Formal analysis of multi-party contract signing

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Abstract. We analyze the multi-party contract-signing protocols of Garay and MacKenzie (GM) and of Baum and Waidner (BW). We use a finite-state tool, Mocha, which allows specification of protocol properties in a branching-time temporal logic with game semantics. While our analysis does not reveal any errors in the BW protocol, in the GM protocol we discover serious problems with fairness for four signers and an oversight regarding abuse-freeness for three signers. We propose a complete revision of the GM subprotocols in order to restore fairness.

1 Introduction

The problem of digitally signing a contract over a network is more complicated than signing a contract by “pen and paper”. The problem arises because of an inherent asymmetry: no signer wants to be the first one to sign the contract because another signer could refuse to do so after having obtained the first signer’s contract.

A simple solution consists in using a trusted party (T) as an intermediary. Signers send their respective contracts to T, which first collects the contracts and then distributes them among the signers. An intermediary is known to be necessary [8]. However, because of the communication and computation bottleneck at T, this solution is inefficient. Other solutions include randomized protocols as well as protocols based on gradual information exchange. More recently, the so-called optimistic approach was introduced in [2,4]. The idea is that T intervenes only when a problem arises, e.g., a signer is trying to cheat or a network failure occurs at a crucial moment during the protocol. Such protocols generally consist of a main protocol and one or several subprotocols, each with a fixed number of messages. The main protocol is executed by the signers in order to exchange their signatures. The subprotocols are used to contact T in order to force a successful outcome or to abort the protocol.

A contract-signing protocol should respect several desirable properties. The first property is fairness. Intuitively, a contract-signing protocol is fair if at the end of the protocol either each signer obtains all the other signers’ contracts or no signer gets any valuable information. A second property, timeliness, ensures that signer has some recourse to prevent endless waiting. Both fairness and timeliness are standard properties that are also important in fair exchange, certified e-mail and fair non-repudiation protocols. A property that is specific to contract signing, abuse-freeness, was introduced in [9]. A protocol is abuse-free if no signer A is able to prove to an external observer that A has the power to choose between successfully concluding the protocol and aborting the protocol. A protocol that is not abuse-free gives an undesirable advantage to one signer, say Alice, who has the power to decide the outcome of the protocol and can prove this to an external observer. If, for instance, Alice wants to sell a house to Charlie, she could initiate a contract with Bob just to force Charlie to increase his offer.

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There have been several applications of formal methods to contract signing, so far only for the special case of two signers. The finite model-checker Murϕ is used in [13] to analyze two contract-signing protocols, discover subtle errors and suggest corrections. In [5] inductive methods are used to reason about contract-signing protocols specified in the multiset-rewriting framework, MSR. Protocol properties are expressed in terms of strategies, which provide a natural framework for the analysis. In [12] the finite model-checker Mocha is used to analyze two contract-signing protocols. The advantage of using Mocha rather than Murϕ is that Mocha allows to specify protocol properties in ATL, a temporal logic with game semantics, which in turn allows reasoning about strategies. The most recent work on contract signing [6] introduces the notion of an optimistic signer, i.e., a signer that prefers to wait for “some time” for messages from the other signers before contacting the trusted party. The main theorem of [6] is that, independently of a specific protocol, if any of the signers is optimistic, then the other signer will at some point of the protocol have the power to decide the outcome.

All the efforts just described consider only two-party protocols. In this paper we analyze multi-party contract-signing protocols [3,10]. The protocol goal in that case is that each signer sends its signature on a previously agreed upon contract text to all other signers and that each signer receives all other signers’ signatures on this contract. In a multi-party framework, fairness, timeliness, and abuse-freeness should hold against any coalition of dishonest parties. Unlike in the two-party case, the complexity level of the multi-party protocols, especially [10], is such that a tool, e.g., a model-checker, is indispensable in the analysis. This partly comes about from an important difference between the two-party and the multi-party case, namely, in the multi-party case T has to be able to overturn its previous abort decisions [9]. As our analysis shows, this feature is particularly difficult to design correctly. We have discovered an essential obstacle in the GM protocol [10], which appears not to be removable without completely changing the subprotocols for T and which leads to the failure of fairness in the case of four signers. We present this attack in detail in the paper and propose a corrected version of the GM protocol, which has been validated by Mocha. Mocha did not find any problems with fairness in the BW protocol [3] nor in the original GM protocol with only three signers. In the latter case, Mocha did find an amusing problem with abuse-freeness, but this problem is easily corrected. We believe that the main reason for robustness of the BW protocol is that overturning the aborts decisions has been designed correctly.

Outline of the paper. The rest of the paper is organized as follows. In section 2, we describe the BW and the GM protocols. In section 3, we present briefly the finite-state tool, Mocha, the temporal logic ATL and its game semantics. Modeling of the protocols and protocol assumptions in the game semantics along with the modeling of fairness in ATL is briefly discussed. In section 4, we report on our analysis of the BW and GM protocols using Mocha, present the fairness attack on four signers in detail and propose a corrected version of the protocol. We discuss briefly how to restore fairness and present the anomaly with respect to abuse-freeness for three signers. In order to detect this anomaly, we had to model optimistic signers and discuss this issue. We summarize our results and discuss directions for future work in section 5.

