Foundations of the Formal Number Concept: How Preverbal Mechanisms Contribute to the Development of Cardinal Knowledge

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INTRODUCTION

An understanding of number is one of the most important capacities to develop in early childhood. It provides the foundation for the uniquely human ability to engage in formal mathematics, and is critical to successful functioning in modern society. Children’s mathematical skills at school entry predict mathematics achievement throughout schooling (Duncan et al., 2007; Geary, Hoard, Nugent, & Bailey, 2013), and continue to predict economic success and social status into adulthood (Bynner, 1997; Ritchie & Bates, 2013). But what skills predict school entry abilities? Where do number concepts come from, and how do they develop?

The answers to these questions have been the focus of substantial debate for decades. In this chapter, I argue that the foundations for the formal mathematical skills children acquire through instruction depend in part on nonverbal systems that are present at birth (Antell & Keating, 1983; Izard, Sann, Spelke, & Streri, 2009) and exist even in nonhuman animals (Brannon & Merritt, 2011; Brannon & Roitman, 2003). In some ways, the ubiquity of informal, nonverbal quantification abilities throughout development and throughout the animal kingdom is not surprising (Agrillo et al., this volume; Beran et al., this volume; Geary et al., this volume; Pepperberg, this volume; Vallortigara, this volume). Quantities like amount, time, and number are basic aspects of perceptual experience that reflect fundamental attributes of the environment (Dehaene & Brannon, 2011; Gallistel, 2009, 2011). In order
for any organism to successfully navigate and function in the environment, it
must represent behaviorally relevant quantitative information. Contexts such
as foraging (e.g., maximizing amount of food per time spent at various loca-
tions; Gallistel et al., 2007; Gallistel, Mark, King, & Latham, 2001), determin-
ing the number of friends versus foes (ordinal judgments), and keeping track
of number of offspring (cardinality), are faced by many species (Gallistel,
1990, 2009). More generally, being able to represent quantitative information
routinely and with reasonable accuracy clearly confers an adaptive advantage
(Gallistel, 1990, 2012).

Historically, psychologists have been reluctant to attribute numerical
knowledge, in particular, to nonhuman animals (Davis & Perusse, 1988).
The assumption was that number is abstract, and therefore, it cannot be repre-
sented without language. Nonetheless, the past few decades have seen a grow-
ing body of evidence suggesting that a wide variety of nonhuman species, as
well as preverbal human infants, not only represent quantity but also perform
quite sophisticated “computations” over these representations (for reviews,
see Gallistel, 1990, and Mou & vanMarle, 2013a). Of course, nonhuman ani-
mals never go on to learn formal mathematics or to verbalize the concepts
they appear to implicitly understand, whereas human children can and often
do learn to read and write numerals and to perform calculations by engaging
in the formal routines they acquire through instruction. One reason for this
difference between humans and other animals may be that humans are capable
(through language or other means) of forming generalizable concepts of card-
dinality (magnitude), ordinality (greater/less than relations), etc. (Penn,
Holyoak, & Povinelli, 2008). The focus here, however, is on the bridge
between the evolutionarily ancient, nonverbal systems, and the formal,
explicit concepts that children begin to acquire when they learn to count
(see also Starr & Brannon, this volume).

In this chapter, I first describe two nonverbal quantity systems and briefly
review evidence that they are available to preverbal human infants. I then argue
that above and beyond being sensitive to the attribute of numerosity, at least one
of the representational systems infants deploy in numerical tasks constitutes a
true concept of number, a claim that has been debated (Gallistel & Gelman,
2000; Gelman & Gallistel, 1978; Rips, Bloomfield, & Asmuth, 2008). Finally,
I describe recent findings showing that both nonverbal systems play a role in the
development of explicit cardinal knowledge and thus directly impact formal
mathematics learning in young children. Ultimately, understanding what
mechanisms underlie the number concept and whether and how they are related
to the acquisition of the earliest formal mathematical skills (e.g., learning to
count) will provide a locus to search for individual differences, potentially help-
ing educators identify children at risk for starting school behind, and providing
a target for early intervention in at risk children.

Before we delve into the argument, it is useful to first define some of the
terminology I use throughout this chapter. In particular, I often describe
preverbal infants’ performance and abilities using terms and phrases such as “compute,” “addition/subtraction,” “division,” and “engage in arithmetic operations.” In using these terms, I am not suggesting that infants can calculate the results of the formal arithmetic problems you might find on a grade school mathematics exam or consciously reason about mathematical operations in the same way educated children and adults do. Infants in their first year (the developmental period that the majority of this chapter focuses on) have not yet developed language; they do not recognize or understand written numerals or number words (spoken or written) and, therefore, are probably not capable of explicitly representing or thinking about numbers at a conscious level.

Instead, it is assumed here that there are algorithms built into the cognitive mechanisms whose function is to combine and evaluate inputs according to the rules of arithmetic (e.g., summing, multiplying, dividing) to produce outputs, which are the results of those processes. The infant likely does not know that it performs computations or how the computations are carried out. Because these processes are occurring at a level below conscious awareness, I refer to them as “implicit,” whereas “explicit” processes are those that are available for conscious reflection and verbalization, such as when a child recites the count words as part of the counting routine or writes down an addition problem using conventional written notation, e.g., \( 4 + 2 = 6 \). Finally, I use the terms “informal,” “nonverbal” or “preverbal,” and “compute” to refer to these implicit processes, and the terms “formal,” “verbal,” and “calculate” to refer to those processes that are explicit and that result from instruction in culturally transmitted knowledge of mathematics (e.g., counting, arithmetic).

