Theoretical Foundations for Design of a Quantum Wigner Interferometer

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***Abstract*— In this paper, we discuss and analyze a new quantum-based approach for gravimetry. The key feature of this design is that it measures effects of gravitation on information encoded in qubits in a way that can provide resolution beyond the de Broglie limit of atom-interferometric gravimeters. We show that it also offers an advantage over current state-of-the-art gravimeters in its ability to detect quadrupole field anomalies. This can potentially facilitate applications relating to search and recovery, e.g., locating a submerged aircraft on the ocean floor, based on the difference between the specific quadrupole signature of the object of interest and that of other objects in the environment.**

***Index Terms*— Quantum sensing, gravimetry, qubits, quadrupole field anomaly, atom interferometry, Wigner gravimeter.**

I. Introduction

QUANTUM information [1], [2] and quantum sensing [3]–[9] have emerged as significant areas of interest as potential successors to current classical computing and sensor technologies, but many theoretical and practical obstacles remain. One proposed quantum-based technology exploits the fact that spin-based quantum information is intrinsically relativistic and therefore subject to gravitational fields according to Einstein's equations for General Relativity [10]. As such, gravitational fields can modify quantum information, which implies a novel basis for designing highly-sensitive *Wigner Gravimeters*. Such devices could have a significant impact on applications such as underwater navigation, target detection, and surveying [17].

II. The Wigner Rotation

Spin-based implementations of qubits are *intrinsically relativistic* and thus are subject to Lorentz transformations induced by gravitational fields in the form of a *Wigner Rotation* [10]. Expressing a qubit in vector notation as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{1}$$

permits the rotational effect of gravity on its state due to spacetime curvature to be expressed as:

$$|\tilde{\psi}\rangle = \begin{pmatrix} \cos\Omega/2 & \sin\Omega/2 \\ -\sin\Omega/2 & \cos\Omega/2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{2}$$

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where Ω is referred to as the *Wigner angle* of rotation. The value of Ω completely encodes the net rotation of the qubit due to interaction with a gravitational field. In most realistic contexts it would be extremely difficult, if not impossible, to estimate/predict with any practical fidelity the value of Ω a priori from extrinsic measurements of the gravitational field. Although the gravitational field could be thought of negatively as "corrupting" the state the qubit, it is also possible to interpret the altered state of the qubit positively as being a measurement of that field. In the case of a qubit orbiting the earth at the radius of a typical GPS satellite, the effect would only be on the order of 10^{-9} radians per day. That's certainly very small but the cumulative effect can be significant, which is why GPS satellites require relativistic corrections.

The critical point is that gravitation affects quantum systems, hence quantum information, in non-classical ways that can be interpreted either as something that can be advantageously exploited or as something deleterious that must be mitigated.

A. Classical vs. Quantum Systems in Gravity

The Wigner angle of a qubit rotates (precesses) in a non-trivial manner as the qubit moves along a curved manifold determined by its local gravitational field. This non-trivial rotation is purely relativistic and would remain strictly zero in a Newtonian model of gravitation. This can be understood by considering the spin of a particle orbiting at a fixed radius from a massive point or spherically-symmetric object. In a Newtonian model of gravitation the direction of spin of a particle is fixed with respect to the direction from the object to the particle, i.e., the direction of spin rotates with the particle. This is depicted in Figure 1, which shows that the spin is unchanged after a complete orbit ($\mathbf{S}_1 = \mathbf{S}_9$). This is the Newtonian principle of conservation of angular momentum which governs the behavior of gyroscopes.

In a relativistic model, by contrast, spacetime curvature affects spin in a way that does not guarantee that it returns to its original value after a complete orbit ($\mathbf{S}_1 \neq \mathbf{S}_9$). Specifically, Figure 1 shows that after the particle completes its orbital motion, the spin no longer points upward and Ω is instead proportional to the angle between $\mathbf{S}_1$ and $\mathbf{S}_9$. In the context of general relativity for classical macroscopic bodies, this effect is known as *geodetic precession* or *de Sitter precession* [15]. An explicit expression for Ω in the simple case of a spherically-symmetric gravitational field will be developed in the following section.