2 Protocol description

In this section we describe the multi-party contract-signing protocols proposed by Baum and Waidner in [3] and Garay and MacKenzie in [10]. Unlike two-party protocols, which generally have similar structures, the two multi-party protocols described below have fundamentally different structures. For this section and for the rest of the paper, we shall assume that each protocol participant has a private signing key and a corresponding public verification key. Each participant shall be identified with this private/public key pair, and if we say that “A can . . .”, we shall mean anyone that possesses the private key of A.
2.1 GM multi-party contract-signing protocol

The protocol allows \( n \) (\( n \geq 2 \)) participants, say \( P_1, \ldots, P_n \), to exchange signatures with the help of a trusted party \( T \) on a preagreed contract text \( m \). \( P_i \) is said to have a contract if it has everybody’s signature on the text \( m \). The order of the participants \( P_1, \ldots, P_n \), henceforth referred to as signers, and the identity of \( T \) are also agreed upon before the protocol begins. The preagreed contract text \( m \) contains an identifier that uniquely identifies each protocol instance. In [10], the communication amongst the participants is assumed to be over a network channel in control of a “Dolev-Yao intruder”, while the communication between the participants and the trusted party is assumed to be over a private channel.

The protocol uses zero-knowledge cryptographic primitives, private contract signatures, that were first introduced in [9]. The private contract signature of \( A \) for \( B \) on text \( m \) with respect to a trusted party \( T \), denoted as \( PCS_A(m, B, T) \) has the following properties:

a) \( PCS_A(m, B, T) \) can be faked by \( A \).

b) \( PCS_A(m, B, T) \) can be created by \( A \).

c) \( PCS_A(m, B, T) \) can be converted into a conventional universally-verifiable digital signature, \( S_A(m) \), by both \( A \) and \( T \). Only \( A \) and \( T \) can do this conversion.

The protocol itself consists of three subprotocols: main, abort, and recovery subprotocols. Usually signers try to achieve the exchange by executing the main subprotocol. They contact \( T \) using one of the other two subprotocols when they think something is amiss. Once a signer contacts \( T \), it no longer takes part in the main subprotocol. \( T \) responds to a request with either an abort token or a signed contract. The decision whether to reply with an abort token or with a signed contract is based on a database maintained by \( T \), which stores all the relevant information of the requests and its responses. Once \( T \) sends back a signed contract, it always replies with the signed contract. As discussed below, a decision to abort may, however, be overturned in order to maintain fairness. We discuss the subprotocols in some detail.

Main protocol. The main protocol for \( n \) signers is divided into \( n \)-levels, that can be described recursively. For each level of recursion, a different “strength” of promise is used. The strength of a promise is denoted by an integer “level”, and an “\( i \)-level promise from signer \( A \) to signer \( B \) on a message \( m \)” is implemented using PCS: \( PCS_A(m, i, B, T) \).

In level \( i \), signers \( P_i \) through \( P_1 \) exchange i-level promises to sign the contract. The \( i \)-level protocol is triggered when \( P_i \) receives \( 1 \)-level promises from \( P_{i+1}, \ldots, P_n \). After receiving these promises, \( P_i \) sends its level promise to signers \( P_{i-1}, \ldots, P_1 \) and waits for \( i-1 \)-level promises to finish. At the end of the \( i-1 \)-level, \( P_1, \ldots, P_{i-1} \) have exchanged \( i-1 \)-level promises and \( P_i \) receives a \( i-1 \)-level promise from each of the signers \( P_1, \ldots, P_{i-1} \). Now \( P_1, \ldots, P_i \) exchange \( i \)-level promises, and close the higher levels.

In order to close level \( a \) where \( a > i \), \( P_i \) sends an \((a-1)\)-level promise to \( P_a \) and waits for \( a \)-level promises from signers \( P_{a+1}, \ldots, P_a \). After receiving these promises, \( P_i \) indicates its willingness to close the level \( a \) to signers \( P_1, \ldots, P_{a-1} \) by sending them its \( a \)-level promise, and in return waits for \( a \)-level promises from them. Upon receiving these, \( P_i \) sends its \( a \)-level promises to \( P_{a+1}, \ldots, P_a \), completing its obligation in the \( a \)-level protocol. \( P_i \) then proceeds to complete \( a + 1 \) level.

Once the \( n \)-levels are completed, each signer has a \( n \)-level promise from everybody else, and the contract exchange is ready to begin. In this exchange, each signer also sends a \( n + 1 \)-level promise to everybody along with its signature on the preagreed text. In order to complete the exchange, signer \( P_i \) waits for the contract and \( n + 1 \)-level promises from \( P_n, \ldots, P_{i+1} \). Upon receiving these, \( P_i \) sends its signature and \( n + 1 \)-level promises to everybody, and waits for the signatures and \( n + 1 \)-level promises from \( P_{i-1}, \ldots, P_1 \). Once these are received, the protocol ends for \( P_i \), and \( P_i \) has the contract.