TWO CORE MECHANISMS FOR REPRESENTING NUMBER

System 1: Analog Magnitude System (ANS)

After more than three decades of research, it is now clear that humans possess a nonverbal system for representing number and other quantities that is evolutionarily ancient and shared across a wide range of nonhuman animal species (Dehaene & Brannon, 2011; Gallistel, 1990). The analog magnitude system (ANS) is present from birth and continues to support numerical reasoning in adulthood (Cordes, Gelman, Gallistel, & Whalen, 2001; Izard et al., 2009; Whalen, Gallistel, & Gelman, 1999). It represents both discrete and continuous quantities as continuous analog magnitudes, in the same way a line can be used to represent a quantity, with its length being proportional to the number. For example, if a line that is 2 inches long represents the number “2,” the line representing “8” would be 8 inches long. Importantly, the magnitude representations are imprecise, and the amount of variability (i.e., error) increases in proportion to the magnitude. This means that the representation for 20 is twice as variable, or “fuzzy,” as the representation for 10. The
consequence of this *scalar variability* is that our ability to discriminate two numbers depends on their ratio, not their absolute difference (Dehaene & Brannon, 2011; Gallistel, 1990; Halberda & Odic, this volume).

In addition to being imprecise, another important feature of magnitude representations is that they are amodal. Adults can represent the number of dots in a visual array, the number of sounds played in a sequence (Barth et al., 2006), or the number of touches on one’s hand (Plaisier, Tiest, & Kappers, 2010). Above and beyond representing number, the ANS also represents continuous spatial and temporal quantities such as surface area and duration (Feigenson, Dehaene, & Spelke, 2004; Walsh, 2003). Representing more than one type of quantity using a single format makes it possible for various quantities to be compared and combined in arithmetical computations, such as generalizing, comparing, and summing quantities across different sensory modalities, as both human and nonhuman animals have been shown to do (Barth, Kanwisher, & Spelke, 2003; Beran, 2012; Meck & Church, 1983), and even across different types of quantities, such as computing rate from representations of time and number (Davison & McCarthy, 1988; Hernstein, 1961). Thus, the ANS represents a variety of quantities from different sensory modalities, and does so using a common mental currency.

There is substantial evidence for the ANS in infants. How do we know this? Because infants cannot speak and tell you, for example, whether two arrays of dots differ in number, the majority of studies exploring these abilities measure infants’ visual attention (i.e., their looking time to different displays) to assess their expectations about physical events or to determine whether they can discriminate two stimuli. Most of the studies reviewed here used one of two common paradigms: habituation (Fantz, 1964) or violation-of-expectation (VOE; Baillargeon, Spelke, & Wasserman, 1985). In habituation studies, infants first complete an habituation phase in which they are shown a particular stimulus repeatedly (e.g., 10 dots) over several trials, and their looking time is measured. Other features of the habituation displays may vary (e.g., the spacing of the dots, the size of the dots, the arrangement of the dots), but the number of dots is held constant. Just like adults, infants’ attention diminishes with repeated exposure, indicating they are becoming familiar with the stimulus (i.e., 10 dots). Once their looking time declines below a criterion level, they are considered “habituated” and they move to the test phase in which they are presented, on alternating trials, with the familiar (i.e., 10 dots) and a novel (i.e., 20 dots) stimulus. If infants can discriminate between the old and new stimulus, they are expected to recover interest, and therefore look longer, on novel compared to familiar trials.

Studies using the VOE method assume that infants generate expectations about physical events and look longer at events that appear to violate their expectations, just like children and adults do when they see a magician pull a rabbit out of an apparently empty hat! In the typical VOE study, infants are presented with events on a puppet stage involving real objects. The same
event usually is shown on every trial. For example, the infant might see an experimenter place two objects on the stage and then raise a screen to occlude them. After a brief pause, the screen is lowered to reveal an “expected” outcome (e.g., two objects), or an “unexpected” outcome (e.g., one object), on alternating trials. Infants’ looking time to the outcomes is measured. If infants hold the expectation that objects cannot magically disappear into thin air, then they should look longer at the unexpected outcome compared to the expected outcome. Looking time measures in general, and habituation and VOE procedures specifically, have transformed the study of infant cognition because of their ability to tap infants’ implicit knowledge (but see Schöner & Thelen, 2006), and continue to be widely used in infant research.

Using these basic methods, researchers have shown that like nonhuman animals and adults, infants can discriminate both continuous and discrete quantities (vanMarle & Wynn, 2006, 2009). Moreover, their number abilities are not limited to the visual domain, but extend to the auditory and tactile domains, as well (Feron, Gentaz, & Streri, 2006; Lipton & Spelke, 2003; Xu & Spelke, 2000). Importantly, their discrimination of quantities is ratio-dependent. Seminal work by Xu and colleagues (Xu, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005) shows that at 6 months of age, infants can discriminate numbers of visual items (i.e., dots) that differ by a 1:2 ratio, but not a 2:3 ratio. This discrimination function is also seen for both small (e.g., 2 vs. 4) and large (e.g., 8 vs. 16) numbers of sounds (Lipton & Spelke, 2003; vanMarle & Wynn, 2009), as well as for duration (vanMarle & Wynn, 2006), suggesting that like nonhuman animals, infants’ ANS uses a common format to represent quantities across different sensory modalities and across different quantitative dimensions. Further, there is a developmental improvement in the precision of infants’ number representations that increases substantially over the first year of life and continues to increase throughout childhood (Halberda & Feigenson, 2008). At birth, infants require a 1:3 ratio to reliably discriminate numbers, but succeed at 1:2 ratios by 6 months and 2:3 ratios by 9 months (Izard et al., 2009; Lipton & Spelke, 2003); the same is true for duration, with infants needing at least a 1:2 ratio at 6 months, but succeeding with 2:3 ratios by 9 months (Brannon, Suanda, & Libertus, 2007; vanMarle & Wynn, 2006).

