected by the change of the tube diameter.

In Table 1, the heat transfer and oscillating flow performances under different operating conditions and assumptions are summarized. As can be seen, case 7 has the highest sensible heat transfer rate and largest amplitude of the liquid slug oscillation. As the temperature difference between evaporator and condenser increases, the proportion of the latent heat transfer in the total heat transfer decreases from 18.6% to 1.5%. When the evaporator temperature is decreased, both latent and sensible heat transfers are decreased. When the diameter decreases by half, the latent heat transfer decreases by half and the sensible heat transfer decreases by two thirds with a slowed down oscillation.
Nomenclature

\[ A \] = area, \( m^2 \)

\[ c_p \] = specific heat at constant pressure, \( J/kg \cdot K \)

\[ c_v \] = specific heat at constant volume, \( J/kg \cdot K \)

\[ d \] = diameter of the minichannel, \( m \)

\[ h_{lv} \] = latent heat of vaporization, \( J/kg \)

\[ k \] = thermal conductivity, \( W/m \cdot K \)

\[ \rho \] = density, \( kg/m^3 \)

\[ \mu \] = dynamic viscosity, \( N \cdot s/m^2 \)

\[ \nu \] = kinematic viscosity, \( m^2/s \)

\[ \rho_v \] = density of vapor, \( kg/m^3 \)

\[ \rho_l \] = density of liquid, \( kg/m^3 \)

\[ \rho_{lv} \] = density of liquid-vapor mixture, \( kg/m^3 \)

\[ \gamma \] = surface tension coefficient, \( N/m \)

\[ \theta \] = contact angle

\[ \theta_c \] = critical angle

\[ \theta_m \] = meniscus angle

\[ \theta_m^c \] = critical meniscus angle

\[ \phi \] = phase angle

\[ \phi_m \] = meniscus phase angle

\[ \phi_m^c \] = critical meniscus phase angle

\[ \chi \] = accommodation coefficient

\[ \chi_c \] = accommodation coefficient for condensation

\[ \chi_e \] = accommodation coefficient for evaporation

\[ K \] = curvature, \( 1/m \)

\[ L \] = length, \( m \)

\[ L_1 \] = length of thin film in the condenser section, \( m \)

\[ L_2 \] = length of thin film in the evaporator section, \( m \)

\[ m \] = mass of vapor plugs, \( kg \)

\[ m_{\dot{\theta}} \] = mass flow rate, \( kg/s \)

\[ N_u \] = Nusselt number

\[ p \] = vapor pressure, \( Pa \)

\[ Q_{cond} \] = condensation heat transfer, \( W \)

\[ Q_{evp} \] = evaporation heat transfer, \( W \)

\[ Q_{in} \] = sensible heat transfer into the liquid plug, \( W \)

\[ Q_{out} \] = sensible heat transfer out of the liquid plug, \( W \)

\[ Q_{s} \] = sensible heat transfer, \( W \)

\[ Re \] = Reynolds number

\[ R_g \] = gas constant, \( J/kg \cdot K \)

\[ R \] = radius of the minichannel, \( m \)

\[ r \] = radial coordinate, \( m \)

\[ s \] = axial coordinate in meniscus region, \( m \)

\[ t \] = time, \( s \)

\[ T \] = temperature, \( K \)

\[ v_p \] = velocity of the liquid plug, \( m/s \)

\[ x_p \] = displacement of the liquid plug, \( m \)

\[ \lambda \] = liquid thermal diffusivity, \( m^2/s \)

\[ \alpha \] = accommodation coefficient

Greek Symbols

\[ \alpha_l \] = liquid thermal diffusivity, \( m^2/s \)

\[ \alpha \] = accommodation coefficient
### References


### Table 1 Summary of heat transfer and fluid flow in a U-shaped minichannel

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<th>$T_e$ ($^\circ$C)</th>
<th>$T_i$ ($^\circ$C)</th>
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<th>$Q_{\text{tep}}$ (W)</th>
<th>$Q_{\text{cool}}$ (W)</th>
<th>$Q_t$ (W)</th>
<th>$Q_f$ (W)</th>
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<th>$Q_{\text{tr}}$ (W)</th>
<th>$Q_{\text{am}}$ (%)</th>
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Subscripts

0 = initial condition
a = advancing
c = condenser
cap = capillary
d = disjoining pressure
e = evaporator
i = index of vapor slugs
in = inlet
l = liquid
p = plug
r = receding
sat = saturated
tr = transient point
v = vapor
w = wall of the tube
$\delta$ = interface

$\gamma$ = ratio of specific heats
$\delta$ = liquid film thickness, m
$\Delta$ = slope of liquid film
$\Delta p_b$ = pressure loss at bend, Pa
$\zeta$ = loss coefficient
$\theta$ = contact angle
$\mu$ = dynamic viscosity, N s/m$^2$
$\rho$ = density, kg/m$^3$
$\sigma$ = surface tension, N/m
$\tau$ = shear stress, N/m$^2$