If some expected messages are not received, \( P_i \) may either quit the protocol or contact \( T \). \( P_i \) may simply quit the protocol if it has not sent any promises or contact \( T \) if it has sent some promises. It may contact \( T \) with a request to abort if it has not received any promise from some signer. It may request \( T \) to recover the protocol if it has a promise from every other signer. A detailed description of the main protocol is given in table 1.
Table 1: GM multi-party contract-signing protocol—Main

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Wait for all higher recursive levels to start</td>
</tr>
<tr>
<td>2.</td>
<td>$P_i \rightarrow P_j: PCS_{P_i}((m, 1), P_j, T)$ ($n \geq j &gt; i$)</td>
</tr>
<tr>
<td></td>
<td>If $P_i$ does not receive 1-level promises from $P_n \ldots P_{i+1}$ in a timely manner, $P_i$ simply quits.</td>
</tr>
<tr>
<td>3.</td>
<td>Start recursive level $i$</td>
</tr>
<tr>
<td>4.</td>
<td>$P_i \rightarrow P_j: PCS_{P_i}((m, 1), P_j, T)$ ($i &gt; j \geq 1$)</td>
</tr>
<tr>
<td>5.</td>
<td>Wait for recursive level $i-1$ to finish</td>
</tr>
<tr>
<td>6.</td>
<td>$P_i \rightarrow P_j: PCS_{P_i}((m, i-1), P_j, T)$ ($i &gt; j \geq 1$)</td>
</tr>
<tr>
<td></td>
<td>If $P_i$ does not receive $(i-1)$-level promises from $P_{i-1} \ldots P_1$ in a timely manner, $P_i$ aborts.</td>
</tr>
</tbody>
</table>

Send $(i-1)$-level promises to all lower-numbered signers

7. $P_i \rightarrow P_j: PCS_{P_i}((m, i), P_j, T)$ ($i > j \geq 1$) |
|      | If $P_i$ does not receive $(i-1)$-level promises from $P_{i-1} \ldots P_1$ in a timely manner, $P_i$ recovers. |

Complete all higher recursive levels

For $a = i + 1$ to $n$, $P_i$ does the following:

8. $P_i \rightarrow P_a: PCS_{P_i}((m, a-1), P_a, T)$ |

9. $P_i \rightarrow P_j: PCS_{P_i}((m, a), P_j, T)$ ($a \geq j > i$) |

If $P_i$ does not receive a-level promises from $P_a \ldots P_{i+1}$ in a timely manner, $P_i$ recovers. |

10. $P_i \rightarrow P_j: PCS_{P_i}((m, a), P_j, T)$ ($i > j \geq 1$) |
|      | If $P_i$ does not receive a-level promises from $P_a \ldots P_{i+1}$ in a timely manner, $P_i$ recovers. |

11. $P_i \rightarrow P_j: PCS_{P_i}((m, a), P_j, T)$ ($i > j \geq 1$) |
|      | If $P_i$ does not receive a-level promises from $P_a \ldots P_{i+1}$ in a timely manner, $P_i$ recovers. |

12. $P_i \rightarrow P_j: PCS_{P_i}((m, a), P_j, T)$ ($a \geq j > i$) |

Finish recursive level $i$ when $(i-1)$-level promises are received

13. $P_i \rightarrow P_j: PCS_{P_i}((m, i), P_j, T)$ ($i > j \geq 1$) |
|      | If $P_i$ does not receive $(i-1)$-level promises from $P_{i-1} \ldots P_1$ in a timely manner, $P_i$ recovers. |

Abort protocol. $T$ maintains two sets, $S_m$ and $F_m$, that are used by $T$ to make decisions when a signer contacts $T$. These sets are created when $T$ is contacted for the first time for $m$ and are initialized to be empty. The set $S_m$ contains the indices of all signers that have contacted $T$ and received an abort token from $T$ in response. The intuitive meaning of the set $F_m$ is not clearly stated in [10], but it contains some additional information that $T$ uses in deciding when to overturn an abort decision that $T$ has taken before.

The details of the abort protocol are given in table 2. Mainly, if $T$ is contacted with a request to abort, then $T$ checks its database. If this is the first request or if the protocol has not already been recovered, $T$ sends back an abort token and updates the sets $S_m$ and $F_m$. If the protocol has already been successfully recovered, $T$ sends back a signed contract.

Recovery protocol. The details of the recovery protocol are given in table 3. For $P_i$ to recover, it sends the message $S_{P_i}((PCS_{P_j}((m, k_j), P_i, T))_{j \in \{1, \ldots, n\} \setminus \{i\}}, S_{P_j}((m, 1)))$, where

- if $j > i$, $k_j$ is the maximum level of a promise received from $P_j$ on $m$,
- if $j < i$, $k_j$ is the maximum level of promises received from all signers $P_{j'}$, with $j' < i$, i.e., the min-max of the level of promises from signers with lower index. (E.g., if the maximum level of the promises received by $P_k$ from $P_5$ and $P_2$ was 6, and the maximum level received by $P_4$ from $P_1$ was 5, then it would send the 5-level promises for $P_1$, $P_2$ and $P_3$.)