Using a single format allows infants to recognize correspondences, or the equivalency in number, across various sensory modalities, such as matching the number of sounds they hear with the number of objects they see, an ability that is available at birth (Izard et al., 2009). Other studies have shown similar cross-modal matching abilities in 5- to 8-month-olds using a variety of stimuli (Feigenson, 2011; Jordan & Brannon, 2006; Starkey, Spelke, & Gelman, 1983, 1990), including comparisons of tactile to visual stimuli (Feron et al., 2006). Beyond recognizing correspondences, infants are also able to engage in nonverbal arithmetic. Work by McCrink and Wynn (2004) showed that infants are sensitive to computer-animated addition (5 + 5) and subtraction
events (10–5), and discriminate between numerically correct (e.g., $5 + 5 = 10$, $10–5 = 5$) and numerically incorrect outcomes (e.g., $5 + 5 = 5$, $10–5 = 10$). For example, when 5 rectangles moved behind a screen, and then 5 more hid behind the same screen, infants looked longer when the screen lowered to reveal only 5 rectangles compared to when 10 were revealed. Building on this finding, McCrink and Wynn (2007) went on to show that 6-month-old infants correctly discriminated ratios that represent a division of two object collections. When familiarized to arrays of blue and yellow dots, where the total number of blue and yellow dots changed across displays (e.g., 10:20, 20:40, 5:10), infants were able to abstract the common ratio (1:2) and generalize this learning to new displays, looking longer at new displays with a novel ratio (15:60), compared to new displays that bore the same, familiarized ratio (15:30). Importantly, infants’ arithmetic performance is ratio-dependent and follows the same discrimination function as for simple number discrimination (success with 1:2 ratios and failure with 2:3 ratios), suggesting that the ANS underlies infants’ sensitivity to the outcomes of arithmetic events involving collections of objects or sounds (i.e., implicit addition, subtraction, and division; McCrink & Wynn, 2007; McCrink, this volume).

Despite the evidence that infants can enumerate entities from various sensory modalities, some researchers remain skeptical about whether infants in these studies are responding on the basis of number, per se, or to some non-numerical, low-level perceptual property of the displays. For example, there are many studies attempting to show that spatial dimensions that naturally co-vary with number (e.g., surface area, density, perimeter) are more salient to infants than the abstract property of number (Clearfield & Mix, 1999, 2001; Mix, Huttenlocher, & Levine, 2002; cf. Cordes & Brannon, 2008, 2009, 2011). Two ongoing studies in my lab address this debate by testing whether infants’ ANS representations are truly abstract. In one study, we explore whether infants can extract ratio information separately from visual and auditory displays, and then detect correspondences across the modalities (Mou & vanMarle, 2013b). To test this, researchers simultaneously presented 6- and 10-month-old infants with two visual displays, each with an array of red and blue dots. The two displays differed only in the ratio of red:blue dots (the total number of dots is held constant, and non-numerical features, such as surface area and density, are controlled). While the displays were visible, an auditory stimulus cycled in the background that contained pitch information matching one of the two visual displays in ratio (i.e., the ratio of high:low pitches was the same as the ratio of red:blue dots). As in previous cross-modal matching studies, infants looked longer at the visual displays that matched the auditory stimulus in ratio, indicating that they not only can extract the ratios from both types of sensory input, but also compare them and recognize when they correspond. Once again, we observe the same ratio-dependent discrimination as found for simple numbers and unimodal ratio discrimination (e.g., success with 1:2 ratios, but not 2:3 ratios, at 6 months, and success with both
at 10 months; Mou & vanMarle, 2013b). Because the low-level perceptual properties of the visual displays and auditory sequences share nothing in common, infants’ ability to detect correspondences must be based on their sensitivity to the abstract relations of the ratios of pitches and dots, suggesting that ANS representations are highly abstract, even in preverbal infants.

In a separate study, we are exploring whether infants can perform implicit computations involving different quantitative dimensions. In particular, we ask whether 11-month-olds can combine their representations of number and time in order to estimate the number of dots that should appear in a given amount of time, which would suggest sensitivity to rate. The ability to estimate rate has been widely observed among nonhuman animals, as related, for example, to foraging behaviors (see Davison & McCarthy, 1988, for review). To test this in infants, we first familiarize them to computerized displays in which dots appear on the screen one at a time, while a tone plays continuously. The rate is the number of dots per the tone duration. During familiarization, the number of dots (e.g., 4) and the tone duration (e.g., 1250 ms) is held constant. Once familiarized, the infant sees new animated displays in which an occluder rises to hide the center of the screen (where the dots appear). A tone that is either 2x or 4x the familiarized duration (e.g., 2500 ms or 5000 ms, respectively) then plays, depending on test condition. Finally, the occluder lowers to reveal either 8 dots or 16 dots. If infants are sensitive to the rate of the appearance of the dots in the familiarization phase, then in the 2x condition they should expect there to be 8 dots behind the occluder and look longer at the unexpected outcome of 16 dots. Infants in the 4x condition should show the opposite pattern because they should be expecting 16 dots, and should find the 8 dots unexpected. Pilot data support our hypothesis that infants, like animals, can compute rate (Seok & vanMarle, 2014). If these results are replicated in the current study, it will strongly suggest that infants are not only sensitive to the numerical ratio of collections of objects (e.g., “4 blue: 8 yellow”), but also to the relation between number of items and the time needed to present them (i.e., the ratio of “4 dots: 2 seconds”), which would imply again that the ANS is highly abstract.

Together, these studies indicate that from birth, infants can engage in sophisticated quantitative reasoning using the ANS. Like adults and nonhuman animals, they can represent both discrete and continuous quantities, detect correspondences across sensory modalities, and even perform arithmetic computations, including combining representations across different quantitative dimensions.