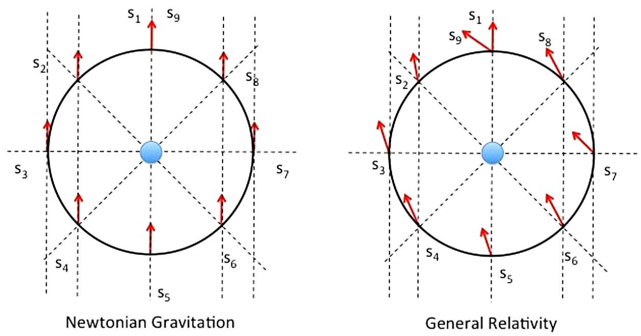

Fig. 1. Conceptual description of spin rotation in Newtonian gravity and in General Relativity. Notice that due to spacetime curvature, the spin does not reach its original value ($\mathbf{S}_1 \neq \mathbf{S}_9$) after completion of a single circular orbit. Once the orbit has been completed, the angle between $\mathbf{S}_1$ and $\mathbf{S}_9$ is proportional to the Wigner angle Ω.

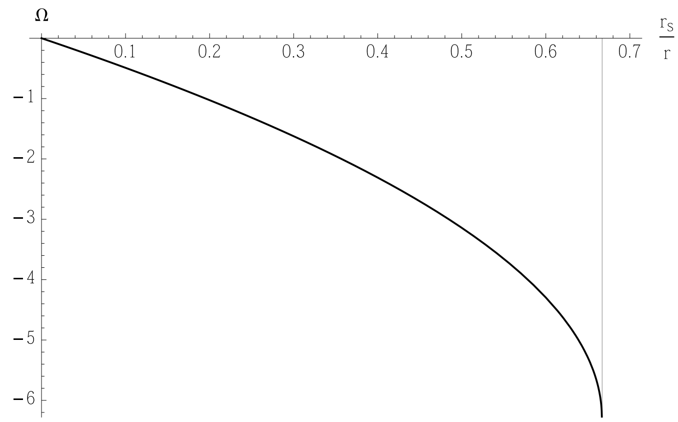

Fig. 2. The Wigner rotation angle Ω of qubit at completion of a full circular orbit of an object of mass $r_s/2$ at Schwarzschild radius r.

The effect of gravity on quantum information is clear. For instance, suppose a computational basis is defined using the direction established by the imaginary vertical line that connects an orbiting satellite with a distant star (see the parallel vertical dotted lines in Figure 1) as a quantization axis. In the case of Newtonian gravity the orientation of states prepared in the computational basis aboard the satellite will remain invariant as the satellite orbits the massive object. That is, if prepared/initialized in state $|0\rangle$ the particle will remain in the $|0\rangle$ state as the satellite performs its orbital motion. In the context of General Relativity, by contrast, the orientation of states prepared in the computational basis aboard the satellite will slowly rotate as the satellite orbits around the massive object. That is, as the satellite moves it becomes possible for an onboard observer to measure states orthogonal to the initial $|0\rangle$ state.

The effect of gravitation on spin-based qubits is of significance to quantum computing because their states will invariably drift during execution of an algorithm, and failure to mitigate this effect can undermine both the correctness and asymptotic computational complexity of the algorithm [16]. In the next section we examine in greater detail precisely how gravitational perturbations can be exploited for development of new quantum-based sensing technologies.