If $T$ is contacted with a request to recover, then $T$ checks its database. If this is the first request for $m$ or if the protocol has already been recovered, $T$ replies with a signed contract.
Table 2: GM multi-party contract-signing protocol—Abort

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$P_i \rightarrow T$: $S_T(m, P_i, \ldots, P_n, \text{abort})$ if not validated$(m)$ then if $S_m = \emptyset$, $T$ stores $S_T(m, P_i, \ldots, P_n, \text{abort})$; $S_m = S_m \cup {i}$; if $i$ is larger than the maximum index in $S_m$, $T$ clears $F_m$; 2. $T \rightarrow P_i$: $S_T(m, P_i, \ldots, P_n, \text{abort})$, $S_T(m, S_m, \text{abort})$; else (validated$(m)$ = true) 3. $T \rightarrow P_i$: ${S_T((m, k_j))}_{j \in {1, \ldots, n} \setminus {i}}$ where $k_j$ is the level of the promise from $P_i$ that was converted to a universally-verifiable signature during the recovery protocol.</td>
</tr>
</tbody>
</table>

which it obtains by converting the promises into conventional digital signatures. Otherwise, if the protocol has already been aborted, $T$ must decide whether to maintain the abort or to overturn it. Overturning of the abort is necessary in order to maintain fairness. Indeed, consider the scenario in which a dishonest $P_{n-1}$ contacts $T$ with an abort request, receives an abort token and dishonestly continues the protocol. After the $n$-levels are completed, $P_n$ sends its signature to others and waits for signatures from other signers. If $P_{n-1}$ does not send back its signature, then $P_n$ will be forced to contact $T$ with a request to recover. Now, $T$ must overturn its previous abort, otherwise $P_n$ will not receive the signature of $P_{n-1}$. The decision whether to overturn is based on the contents of the sets $S_m$ and $F_m$, as described in table 3.

2.2 BW multi-party contract-signing protocol

The protocol allows $n$ ($n \geq 2$) participants or signers, say $P_1, \ldots, P_n$, to exchange signatures with the help of a trusted party $T$ on a preagreed contract text $m$. In our description, we suppose $n - 1$ potentially dishonest signers. The original protocol is actually parameterized with respect to a threshold $t$, the maximum number of possibly dishonest signers. In our analysis however we assume the worst possible scenario for an honest signer, namely that all the other signers are dishonest (i.e., $t$ is $n - 1$).

The protocol consists of two subprotocols: main and recovery. Usually signers try to achieve the exchange by executing the main subprotocol. They contact $T$ using the recovery subprotocol when they think something is amiss.

Main protocol. A detailed description of the main protocol for each signer $P_i$ is given in table 4. The protocol is composed of $n+1$ rounds\(^1\). The main protocol is symmetric for each signer, and in round $i$ a signer sends an $i$-level promise to other signers. In [3], the $i$-level promise is implemented using a universally-verifiable digital signature and includes all the promises (through the vectors $M$ and $X$, as defined in table 4) that the signer has received till round $i - 1$. Note that these promises are based on universally verifiable signatures and that the protocol does not intend to provide abuse-freeness. The $n + 1$ level promises from everybody is considered to be the signed contract. If any expected message is not received, a signer can launch the recovery protocol. Once the signer launches the recovery protocol, it is not allowed to continue the main subprotocol.

Recovery protocol. The details of the recovery protocol are given in table 5. To recover, $P_i$ sends a recovery request. If the recovery request is launched in the first round, i.e. $P_i$ did not receive a message from all the signers, the recovery request consists in the first level promise. Otherwise, if $r > 1$, the recovery request contains, via the vector $X_{r-1,i}$, the set of received messages.

$T$ maintains for each contract for which it has been contacted a variable recovered indicating whether the given contract has already been recovered or not, a set con, containing the indexes of the signers that contacted $T$ for the given contract and the set abort set containing the indexes of the signers for whom $T$ aborted the protocol.

\(^1\) In [3], the protocol has $t + 2$ rounds.
Table 3 GM multi-party contract-signing protocol—Recovery