**System 2: Object Tracking System (OTS)**

The ANS has undoubtedly received the lion’s share of the attention in the infant quantity representation literature, and for good reason. If the animal
numerical cognition literature can be used as a guide, there is no need to posit a second system (see also Posid & Cordes, this volume). In animals, there is clear evidence that the ANS represents values throughout the number range, and with the exception of a handful of studies (for review, see Mou & vanMarle, 2013a), there is comparatively little evidence that nonhuman animals routinely use a limited-capacity object tracking system (OTS) to represent and reason about quantities (but see Agrillo and colleagues, this volume). The data for infants is less clear, however, with more support for the OTS in quantitative contexts than seen in the animal literature. And, given the recent hypothesis forwarded by Mou and vanMarle (2013a) that the OTS’s role in numerical reasoning is limited primarily to early development, largely disappearing as the ANS gains in precision over the course of the first few years of life, it is important to objectively evaluate the possibility that the OTS may contribute to the development of the number concept.

The object tracking system (OTS) is a recent model that is derived from two well-known theories in the literature on visual attention in adults: the FINST model (Pylyshyn & Storm, 1988) and the object file model (Kahneman, Treisman, & Gibbs, 1992). The OTS consists of a set of indexes that can be used to “point” to objects in the world. Importantly, the indexes are “sticky” and remain fixed on an object as it moves around in space. The indexes can also store featural information about the object (e.g., color, size, orientation, kind), but the location information (i.e., where the index is pointing) is prioritized (Flombaum, Scholl, & Santos, 2009; Scholl, 2001). Functionally, the OTS makes it possible to keep track of the same individual over time, even when that individual goes out of sight for brief periods of time (e.g., when a squirrel runs behind a tree), and even when that individual undergoes radical feature changes (e.g., “it’s a bird, it’s a plane, it’s Superman!”; Kahneman et al., 1992, p. 17).

The most important feature of the OTS is its limited capacity. It can track only as many objects as it has indexes, which in human adults appears to be about four (Pylyshyn & Storm, 1988; Scholl, 2001). However, unlike the ANS, the OTS did not evolve to represent quantity. Indeed, there is no provision in the model for generating representations of continuous spatial quantities (e.g., surface area, volume), although they may be bound to active indexes as features, and it represents discrete quantity (i.e., number) at best only indirectly. For example, you might have three active indexes tracking three dogs running across the street. If all three dogs run behind a house, but only two reappear, you would likely infer that the third was still behind the house. At a mechanistic level, you would make this inference based on the fact that only two of your active indexes were able to reacquire their targets once the dogs reappeared, creating a mismatch between your mental representation (three active indexes pointing at three different dogs) and the actual state of the world (two dogs visible). Importantly, it is only by means of this one-to-one correspondence process that the OTS can be said to represent number at all.
Even so, in contrast to the fuzzy nature of ANS representations, the OTS represents small sets precisely, indexing exactly one or exactly two items, up to its capacity limit.

The evidence for the object tracking system in infants comes primarily from two counterintuitive findings. First, several elegant studies by Feigenson and colleagues (Feigenson, Carey, & Spelke, 2002), as well as work by Xu and colleagues (Xu, 2003; Xu, Spelke, & Goddard, 2005), have shown that 6- and 7-month-old infants often fail to discriminate small numbers of visual items (e.g., 1 vs. 2), even when the same ratio, but for larger sets, would be highly discriminable. Such a finding is inconsistent with ratio-dependent performance and therefore is inconsistent with the ANS model. A second striking finding is based on infants’ failure in some cases to represent values beyond three. In particular, Feigenson, Carey, and Hauser (2002) showed that when 10- to 12-month-old infants were allowed to choose between two sequentially hidden sets of food items, they chose the larger amount, but only as long as neither set was larger than 3. They reliably chose 2 crackers over 1, and 3 over 2, but chose randomly when faced with comparisons like 3 vs. 4, 2 vs. 4, 3 vs. 6, and remarkably, even when choosing between 1 vs. 4 crackers, which is a ratio even newborn infants can easily discriminate (Izard et al., 2009). This performance pattern, in which infants succeed with small sets and fail when either set is outside the capacity limit of the OTS, is termed the set size signature, and is thought to indicate that the infant used the OTS and not the ANS to represent and compare the sets.

Several other studies also suggest that infants possess and use the OTS to track small sets of visual objects. Wynn’s (1992a) classic finding in which 5-month-old infants were reported to be sensitive to simple additions and subtractions of objects from a small set of objects has been explained by appealing to this mechanism. In her experiment, infants tracked visual objects on a puppet stage as they moved around and were briefly occluded. In the “addition” events, infants initially saw one object that was subsequently hidden behind a screen, and then a second object was added behind the screen. When the screen was removed to show the outcome, infants looked longer when only one object was revealed (unexpected, $1+1=1$), compared to when two objects were revealed (expected, $1+1=2$). The “subtraction” condition was the same except two dolls were initially placed and then hidden, and then one was removed from the hidden set before the screen was lowered to reveal one object (expected, $2–1=1$) or two (unexpected, $2–1=2$). Infants in both conditions looked longer at the unexpected outcomes, suggesting they kept track of the objects and detected when one had magically appeared ($2–1=2$) or disappeared ($1+1=1$). Infants’ performance in this task is consistent with the OTS. Like adults, infants presumably assigned an index for each object as it was brought out on the stage. When the screen hid the objects, the indexes continued to point to them. And when the screen was finally lowered, the active indexes were put in one-to-one correspondence...
with the revealed object(s). Mismatches were detected when the wrong number of objects was revealed, leading to longer looking to unexpected outcomes compared to expected outcomes.

Wynn’s (1992a) findings have been replicated and extended in various ways that are consistent with the OTS. For example, infants appear to prioritize spatiotemporal information (i.e., an objects’ location), looking longer when the wrong number of objects is revealed even if the items changed features behind the screen (Simon, Hespos, & Rochat, 1995), and even when the objects’ locations were constantly changing (Koechlin, Dehaene, & Mehler, 1997). Because the objects in Wynn’s original study were always placed in the same locations on the stage, infants’ longer looking to incorrect outcomes may have been due to attention to whether a given location is empty or filled, rather than attention to the number of objects, per se. Koechlin et al.’s study showed this was not true, and that infants were tracking the individual objects, not the locations on the stage. Thus, infants appear to respond on the basis of number, and not on the basis of missing or extra features, or filled or empty locations. In addition, infants in the original study represented the number of objects precisely. Not only did they look longer when \(1 + 1 = 1\) compared to when \(1 + 1 = 2\), but they also looked longer when \(1 + 1 = 3\) (Wynn, 1992a). This suggests they were not merely expecting there to be more objects behind the screen following the addition event but were expecting “exactly two,” reflecting the characteristic precision with which the OTS represents small sets.