III. Gravitational Drifting of Qubit States

Given a qubit initialized to a uniform superposition, i.e., the states "0" and "1" have equal amplitude and thus have equal probability of being the measured state:

$$|\psi_1\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \tag{3}$$

then the effect of gravity can be parameterized with the Wigner angle Ω representing the drift of the qubit's state:

$$|\psi_1\rangle \longrightarrow \frac{\cos\frac{\Omega}{2} + \sin\frac{\Omega}{2}}{\sqrt{2}}|0\rangle + \frac{\cos\frac{\Omega}{2} - \sin\frac{\Omega}{2}}{\sqrt{2}}|1\rangle \tag{4}$$

Thus, as the value of Ω varies from 0 to $\pi/2$ the probability of measuring "1" is amplified while the probability of measuring "0" is commensurately diminished, and the reverse occurs as Ω exceeds π.

As a function of time, the Wigner rotation $\Omega(t)$ represents the change in a qubit's state as it traverses a gravitational field. The simplest example that can be analyzed is the case of a qubit orbiting a uniform sphere of mass M defined in the Schwarzschild metric [10], [12], [13]. If the orbit is circular with radius r then the Wigner rotation angle induced over each complete orbit can be expressed (assuming natural units $G = 1$ and $c = 1$) as:

$$\Omega = 2\pi\sqrt{f}\left(1 - \frac{K r_s}{2rf}\frac{1}{K + \sqrt{f}}\right) - 2\pi \tag{5}$$

where

$$K \equiv \frac{1 - \frac{r_s}{r}}{\sqrt{1 - \frac{3r_s}{2r}}} \quad f \equiv 1 - \frac{r_s}{r} \quad r_s \equiv 2M \tag{6}$$

Thus Ω here gives the net relativistic rotation due to interaction with the gravitational field, i.e., the effect of a spin-$\frac{1}{2}$ quantum field interacting with a classical gravitational field.

The value of Ω per circular orbital period of a qubit as a function of r_s/r (Schwarzschild spacetime) is shown in Figure 2. The probability of the qubit being measure in state "1" or "0" as a function of r_s/r in this scenario is shown in Figure 3. These probabilities can be approximated in the limit of small Ω as:

$$\begin{aligned} P_0 &= |\langle 0|\psi_1\rangle|^2 \approx \frac{1}{2} + \frac{\Omega}{2} \\ P_1 &= |\langle 1|\psi_1\rangle|^2 \approx \frac{1}{2} - \frac{\Omega}{2} \end{aligned} \tag{7}$$

where P_0 and P_1 are the probabilities of measuring "0" and "1", respectively. Thus, in a weak gravitational field the measurement error, e.g., the change in the probability of measuring "1," is approximately $\Omega/2$.

IV. Quantum Gravimetry

Although the effects of gravity on qubits are problematic for quantum computational applications [11], [16], they can in principal be exploited simply by interpreting them not as errors due to gravitation but rather as information about the local gravitational field of the qubits. In this section we show how

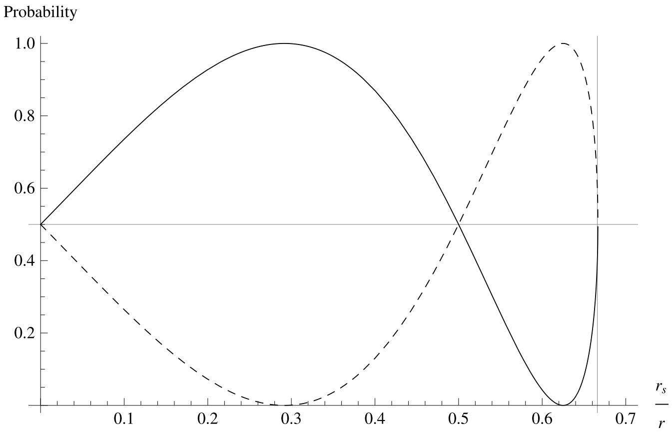

Fig. 3. Probability that a qubit (initialized to a uniform superposition) is measured in state "1" (solid line) and the probability it is in state "0" (dashed line) at completion of a full circular orbit of an object of mass $r_s/2$ at radius r in Schwarzschild spacetime.

this can be applied to design a new technology for quantum gravimetry, *Wigner gravimeters*, using amplitude amplification techniques, originally developed for quantum search in the form of Grover's algorithm, to *amplify* the effects of Wigner rotation [10], [18].