1. \( P_i \rightarrow T: S_P((PCS_P((m, k_j), P_i, T))_{j \in (1 \ldots n)} \cup (m, 1))) \)
   if \( i \in S_m, T \) ignores the message
   else if validated(m)
   2. \( T \rightarrow P_i: S_P((m, k_j))_{j \in (1 \ldots n)} \cup (m, 1)) \)
      where \( k_j \) is the level of the promise from \( P_j \) that was converted to a universally-verifiable signature.
      else if \( S_m = \emptyset \)
      validated(m):=true
      3. \( T \rightarrow P_i: S_P((m, k_j))_{j \in (1 \ldots n)} \cup (m, 1)) \)
      else (validated(m)=false \&\& S_m \neq \emptyset)
      a. if \( i \notin F_m \), then
         (i) if for any \( \ell \in S_m \) there is a \( j \in S_m \) such that \( j > k_\ell \)
            \( S_m := S_m \cup \{\ell\} \)
            let \( a \) be the maximum value in \( S_m \)
            if \( a > i \) then \( \forall j, \text{ such that } k_j = a - 1 \cdot F_m := F_m \cup \{j\} \)
            else \( F_m := \emptyset \)
            4.1.1. \( T \rightarrow P_i: S_T(S_P((m, P_j, (P_1, \ldots, P_n), abort)), S_T(m, S_m, abort)) \)
               where \( S_T(S_P((m, P_j, (P_1, \ldots, P_n), abort)) \)
               corresponds to the stored abort token
               (ii) else
                   validated(m):=true
                   4.1.2. \( T \rightarrow P_i: S_P((m, k_j))_{j \in (1 \ldots n)} \cup (m, 1)) \)
               b. else (\( i \in F_m \))
                  let \( a \) be the maximum value in \( S_m \)
                  (i) if \( \forall j, \text{ such that } i < j \leq a \cdot k_j < a \) \&\& \( \forall j < i \cdot k_j \geq a \)
                     validated(m):=true
                     4.2.1. \( T \rightarrow P_i: S_P((m, k_j))_{j \in (1 \ldots n)} \cup (m, 1)) \)
                  (ii) else
                     \( S_m := S_m \cup \{\ell\} \)
                     if \( a > i \) then \( \forall j, k_j = a - 1 \cdot F_m := F_m \cup \{j\} \)
                     if \( a = i \) then \( F_m := \emptyset \)
                     4.2.2. \( T \rightarrow P_i: S_T(S_P((m, P_j, (P_1, \ldots, P_n), abort)), S_T(m, S_m, abort)) \)
                     where \( S_T(S_P((m, P_j, (P_1, \ldots, P_n), abort)) \)
                     corresponds to the stored abort token

When \( T \) receives a valid recovery request, it first verifies whether this signer already contacted the \( T \) or not. If so, \( T \) ignores the request. If not, it checks whether the contract has already been successfully recovered or not. A successful recovery is always maintained. Otherwise, we distinguish two cases. If the recovery request is sent in the first round, \( T \) must abort the protocol, as the request does not contain a proof that all signers actually started the protocol. If the recovery request is sent during any later round, \( T \) verifies whether all the requests that were aborted previously occurred at least two rounds before. If so \( T \) can be sure that the previous abort replies were sent to dishonest players, as they continued the protocol and sent out higher level promises. Hence, the previous abort decision is overturned. Otherwise, \( T \) replies with an abort token.

3 Model

ATS, ATL and Mocha. The desired properties of contract signing are easily described using games, and hence we chose a game-variant of Kripke structures, alternating transition systems (ATS) [1], to model the protocols. An ATS is composed of a set of players \( \Sigma \), a set of states \( Q \) that represents all possible game configurations, a set \( Q_0 \subseteq Q \) of initial states, a finite set of propositions \( \Pi \), a labeling function \( \pi : Q \rightarrow 2^\Pi \) that labels states with propositions, and a game transition function \( \delta : Q \times \Sigma \rightarrow 2^{2Q} \). For a player \( a \) and a state \( q \), \( \delta(q, a) \) is the set of choices that \( a \) can make in the state \( q \). A choice is a set of possible next states. One step of the game at a state \( q \) is played as follows: each player \( a \in \Sigma \) makes its choice and the next state of the game \( q' \) is the intersection (required to be a singleton) of the choices made by all the players of \( \Sigma \), i.e.,
{q'} = ∩a∈Σδ(q, a). A computation is an infinite sequence λ = q_0 q_1 ... q_n ... of states obtained by starting the game in q_0, where q_0 ∈ Q_0.

In order to reason about ATS, we use alternating-time temporal logic (ATL) [1]. For a given set of players A ⊆ Σ, a set of computations Λ, and a state q, consider the following game between a protagonist and an antagonist starting in q. At each step, to determine the next state, the protagonist selects the choices controlled by the players in the set A, while the antagonist selects the remaining choices. If the resulting infinite computation belongs to the set Λ, then the protagonist wins. If the protagonist has a winning strategy, we say that the ATL formula ⟨⟨A⟩⟩ is satisfied in state q. Here, ⟨⟨A⟩⟩ is a path quantifier, parameterized by the set A of players, which ranges over all computations that the players in A can force the game into, irrespective of how the players in Σ \ A proceed. The set Λ is defined using temporal logic formulas. For those familiar with branching time temporal logics, the parameterized path quantifier ⟨⟨A⟩⟩ can be seen as a generalization of the CTL path quantifiers: the existential path quantifier ∃ corresponds to ⟨⟨Σ⟩⟩ and the universal path quantifier ∀ corresponds to ⟨⟨∅⟩⟩.