More recent studies using the same addition/subtraction paradigm suggest interesting limitations to infants’ object tracking capacity. For instance, similar to adults (vanMarle & Scholl, 2003), infants’ performance breaks down when a tracked object violates cohesion (i.e., falls apart or disintegrates, like a pile of sand when moved; Chiang & Wynn, 2000; Huntley-Fenner, Carey, & Solimando, 2002), indicating that the OTS may only track objects that obey Spelke’s object principles (Spelke, 2000) of cohesion (i.e., objects maintain boundedness; Cheries, Mitroff, Wynn, & Scholl, 2008), solidity (i.e., objects cannot pass through each other; Mitroff, Wynn, Scholl, Johnson, & Shuwairi, 2004), and continuity (i.e., objects must travel continuous paths through space and time; Cheries, Mitroff, Wynn, & Scholl, 2009).

Another limitation affects infants’ ability to apply one-to-one correspondence when some features of the objects change, and in particular when continuous spatial features (e.g., surface area, perimeter) change while the objects are occluded. In a study by Feigenson, Carey, and Spelke (2002), 7-month-old infants successfully discriminated 1 from 2 objects, but only when the size of the objects did not change. If the objects changed size, infants failed. The same was true in the addition-subtraction paradigm. If one medium-sized object was added to one medium-sized object, and then one large object or two small objects were revealed (equated for total surface area), infants looked equally at the expected and unexpected outcomes. The fact that infants
are able to use one-to-one correspondence successfully despite changes in surface features (i.e., color/pattern; Simon et al., 1995), but not changes in spatial dimensions, provides further evidence for the processing preference for spatiotemporal information in the OTS.

In sum, the OTS appears to be an important part of infants’ early ability to track objects, and at least indirectly, represents the number of items up to the capacity limit of three items. However, in order to be useful as a foundation for the formal number concept, the OTS needs to do more than just track individuals.

“NUMBER CONCEPT” DEFINED

Before I discuss whether either of the two systems might support a number concept, it is useful to briefly discuss what is meant here by “number concept.” Developmentalists have long argued about whether children possess a true concept of number prior to formal schooling, and if so, when it emerges and by what processes. The various accounts reach to both extremes and everything in between, from suggesting that the number concept is innate (Leslie, Gallistel, & Gelman, 2007; Leslie, Gelman, & Gallistel, 2008), to suggesting it is built over the course of the first few years through abstraction and induction of principles available in the two innate core mechanisms (Carey, 2004; Carey & Sarnecka, 2006; LeCorre & Carey, 2007; Spelke, 2011; Spelke & Tsivkin, 2001), to suggesting that it is not until children receive schooling and learn the principles of mathematics as a formal system that they come to have a true concept of number (Rips, Asmuth, & Bloomfield, 2006, 2008; Piaget, 1952).

To my mind, the extreme views in which children have to explicitly understand formal mathematical axioms (e.g., that there is a unique first number, that each number has a unique successor, etc.; Rips et al., 2008) are too stringent. Rips et al., (2008), for example, claim that their view requires only that children be competent of the formal axioms at an implicit, subconscious level. Yet, they go on to assert that it is not enough for children to possess the various axioms themselves. Instead, children must possess a representation of the full set of axioms; that is, they have to know that they know the axioms. Similarly, views suggesting that children do not have a number concept unless they can engage in explicit verbal counting (e.g., Carey, 2004; LeCorre & Carey, 2007) are too stringent because there may be an age at which children possess a number concept and all the necessary principles, but still be unable to act on it, a typical competence-performance dilemma. Indeed, as Gelman and colleagues (Gelman, 1972, 1993; Leslie et al., 2008) have pointed out repeatedly, the principles within a conceptual domain should not have to be represented explicitly, either symbolically or linguistically, particularly for an evolved domain. In Gelman’s view, it is enough for the principles to be structurally inherent or implicit in the developmental mechanism, guiding
reasoning and behavior, but unavailable to conscious reflection or verbalization. This is the view I adopt here, precisely because it is rigorous and requires the organism to possess all the necessary axiomatic principles (those that define the natural numbers), but also because it is not overly strict. Because the principles may be realized implicitly, and their existence is observed through behaviors guided by these implicit processes, Gelman’s view does not a priori rule out species for whom, or ages at which, explicit reproduction of the principles is impossible or improbable. It leaves open as an empirical question the possibility that nonverbal animals and preverbal human infants may possess the number concept, but not as explicitly as evaluated in a school setting. As will become clear in the following sections, this claim is central to the present chapter.

I therefore assume Gallistel and Gelman’s (2000) view (see also Cordes, Williams, & Meck, 2007) that a nonverbal mechanism can instantiate the number concept inasmuch as the principles that define the domain are part of the structure and process of the mechanism; the child need not be able to reflect on them or consciously implement them in order for the concept to be in place. In the case of number, this means that the mechanism(s) must adhere to the basic counting (arithmetic) principles, including the stable order and abstraction principles. That is, the representations must meet all of the following criteria: (1) Abstract. The representations cannot be limited to specific sensory modalities or operate over a restricted set of entities. (2) Cardinality. Cardinality is an abstract property of sets that represents the total number of objects in a set, and allows sets to enter into arithmetic computations (i.e., addition, subtraction, multiplication, division). Number representations must denote the cardinality of the sets they represent. (3) Ordinality. Numbers form an ordered system; representations of number must be similarly ordered according to their cardinalities to allow judgments of “larger than” and “smaller than.” (4) Arithmetic. The representations must be able to enter into arithmetic operations. In particular, the representations must be subject to addition, subtraction, multiplication, and division. Meeting all four of these criteria will be taken to indicate that the system in question is capable of supporting a true number concept.