Current state-of-the-art approaches to gravimetry use atom interferometry ([19], [20]) to infer gravitational field characteristics from the motion of heavy atoms. As such, the information resolution is limited by the atom's de Broglie wavelength $\lambda = h/mv$, where h is Planck's constant and m and v are respectively the mass and velocity of the atom. In practice this implies resolution limit of around $\delta g/g \lesssim 10^{-9}$; however, this limit can potentially be exceeded by incorporating information about other quantum properties such as the Wigner rotation and entanglement. Specifically, assume a qubit is given in an initial state $|\psi\rangle = |0\rangle$, then once it has interacted with the environment the probabilities of measuring "0" and "1" are given by:

$$\begin{aligned} P_0(\Omega) &= |\langle 0|\hat{W}_\Omega|\psi\rangle|^2 \\ P_1(\Omega) &= |\langle 1|\hat{W}_\Omega||\psi\rangle|^2 \end{aligned} \tag{8}$$

where $\hat{W}_\Omega$ is the Wigner rotation operator. Thus if $\Omega = 0$ then $P_0 = 1$ and $P_1 = 0$, which correspond to the measurement probabilities of the undisturbed state, but then will change as the qubit interacts with the environment.

Given a sufficient number of single-qubit sensors it becomes possible to statistically estimate information about the rotation angle, which is related to the physical interaction being measured by the sensor. For this example, the value of Ω is approximately:

$$\Omega \approx 2\arccos\left(\sqrt{\frac{n_0}{n}}\right) \tag{9}$$

where n is the total number of qubit sensors and n_0 is the number of times that the measurement of the qubit sensor state resulted in "0".

As a consequence, the n qubits can be used to determine the angle Ω. The sensitivity of each qubit sensor can be expressed as:

$$\mathcal{S}(\Omega) \equiv \left|\frac{dP_0}{d\Omega}\right| \tag{10}$$

which shows that it can exhibit large changes in probabilities with respect to small changes in the rotation angle.

If an n-qubit system is in the initial "0" state:

$$|\Psi\rangle = |0\rangle^{\otimes n} = |00...00\rangle \tag{11}$$

and after interacting with the environment becomes:

$$|\Psi'\rangle = (\hat{W}_\Omega|0\rangle)^{\otimes n} \tag{12}$$

then the sensitivity of the system is:

$$\begin{aligned} \mathcal{S}_n &= \left|\frac{dP_0}{d\Omega}\right| \\ &= \left|\sum_{i,j,..,k=0}^{1} \frac{dP_{0ij...k}}{d\Omega}\right| \\ &= \left|\frac{dP_{00...00}}{d\Omega} + \frac{dP_{00...01}}{d\Omega} + ... + \frac{dP_{01...11}}{d\Omega}\right| \end{aligned} \tag{13}$$

where P_0 is the independent probability of measuring a "0" in the first qubit, i.e., regardless of what is measured in the subsequent qubits.

It is important to note that the expression for $\mathcal{S}_n$ is purely algorithmic in nature and only describes the operations required by the proposed protocol to estimate the sensitivity of the device. At this level of abstraction the system is essentially noiseless/perfect up to intrinsic quantum randomness, i.e., the expression of $\mathcal{S}_n$ implicitly assumes that the implemented measurement process does not introduce any unmodeled anisotropic dependencies with respect to the gravitational gradient. Thus, any concrete system specification would demand additional sensitivity analysis to be performed.

Clearly the *precision* required to compute the rotation angle will increase with n, but the *sensitivity* of an n-qubit system is exactly the same as the sensitivity of a single qubit. In other words, simply processing the bits that result from measuring the state of each of the qubit sensors can provide no sensitivity advantage whatsoever compared to having many sensors measuring the same physical property. As will be seen, however, the sensitivity of the system *can* be increased by adapting a quantum computer to process the qubit states of the quantum sensors.