We now illustrate the expressive power of ATL that allows to model both cooperative as well as adversarial behavior amongst the players. Consider the set of players Σ = {a, b, c} and the following formulas with their verbal reading:

- ⟨⟨a⟩⟩ ∨ p, player a has a strategy against players b and c to eventually reach a state where the proposition p is true;
- ¬⟨⟨b,c⟩⟩ ∨ p, the coalition of players b and c does not have a strategy against a to make p true forever;
- ⟨⟨a,b⟩⟩ ∨ (p ∧ ¬⟨⟨c⟩⟩ ∨ p), a and b can cooperate so that the next state satisfies p and from there c does not have a strategy to impose p forever.

The details of ATS and ATL can be found in [1]. Instead of modeling protocols directly with ATS, we use a more user-oriented notation: a guarded command language a la Dijkstra. The details about the syntax and semantics of this language (given in terms of ATS) can be found in [11]. Intuitively, each player a ∈ Σ disposes of a set of guarded commands of the form guard_ξ → update_ξ. A computation-step is defined as follows: each player a ∈ Σ chooses one of its commands whose boolean guard evaluates to true, and the next state is obtained by taking the conjunction of the effects of each update part of the commands selected by the players. Given an ATS described in terms of guarded commands, the finite state tool MOCHA automates the model-checking of ATL formulae over the specified ATS.

**Modeling protocols.** Unlike the classical security protocols aiming at secrecy and authentication, optimistic contract-signing protocols usually consist of subprotocols that can be invoked at specified moments. Running a protocol at a time not foreseen by the designer, may have unexpected effects.
### Table 5 Baum-Waidner multi-party contract signing protocol—Recovery

1. $P_i \rightarrow T$: $\text{resolve}_{r,i}$
   - where $\text{resolve}_{r,i} = \begin{cases} (1, i, S_{P_{r,1,i}}(m_{1,i}, \text{resolve})) & \text{if } r = 1 \\ (r, i, S_{P_{r-1,i}}(X_{r-1,i}, \text{resolve})) & \text{otherwise} \end{cases}$
   - if $i \in \text{con}$ then stop
   - else if recovered
   - $\text{con} := \text{con} \cup \{i\}

2. $T \rightarrow P_i$: $\text{signed}_{r,i}$
   - where $\text{signed}_{r,i} = \text{first}_{\text{signed}}$
   - else ($\neg \text{recovered}$)
   - a. if $r = 1$
     - $\text{abort}_{\text{set}} := \text{abort}_{\text{set}} \cup \{(r, i)\}$
     - $\text{con} := \text{con} \cup \{i\}$
   - b. else ($r > 1$)
     - (i) if $\forall (s, k) \in \text{abort}_{\text{set}} \cdot s < r - 1$
       - if $\text{con} = \emptyset$ then $T$ sets $\text{first}_{\text{signed}} := (\text{resolve}_{r,i}, S_{T}(c, r, i, \text{recovered}))$
       - recovered := true
       - $\text{con} := \text{con} \cup \{i\}$
     - 4. $T \rightarrow P_i$: $\text{signed}_{r,i}$
       - where $\text{signed}_{r,i} = S_{T}(c, r, i, \text{aborted})$
     - (ii) else
       - $\text{abort}_{\text{set}} := \text{abort}_{\text{set}} \cup \{(r, i)\}$
       - $\text{con} := \text{con} \cup \{i\}$
   - 5. $T \rightarrow P_i$: $\text{aborted}_{r,i}$
     - where $\text{aborted}_{r,i} = S_{T}(c, r, i, \text{aborted})$

Side-effects. This may be used by a signer to gain an advantage over other signers. We believe that such concurrency issues are a major source of problems. Therefore, and since the high number of messages would create a serious state explosion, we only analyze the structure of the protocols and concentrate only on single protocol instance. We now discuss our model in detail.

The protocol instance is modeled as an ATS and each protocol participant is modeled as a player in the ATS using the above introduced guarded command language. Besides, the branching aspect, another notable difference with more classical secrecy and authentication protocols, is that contract-signing protocols must be secure against malicious signer, rather than an external intruder. In order to model this we have two process for each signer, one describing its honest behavior and the other the dishonest one. Communication is modeled using shared variables. Each protocol message is modeled using a boolean variable, initialized to false and set to true when it is sent. Sending of a message is modeled using guarded commands, where the guard depends on previously sent out messages. When modeling the honest behavior of a participant, we ensure that a given message is sent out only when specified by the protocol. In contrary, the guards are relaxed in the malicious version of the signer so that each message can be sent out, as soon as possible, i.e., as soon as all messages needed to compose the given message are received. We do not explicitly model any cryptographic primitives, but only the fact that protocol messages can be sent out of order. Hence, a dishonest signer can send messages out of order and continue the protocol, even if it is supposed to stop. We manually decide which messages must be known in order to send some other message. Moreover, the communication between any two signers is assumed to be on private channels and we do not model the possibility to spy other channels. The trusted party is modeled to be always honest.