DOES THE ANS AND/OR OTS MEASURE UP?

Abstract Representations

The first requirement for a number concept is that the representations are abstract. The OTS fails to meet this criterion in every sense of the word—in terms of being modality-general, in the sense of representing more than one type of quantity, and in the sense of abstracting away from individual items, which is the very essence of what it means for numbers to be abstract. The OTS is not abstract in the sense of being “amodal” because it is, by definition,
a mechanism of visual attention. It is true that auditory information can be bound to object indexes in both adults and infants (Jordan, Clark, & Mitroff, 2010; Kobayashi, Hiraki, & Hasegawa, 2005; Kobayashi, Hiraki, Mugitani, & Hasegawa, 2004). However, the OTS does not track auditory individuals, but rather, uses auditory information as an additional cue in the service of its primary function, which is to track visual objects. Nor can the OTS be said to be abstract in the sense of representing more than one type of quantity. Even if we grant that it represents number indirectly (which has been challenged by Gallistel, 2007, and Leslie et al., 2007), it does not itself generate representations of continuous spatial quantities (surface area, volume, etc.) or duration, making it moot to even ask whether it can compute an abstract quantity such as rate. And, perhaps even more important, is that because the primary evolved function of the OTS to maintain the identity of individuals over time, the system is inherently non-numerical. As Gallistel (2007) so clearly stated: “The essence of numerical meanings is their abstraction from the particular. It is that abstraction that enables us to judge that the number of chairs is sufficient for the number of people present” (p. 445).

Clearly then, the OTS falls far short of meeting the criterion of abstraction. The ANS, on the other hand, meets the criterion in all respects. Infants represent numbers of visual items (Xu & Spelke, 2000), auditory items (Lipton & Spelke, 2003; vanMarle & Wynn, 2009), and also actions (i.e., puppet jumps; Wood & Spelke, 2005; Wynn, 1996), and already at birth, they can match the number of stimuli they hear with the number they see or feel (Feron et al., 2006; Izard et al., 2009). The ANS represents both discrete and continuous quantities, including duration (Feigenson et al., 2004), and I described preliminary evidence that infants are able to combine their representations of time and number allowing them to compute a rate (Seok & vanMarle, 2014). Together, such findings indicate a high level of abstraction, and underscore the fundamental utility of having a single representational currency. The ANS also captures the very essence of what it means for a number to be abstract. After enumerating a set of individuals, the resulting magnitude carries information only about the total number in the set, collapsing over the individuals that make up the set. Thus, the ANS stands as the paragon of abstraction, fulfilling the first criterion of the number concept, while the OTS does not.

Cardinality

A critical part of the number concept is the notion of cardinality, which in layman’s terms might be thought of as “number sense.” Cardinality is a property of sets and a cardinal representation refers to the total number of items in a set. To illustrate the nature of cardinality, consider that on some models (e.g., Gallistel & Gelman, 1992; Meck & Church, 1983), analog magnitude representations of number are formed through an iterative process analogous
to filling a beaker with water, one cupful for each item counted. After all the items have been counted, the height of water in the beaker will be proportional to the number of items counted; that is, the height is a representation of the cardinality of the set (Gallistel & Gelman, 2000). In fact, as continuous representations (e.g., height of water, length of a line), analog magnitudes carry information only about cardinality, and not any of the enumerated individuals. This is in stark contrast to the OTS, whose most central function is to maintain distinct representations of individuals. In fact, Gallistel (2007) has argued that the OTS cannot play any role in the acquisition of counting in children precisely because it does not have separate symbols to indicate how many indexes are active at any given time. Thus, it can detect a mismatch between a set of indexes and a set of objects, but it cannot represent how many objects there are in a set, making it unclear how it could ever imbue the number words with cardinal meaning, as some have suggested (e.g., Carey, 2004; LeCorre & Carey, 2007).

**Ordinality**

Because numbers form an ordered series, it is possible to judge whether one number is larger or smaller than another by knowing where they are located within the series. Children come to learn this explicitly only after they can recite the count list (Geary, 1994). But long before they have explicit knowledge of ordinality, they appear to understand ordinality implicitly (Brannon & Van de Walle, 2001; Bullock & Gelman, 1977; Huntley-Fenner & Cannon, 2000; Rousselle, Palmers, & Noel, 2004; Siegel, 1974; Sophian & Adams, 1987; Strauss & Curtis, 1981). It is well established that infants are sensitive to ordinal relations. For example, in their first year, they can discriminate ascending and descending sequences for both number of objects and other quantitative stimulus dimensions, such as object size (Brannon, 2002; Macchi-Cassia, Picozzi, Girelli, & de Hevia, 2012; Picozzi, de Hevia, Girelli, & Macchi-Cassia, 2010; Suanda, Tompson, & Brannon, 2008). It has been assumed that infants’ ordinal skills rely on the ANS, but no published studies have specifically tested whether their performance is ratio-dependent. In addition, as reviewed earlier, there are data to suggest that the OTS may support ordinal judgments, at least for small sets of visual objects (Feigenson, Carey, & Hauser, 2002). Recent work in my lab, however, shows that infants are not limited by set size when making ordinal judgments, as suggested by Feigenson, Carey, and Hauser (2002). When both sets are large, infants succeed and their performance is clearly ratio-dependent (vanMarle, 2013; vanMarle, Mou, & Seok, 2014; vanMarle & Wynn, 2011). The apparent set size signature instead may reflect the incommensurability of OTS and ANS representations. Mou and vanMarle (2013a) proposed such an idea, suggesting that the precision offered by the OTS allows it to trump the ANS for small visual sets early in life (roughly until about 2 years; vanMarle, Seok, &
Mou, 2014). Once the ANS becomes precise enough to reliably discriminate sets within the small number range, infants may abandon the OTS for making judgments that involve cardinality and ordinality. Further research is needed to test this hypothesis. An alternative possibility, not mutually exclusive to the first, is that the OTS may have attentional priority in infancy, perhaps being activated earlier than the ANS for small sets (Hyde & Spelke, 2011).