Assume a collection of qubit sensors, as previously described, where the physical response of each sensor is represented by the rotation of a qubit state. If each sensor performs an independent measurement then it would only be possible to compute a simple mean of the measurements, which would yield no quantum advantage even though each individual quantum sensor may be superior to its classical counterpart. However, if each quantum sensor state is treated as an evolving qubit in a quantum computer then amplitude amplification can be performed before the measurements are completed, i.e., non-local quantum operations can amplify the response of the overall system via entanglement across the multiple sensors [18]. In other words, the system becomes a

TABLE I
EXPECTED PERFORMANCE OF THE TWO SENSITIVITY AMPLIFICATION ALGORITHMS DISCUSSED IN THE TEXT

Algorithm	Qubits	Oracle	Iterations	Sensitivity Amplification
1	n	N	$\mathcal{O}(n)$	$\mathcal{O}(n)$
2	n	Y	$\mathcal{O}(2^n)$	$\mathcal{O}(2^n)$

quantum computer for which the physical interaction acts as a computational oracle.[1]

Thus, the core of the proposed quantum gravimetry concept is a pair of quantum algorithms for performing operations on a system of single-qubit sensors that are immersed in a gravitational field. The result of these algorithms is an overall sensitivity amplification of the qubit sensor system. The expected performance of the system in an idealized noisless setting is summarized in Table I.

Numerical simulations suggest that the number n of qubits required to measure a gravitational potential ϕ is:

$$n \gtrsim \log_2 \left(|\phi| \times 10^{32} \right) \tag{14}$$

which suggests a simple logarithmic scaling in the number of qubits.[2]

V. MULTIPOLE GRAVITATIONAL SIGNATURES

The integral representation of the gravitational potential due to a mass distribution of density ρ over a volume V is given by:

$$\Phi(\mathbf{r}) = G \int_{V'} \frac{\rho(\mathbf{r}')}{|\mathbf{r}' - \mathbf{r}|} dV' \tag{15}$$

Analogous to the electromagnetic field potential, a multipole expansion can be used to facilitate calculation of this integral for complex mass distributions [21], [22]. Specifically, if proximity of the test particle to the mass distribution is assumed bounded, then $|\mathbf{r}'| \ll |\mathbf{r}|$ and the following Taylor expansion can be used:

$$\begin{aligned} \frac{1}{|\mathbf{r}' - \mathbf{r}|} \approx \frac{1}{r} &- r'^a \nabla_a \left(\frac{1}{r} \right) \\ &+ \frac{1}{2} r'^a r'^b \nabla^a \nabla^b \left(\frac{1}{r} \right) \\ &- \frac{1}{6} r'^a r'^b r'^c \nabla^a \nabla^b \nabla^c \left(\frac{1}{r} \right) + \ \dots \end{aligned} \tag{16}$$

where Einstein's summation notation is used for repeated indices in tensor expressions. Thus, the gravitational potential can be expressed as a multipole expansion:

$$\begin{aligned} \Phi(\mathbf{r}) = GM \frac{1}{r} &- G P^a \nabla^a \left(\frac{1}{r} \right) \\ &+ G \frac{1}{2} Q^{ab} \nabla^a \nabla^b \left(\frac{1}{r} \right) \\ &- G \frac{1}{6} S^{abc} \nabla^a \nabla^b \nabla^c \left(\frac{1}{r} \right) + \ \dots \end{aligned} \tag{17}$$

where the *multipole moments* are given by:

$$\begin{aligned} M &= \int_V \rho(r) dV \\ P^a &= \int_V r^a \rho(r) dV \\ Q^{ab} &= \int_V r^a r^b \rho(r) dV \\ S^{abc} &= \int_V r^a r^b r^c \rho(r) dV \end{aligned} \tag{18}$$

and all the integrals are taken over the $\mathbf{r}'$ coordinates. The quantities M, P^a, and Q^{ab} are known as the *monopole moment*, *dipole moment*, and *quadrupole moment*, respectively, of the mass distribution ρ.