As an example, consider the short extract of the modeling of the three-party GM protocol depicted in figure 1. In the extract, the integer variable $Pr_{i,j,k}$ models the promises that $P_i$ has sent to $P_j$, and $Pr_{i,j,k} = k$ means that $P_i$ has sent out up to $k$-level promises to $P_j$. (For
efficiency reasons, we use the logarithmic encoding of a ranged integer variable, rather than having one boolean variable for each level of promise). In the extract, the first rule of honest $P_1$ says that $P_1$ may quit the protocol, if it has not contacted the trusted party, and has neither received nor sent any promises. The corresponding modeling of dishonest behavior of $P_1$ states that $P_1$ may quit the protocol at any moment. The second rule of honest $P_1$ gives the exact condition when the first level promise has to be sent to $P_2$. The corresponding dishonest rule, merely requires that $P_1$ has not quit the protocol before sending the promise.

![Extract of honest modeling of $P_1$ for the three-party GM protocol:](image)

The corresponding actions of a dishonest modeling:

- $P_1$.stop -> $P_1$.stop'=true
- $P_1$.stop & $P_1$.contacted_T & $P_1$.stop'=false & $P_2$.stop'=false & $P_3$.stop'=false

Fig. 1: Extract of the three-party GM protocol modeling

As we are unable to verify parametric systems with MOCHA, we simplify our task and verify the protocols only for a given number $n$ of signers. In order to avoid encoding each instance of the protocol using guarded commands, we have written a dedicated C++ program for each protocol which takes the number $n$ of signers as a parameter and generates the protocol specification. Although our model is restricted with regard to several aspects, the model seems to be of interest as several unknown anomalies have been revealed.

**Modeling properties.** We express the desired security guarantees using ATL. We concentrate on modeling of fairness. Consider an instance of the protocol with $n$ signers, which we denote as $P_1,\ldots,P_n$. In the following, we assume that only one of the signers, say $P_1$ is honest, and the other dishonest signers are colluding to cheat the honest signer.

A protocol is fair for an honest $P_1$, if at the end of the protocol, either $P_1$ receives signed contracts from all the other signers or it is not possible for any other signer to obtain $P_1$’s signed contract. One possible ATL formula for modeling this says that if any signer receives $P_1$’s signed contract, then $P_1$ has a strategy to get the signed contracts from other signers:

$$\forall \diamond ((P_2.S_{P_1}(m) \lor \ldots \lor P_n.S_{P_1}(m)) \rightarrow \langle P_1 \rangle \diamond (P_1.S_{P_2}(m) \land \ldots \land P_1.S_{P_n}(m)))$$  \hspace{1cm} (1)

where $P_i.S_{P_j}(m)$ denotes that player $P_i$ received $P_j$’s signature on the contract text $m$.

It can of course be argued that $P_1$ having a strategy to receive the signed contracts is not a sound modeling of fairness: $P_1$ may have this strategy but if it is ignorant or if it mistakenly does not follow this strategy, then the protocol may end in an unfair state. Therefore one could require the following stronger property: in whatever way $P_1$ resolves the remaining choices specified by the protocol, $P_1$ receives all the signed contract.

$$\forall \diamond ((P_2.S_{P_1}(m) \lor \ldots \lor P_n.S_{P_1}(m)) \rightarrow \forall \diamond (P_1.S_{P_2}(m) \land \ldots \land P_1.S_{P_n}(m)))$$  \hspace{1cm} (2)

In the same vein, a third formulation only requires that there exists a path where $P_1$ receives the signed contracts.

$$\forall \diamond ((P_2.S_{P_1}(m) \lor \ldots \lor P_n.S_{P_1}(m)) \rightarrow \exists \diamond (P_1.S_{P_2}(m) \land \ldots \land P_1.S_{P_n}(m)))$$  \hspace{1cm} (3)

Formula (2) implies formula (1), which in turn implies formula (3). We concentrate on the last, weakest version of fairness. As we show that fairness is violated even in this weakest version, the other stronger versions are also violated. Now, we are ready to discuss our analysis.
It is also possible to express abuse-freeness in our formalism. We are going to discuss this property when analysing the GM protocol.

4 Analysis

We have verified the BW protocol with two and up to five signers, but the model-checker MOCHA did not find any flaw. Due to lack of space we do not go into the details of our analysis of the BW protocol. However, the design of the BW protocol is much simpler than the GM protocol and the decision to overturn an abort is based on the following argument: \( T \) overturns only if it can infer that no previous abort reply has been sent to a potentially honest principal. This seems to ensure the robustness of the protocol. Note that we only analyzed the structure of the protocol. Hence, our results only prove that the protocol is correct in the given model. Nevertheless, we believe that the protocol would also be correct in a more general model as all messages are signed and a unique contract identifier is used.

We now report in more detail on our analysis of the GM contract-signing protocol. The protocol has several peculiarities. The most notable one is that the protocol changes with the number of signers, e.g., the protocol specification of \( P_i \) differs when the value of \( n \) changes. The number of protocol messages increases considerably with the number of signers. For instance, if we have \( n = 3 \), the main protocol has 20 messages and there are 14 different recovery requests. When \( n = 4 \), the corresponding numbers are 41 and 36. Moreover, the protocol is not symmetric for the signers: the protocol specification for \( P_i \) is different from that for \( P_j \), for all \( i \neq j \). For instance, when \( n = 4 \), \( P_1 \) can launch 18 different recovery requests and \( P_4 \) only 2.