Arithmetic

As long argued by Gelman and colleagues (Gallistel & Gelman, 1992; 2000; Gelman & Gallistel, 1978; Leslie et al., 2007), in order for a representational system to constitute a number concept, the symbols it generates must be subject to arithmetic manipulation (see also McCrink, this volume). That is to say, they must count as inputs to computational processes that are isomorphic to addition, subtraction, multiplication, and division. In other words, the ANS supports nonverbal addition and subtraction (McCrink & Wynn, 2004), as well as division (ratio, McCrink & Wynn, 2004; Mou & vanMarle, 2013b; rate, Seok & vanMarle, 2014). Nonverbal multiplication has also been shown in 5- to 7-year-old children before they received formal schooling in mathematics (Barth, Baron, Spelke, & Carey, 2009; McCrink & Spelke, 2010). Researchers have also claimed that the OTS supports addition and subtraction as in, for example, Wynn’s (1992a) task described earlier (Feigenson, Carey, & Spelke, 2002; Scholl & Leslie, 1999; Simon, 1997; Wynn & Chiang, 1998). While the kinds of events in this task may be conceived as “addition/subtraction” events, they are not true arithmetic. True arithmetic requires operating over cardinalities. Although the process of one-to-one correspondence may very well drive longer looking to displays that mismatch the number of active indexes, or searching for objects in a box when three went in but only two came out, that does not make it, in and of itself, arithmetic (Gallistel, 2007).

So, where does this leave us? The ANS meets all four criteria and therefore, by the present definition, instantiates a true concept of number. It is abstract, in every relevant sense of the word; it generates representations that are cardinalities, albeit imprecisely; the cardinal representations it generates are inherently orderable and support ordinal judgments; and its magnitudes are subject to arithmetic manipulation. The OTS, on the other hand, meets none of the criteria. The mechanism is situated within the visual modality and represents other information (auditory, surface area, volume, color, etc.) only as features of visual objects. In addition, the OTS has no means for representing the cardinality of the sets it indexes and thus cannot order sets on the basis of their cardinality. It also does not support true arithmetic, at most, only detecting mismatches between objects in the world and active indexes via a one-to-one correspondence operation. It appears, therefore, that the ANS has everything to offer, and the OTS nothing (except
precision, perhaps), in the way of a preverbal number concept. But does it support the development of the formal number concept?

FOUNDATIONS OF THE FORMAL NUMBER CONCEPT

Children are not born knowing formal mathematics. Explicit knowledge of the verbal number system, arithmetic, algebra, geometry, and so on requires years of formal instruction and indoctrination into the relevant symbolic systems, theorems, and computational rules. The first step in this process begins before preschool when children start to learn number words and how to count. One important empirical question is whether the preverbal magnitude systems contribute anything to the acquisition of this knowledge (Starr & Brannon, this volume). As mentioned earlier, Rips and colleagues (Rips, Asmuth, & Bloomfield, 2006, 2008; Rips, Bloomfield, & Asmuth, 2008) argue that they do not. Other researchers argue that the preverbal systems have a very direct influence by providing the foundation on which the formal knowledge is built. There is substantial debate, however, about which system plays the lead role. Gelman and colleagues (Gallistel & Gelman, 1992, 2000; Gelman and Gallistel, 1978) suggest the ANS alone provides the foundation, whereas Carey and colleagues (Carey, 2004; LeCorre & Carey, 2007) claim that the OTS is the primary mechanism. And finally, Spelke and Tsivkin (2001; also Spelke, 2011) take a middle-of-the-road approach, suggesting that the two systems are complementary and together provide children the representational bases for learning to count.

Learning to count does not happen overnight. In fact, although children often can recite the verbal count list (i.e., “one, two, three, four, . . . ten”) as early as age 2, the number words do not initially carry any numerical meaning (Wynn, 1992b). When children are this age, reciting the count list is much like reciting the alphabet; it is just another ordered list that has been committed to memory. In time, children come to realize that number words are special and used in specific contexts (e.g., counting), after which the number words become imbued with meaning (i.e., cardinality), and children slowly, over the course of approximately 1½ to 2 years, explicitly come to understand the rules of counting (i.e., the counting principles, Gelman & Gallistel, 1978) and how to apply them (LeCorre & Carey, 2007; Wynn, 1992b). This slow learning progression is thought to be observable through their performance in the GiveN task (Wynn, 1992b). In this task, children are asked to give a puppet a specific number of items, e.g., “exactly one cookie.” Children start on set size 1 and move forward in the count list following a correct response. If they answer incorrectly (e.g., giving “three” cookies when asked for “two”), they move back to the previous set size. This continues until they successfully reach set size 6, or respond incorrectly on 2/3 attempts at a given set size. The highest set size for which they respond correctly at least 2/3 times is considered their “knower status.” For example, a child who can give
exactly one, two, and three items, but fails on set size four, would be considered a “three-knower.”