As a simple example, consider a system of two point masses m_1 and m_2 separated by a distance L along the z axis with the coordinate origin at the center of mass. The multipole moments can then be easily calculated as:

$$\begin{aligned} M &= m_1 + m_2 \\ P^a &= \left(\frac{m_1 m_2}{m_1 + m_2} - \frac{m_2 m_1}{m_2 + m_1} \right) L \ k^a = 0 \\ Q^{ab} &= \left(\frac{m_1 m_2}{m_1 + m_2} \right) L^2 \ k^a k^b \\ S^{abc} &= \left(m_1 z_1^3 + m_2 z_2^3 \right) k^a k^b k^c \end{aligned} \tag{19}$$

The gravitational potential in this example is given by:

$$\Phi(\mathbf{r}) = \Phi_M(r) + \Phi_Q(r, \theta) + \dots \tag{20}$$

where the monopole and quadrupole moments are given by:

$$\begin{aligned} \Phi_M(r) &= G \ \frac{m_1 + m_2}{r} \\ \Phi_Q(r, \theta) &= G \ \frac{L^2}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) \frac{1 - 3\cos^2\theta}{r^3} \end{aligned} \tag{21}$$

Notice that because of the symmetry of the problem, and the way we have chosen the coordinate system, we obtain a vanishing dipole moment. Furthermore, it can also be shown that the odd moments of the gravitational potential for all mass distributions with plane symmetry are zero. This means that the next non-zero term in the multipole potential expansion is $\mathcal{O}(1/r^5)$.

From these expressions it can be seen that the monopole field has spherical symmetry because it only depends on the field point distance r. On the other hand, the quadrupole moment has cylindrical symmetry, as it only depends on the the field point distance r and one angular coordinate θ.

It is insightful to examine the difference between the monopole and quadrupole moments in a simple example. Assume two masses $m_1 = 0.7$ and $m_2 = 0.3$, separated a distance $L = 100$, and in units such that $G = 1$. Contour plots of the gravitational potential and its associated vector field (i.e., the force field $\mathbf{F} = \nabla\Phi(\mathbf{r})$) are shown in Figures 4 and 5 for the monopole and quadrupole moments, respectively. As expected, the magnitude of the quadrupole moment is much smaller than the monopole moment. More important, however, the

[1]We have previously described such an amplification algorithm, based on Grover's algorithm, which uses the environment as a computational oracle [18]. It is also possible to design an oracle which further assists the algorithm in the amplification of the gravitational signal.

[2]This scaling behavior appears to be a consequence of the exponentially large computational space offered by quantum computation.

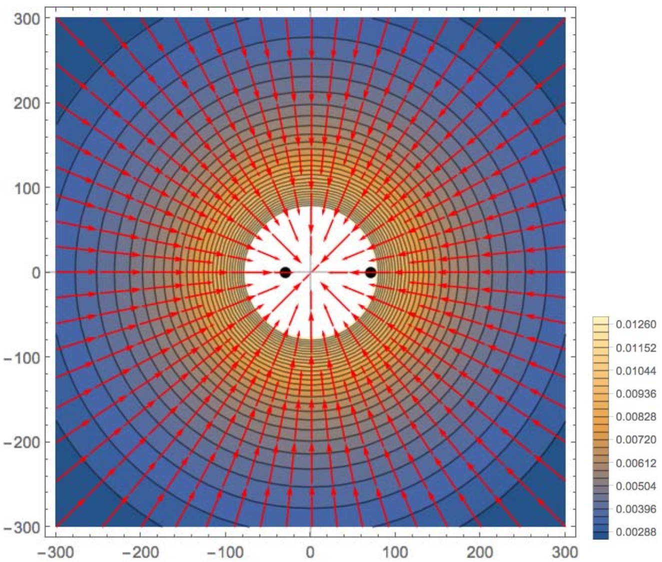

Fig. 4. Contour plot and associated vector field of the monopole gravitational moment of a two point-like massive particles system.

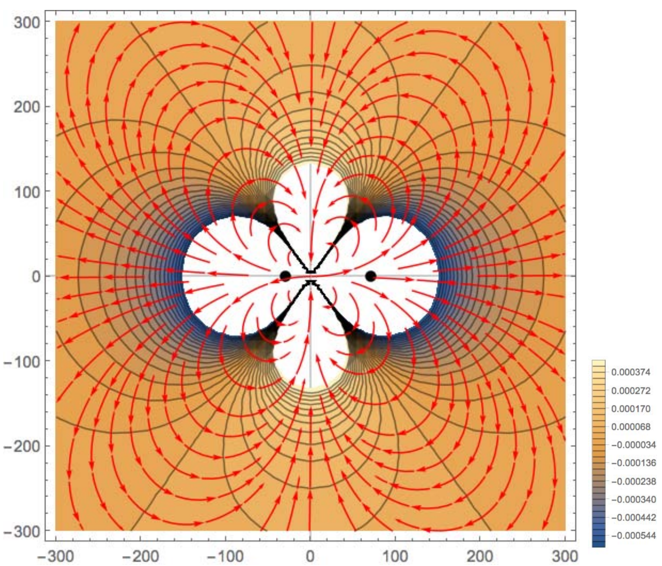

Fig. 5. Contour plot and associated vector field of the quadrupole gravitational moment of a two point-like massive particles system.

gravitational potential and associated force field of the quadrupole moment do not have the rotational symmetry of the monopole moment. This deviation from spherical symmetry can be exploited as a potential gravitational signature to detect and identify targets based on their mass distribution.

We now consider the theoretical performance of our proposed Wigner gravimeter to measure gravitational multipole moments. The monopole gravitational field anomaly of a mass M is given by:

$$\delta g = \frac{GM}{r^2} \tag{22}$$

where G is Newton's gravitational constant and r is the distance to the mass. As a trivial example, consider the field anomaly produced by Mount Everest, which is estimated to

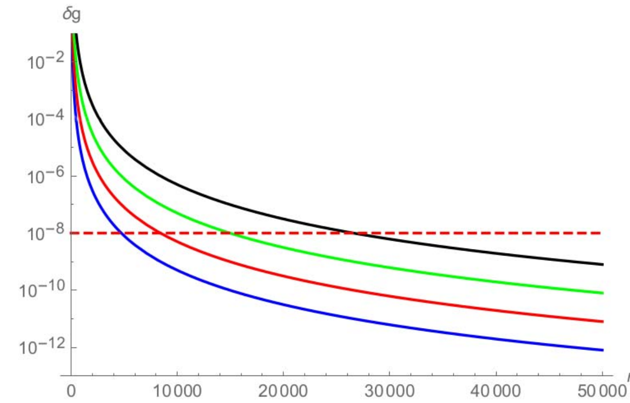

Fig. 6. Quadrupole gravitational field anomaly δg with respect to the range r for different values of α. The dotted red line represents the range detection limit for classical gravimeters.

be $M \approx 10^{13}$ kg and thus can only be detected at a range of 100 km using classical methods [10]. The number of qubits required to achieve a prescribed detection level using the proposed quantum-based approach can be estimated as:

$$n \gtrsim \log_2 \left(\frac{r}{M} \times 10^{32} \right) \tag{23}$$

which leads to an estimate of 80 qubits to match what can be achieved using current classical methods.

Now consider the quadrupole field anomaly produced by two masses m_1 and m_2 separated by a distance L such that:

$$M = m_1 + m_2 = M(\alpha_1 + \alpha_2) \tag{24}$$

where:

$$\alpha_1 \equiv \frac{m_1}{M} \quad \alpha_2 \equiv \frac{m_2}{M} \tag{25}$$

In this case, the quadrupole gravitational field anomaly is given by:

$$\delta g = G\frac{\alpha M}{2}\frac{1 - 3\cos^2\theta}{r^4} \tag{26}$$

where θ is the azimuth angle in spherical coordinates with axis symmetry of the mass system parallel to the z axis, and α is defined as:

$$\alpha \equiv L^2\alpha_1\alpha_2 \tag{27}$$

The behavior of the quadrupole gravitational field anomaly of a mass $M = 10^7$ kg with respect to the range r for different values of α is shown in Figure 6. The curves represent $\alpha = 10^{14}$ (black), $\alpha = 10^{13}$ (green), $\alpha = 10^{12}$ (red), and $\alpha = 10^{11}$ (blue). The horizontal red line corresponds to the range detection limit for classical gravimeters (about $\delta g \approx 10^{-8}$ m/s^2) [10].

The number of qubits necessary to measure the quadrupole gravitational field anomaly is given by:

$$n \gtrsim \log_2 \left(\frac{r^3}{\alpha M} \times 10^{32} \right) \tag{28}$$

The scaling of the number of qubits necessary to measure the quadrupole gravitational field anomaly of a mass $M = 10^7$ kg with respect to the range r for different values of α is shown

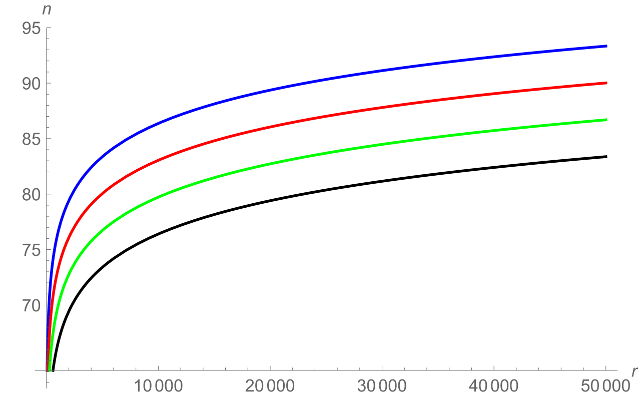

Fig. 7. Scaling of the number of qubits for the detection of the quadrupole gravitational field anomaly with respect to the range r for different values of α.

in Figure 7, where the curves represent $\alpha = 10^{14}$ (black), $\alpha = 10^{13}$ (green), $\alpha = 10^{12}$ (red), and $\alpha = 10^{11}$ (blue).

For the specific case of the Mount Everest-Earth system, Everest's mass is approximately $M_{ev} = 10^{13}$ kg and the Earth's mass is approximately $M_e = 10^{25}$ kg. Considering that the radius of Earth is of about $R_e \approx 10^7$ m, then $\alpha \approx 100$. Therefore, at an arbitrary point on the Earth's surface it is possible to detect the monopole field anomaly of Earth-Mount Everest system using traditional systems, but not the quadrupole component. However, with about 90 qubits it should be possible to detect the quadrupole field anomaly of the Everest-Earth system at any point on the surface of the Earth.[3]

VI. Conclusions

In this paper we have examined a spin-based quantum model for design of a *quantum Wigner gravimeter*. The key feature of this design is that it measures effects of gravitation on information encoded in qubits in a way that can provide resolution beyond the de Broglie limit of atom-interferometric gravimeters. We have shown that a Wigner gravimeter also offers advantages over current state-of-the-art gravimeters in its ability to detect quadrupole field anomalies. This can allow it to potentially detect a target, e.g., an underwater vehicle or submerged aircraft on the ocean floor, based on the difference between the specific quadrupole signature of the target and that of other objects in the environment. The range of possible applications strongly motivates implementation and testing of the proposed technology.

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[3]It is important to note that as the number of qubits is increased, the amount of noise affecting the qubit states will also increase. In the case of large quantum systems it may be necessary to use more sophisticated error-resilient representations of qubit states [23]–[27]. Wigner gravimeters with a large number of qubits will also demand sophisticated signal processing algorithms to eliminate seismic noise and other sources of gravitational variation. Methods for mitigating seismic noise are known and in fact are used in the context of the LIGO experiments for the detection of gravitational waves [28]–[30].

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