Performance on the GiveN task is characteristically stage-like (Wynn, 1992b). First, children learn the meaning of “one” but seem to think all the other number words mean some indeterminate number that is >1. Thus, they can give the puppet “exactly one” but typically give a handful of cookies when asked to give any other set size. After about 6 months, children progress to being “two-knowers” and can give exactly one and two cookies, but give a large and random number for all other set sizes. Several months later, they become “three-knowers.” And then suddenly, they seem to understand counting. Once they can give “exactly four,” they can typically also give “exactly five,” “six,” and any other number up to the limit of their verbal count list and are categorized as “CP-knowers” (i.e., cardinal principle knowers). This sudden shift is considered by some to be a genuine instance of conceptual change (Carey, 2004; Spelke & Tsivkin, 2001), with children moving from their limited, rudimentary, preverbal number capacities to a genuine, explicit (verbal) knowledge of counting, which equips them with a qualitatively different and more powerful means of conceptualizing number.

Although many studies have explored the nature of children’s early counting abilities, those arguing for the ANS-only or OTS-only models have depended primarily on indirect data (e.g., Carey, 2004; Gallistel & Gelman, 1992; LeCorre & Carey, 2007; Wynn, 1992b), and almost exclusively on children’s performance in a single task—the GiveN task. Because these studies did not include independent measures of ANS and OTS performance, they are limited to basing their conclusions on whether children’s performance in GiveN is consistent or inconsistent with either model. Recent work conducted by David Geary and myself (Chu, vanMarle, & Geary, 2013; vanMarle, Chu, Li, & Geary, 2014) moves beyond previous studies by exploring children’s performance on a range of quantitative tasks, including independent measures of the ANS and OTS, while controlling for a host of other general cognitive capacities (e.g., IQ, executive function, preliteracy aptitude). In addition, we followed children longitudinally, which allowed us to examine the relations between the ANS, the OTS, and cardinal knowledge over time.

Children in our sample completed each of three tasks (ANS, OTS, and GiveN) twice, at the beginning and near the end of their first year of preschool (roughly 4 years of age), allowing us to examine concurrent and prospective relations between the two systems and children’s cardinal knowledge. Our findings support a “dual-mechanism” view. At the beginning of preschool, both the ANS and OTS were related to children’s cardinal knowledge as measured by the GiveN task; however, by the end of the year, only the ANS remained a significant predictor of cardinal knowledge. Importantly, the likelihood that children transitioned from “non-CP-knower” (i.e., one-, two-, or three-knower) at the first time point, to “CP-knower” (i.e., four-, five-, or six-knower) at the second time point, was predicted by their ANS, but not OTS,
performance at the beginning of the year. In addition, the relations to cardinality at the first time point were stronger for the ANS than the OTS, suggesting that although both systems contributed to children’s learning the meanings of the first few number words, the ANS played a more substantial role (vanMarle, Chu, Mou, & Geary, 2014).

Our findings are clearly inconsistent with both the ANS-only (Gallistel & Gelman, 1992, 2005; Gelman & Gallistel, 1978) and OTS-only (Carey, 2004; LeCorre & Carey, 2007) models. If one mechanism was solely responsible for imbuing the count words with meaning, then it alone should be related to cardinal knowledge, especially at the first time point. Instead, our data support a “dual-mechanism” view, akin to that proposed by Spelke and Tsivkin (2001; Spelke, 2011) in which both systems are believed to play a role. Our view, however, termed the Merge model, differs from Spelke’s in at least two important respects. First, because language (i.e., the verbal count list) links the ANS and OTS, Spelke’s model predicts that both systems should continue to support cardinal knowledge over time, perhaps even into adulthood (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). Our data, however, suggest that only the ANS remains related to cardinal knowledge, with the OTS falling away relatively early, by the end of the first year of preschool.

Second, the Merge model (vanMarle, Chu, Mou, & Geary, 2014) posits that the transition to CP-knower status occurs not as the result of the ANS and OTS being combined through language, but rather as the result of the OTS fading into the background. It is our contention that rather than being complementary and cooperative, the OTS’s role early on is antagonistic, as it is in infancy (Feigenson, Carey, & Hauser, 2002; vanMarle, 2013; Xu, 2003; Xu, Spelke, & Goddard, 2005; for review, see Mou & vanMarle, 2013a). Our model explains the slow, piecemeal learning of the meanings of the number words as a consequence of this antagonism. Once the capacity limit of the OTS is breached, the OTS no longer competes with the ANS to represent the sets, and the cardinal meaning available through the ANS becomes more easily accessible, making it possible for the mapping mechanism to easily link the words with their meanings (vanMarle, Chu, Mou, & Geary, 2014). Part of the motivation for the Merge model comes from the fact that the ANS, but not the OTS, embodies the necessary attributes of a true concept of number. Given this, it is perhaps not surprising that the ANS is important for cardinality at both time points, while the OTS plays a role only at the beginning. Because the OTS really has just one advantage over the ANS—its precision—it plays an accordingly limited role in the building of the verbal number concept.

CONCLUSIONS

There is a wealth of data suggesting that infants possess two nonverbal systems that are capable of representing number. However, only the ANS
possesses the necessary qualities to be considered a system that instantiates a true concept of number. The OTS falls short in almost every respect. One question that has been debated for decades is which of these systems provides the foundation for children’s first lesson in formal mathematics, which involves developing an explicit understanding of the meaning of number words (their cardinal value), the rules of counting, and how to apply them. When children begin to learn the meanings of the count words, our findings suggest that both mechanisms take part in the process. However, unlike dual-mechanism views in which the two systems are complementary partners aiding in the development of cardinal knowledge, the Merge model suggests that the OTS is antagonistic to the ANS. This may turn what should be a relatively straightforward task of mapping number words onto the cardinal representations within the ANS into a lengthy and difficult process. The Merge model has yet to be directly tested. However, if it or a similar model were to be true, it would suggest that children could benefit from learning contexts that make it more difficult for the OTS to compete. For example, counting exercises that utilize two-dimensional images on cards rather than three-dimensional objects (i.e., manipulatives), or counting large as opposed to small sets, may help children by increasing the probability of engaging the ANS rather than the OTS. Further research will no doubt shed light on whether the Merge model proves useful as a theory, as well as on whether there are ways to enhance children’s ANS abilities to make it a stronger competitor in the early years.

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