As mentioned in section 3, we have written a dedicated C++ program that takes the number of signers, \( n \), as a parameter and generates the protocol specification. Our analysis revealed problems with fairness, when \( n = 4 \). Although, we did not discover any fairness problems when \( n = 3 \), we did find an amusing problem with abuse-freeness. We did not discover any problems with timeliness in the protocol. All these anomalies are novel and the protocol was believed to be secure since it was first published. We discuss our results in detail. The sources codes of our analysis of both protocols are also available at the following website http://www.ulb.ac.be/di/scsi/skremer/MPCS/.

Fairness. We did not discover any problems with fairness when \( n = 3 \). The formulas representing fairness for \( P_1 \), \( P_2 \) and \( P_3 \), introduced in section 3, are validated by MOCHA. However, as we use a restricted model and consider single runs, we can only conclude that the protocol does not present any structural weakness for \( n = 3 \). Indeed, if we relax the assumption of the private channels, the anomaly presented by Shmatikov and Mitchell in [13] on the two-party GJM protocol can be adapted to the multi-party version. In this scenario, a malicious signer eavesdrops on the channel between the honest signer and \( T \), and succeeds in compromising fairness. With our present modeling, we do not find such flaws, as this requires to eavesdrop channels and to decompose messages. The fix proposed by Shmatikov and Mitchell applies to the multi-party protocol too. However, we should emphasize that the authors of the GM protocol require the channels to \( T \) to be private and hence this scenario does not represent a valid attack on the protocol.

We discovered several scenarios that compromised fairness when \( n = 4 \). The first scenario was discovered by hand, when we found an error in the proof of correctness given in the original paper [10]. A detailed analysis using MOCHA detected seven other scenarios. An analysis of these revealed that the proof also did not cover a case. In each scenario, an honest signer is cheated by the coalition of three malicious signers. These scenarios follow the general outline:

1. A dishonest signer contacts the trusted party, \( T \), at the beginning of the protocol, gets an abort token, and dishonestly continues participating in the main protocol.
2. A second dishonest signer tries to recover at some later point. It does not succeed, but manages to put the honest signer in the list \( F_m \). It dishonestly continues the main protocol.
3. The honest signer is forced to recover, but is not successful in getting the abort decision overturned since it is in the list \( F_m \).
4. The third dishonest signer contacts $T$ and manages to overturn the decision. Hence, while the honest signer does not get any signed contract, the honest signer’s contract is obtained.

For lack of space, we just describe one of these scenarios in detail. In this scenario, $P_1$, $P_3$ and $P_4$ collude to cheat $P_2$. The scenario proceeds as follows:

- At the beginning of the protocol, $P_1$ aborts the protocol and $T$ updates $S_m = \{3\}$. However, unlike specified by the protocol, dishonest $P_3$ continues the main protocol execution.
- As soon as $P_1$ receives the second level promise from $P_2$, it asks $T$ to recover by sending
  \[ S_{P_1}((PCS_{P_2}((m, 2), P_1, T), PCS_{P_3}((m, 1), P_1, T), PCS_{P_4}((m, 1), P_1, T)), S_{P_1}((m, 1)))) \]
  $T$ refuses this request, answers with an abort message and updates $S_m = \{1, 3\}$ and $F_m = \{2\}$. As $P_3$ did before, $P_1$ also continues the protocol.
- The main protocol is executed normally until signer $P_2$ reaches point 6.2. (see table 1) of the protocol with $a = 4$. At that point $P_2$ has sent out the set of message
  \[ \{PCS_{P_2}((m, 1), P_1, T), PCS_{P_3}((m, 2), P_1, T), PCS_{P_4}((m, 2), P_3, T), PCS_{P_2}((m, 3), P_1, T), PCS_{P_3}((m, 3), P_3, T), PCS_{P_4}((m, 3), P_4, T)\} \]
  and has received the set of messages
  \[ \{PCS_{P_1}((m, 1), P_2, T), PCS_{P_2}((m, 1), P_2, T), PCS_{P_2}((m, 2), P_2, T), PCS_{P_3}((m, 3), P_2, T), PCS_{P_4}((m, 3), P_2, T))\} \]
  $P_2$ is at position 6.2, with $a = 4$ and is waiting for 4-level promises from $P_3$ and $P_4$. $P_3$ and $P_4$ do not reply and $P_2$ is forced to send the following recovery request to $T$.
  \[ S_{P_2}((PCS_{P_3}((m, 3), P_2, T), PCS_{P_3}((m, 3), P_2, T), PCS_{P_4}((m, 1), P_2, T)), S_{P_2}((m, 1)))) \]
  $P_2$ is in $F_m$, the tests in the protocol description (see table 3) indicate that $T$ refuses the request, updates $S_m = \{1, 2, 3\}$ and replies with an abort message. $F_m$ remains unchanged.
- $P_4$ launches a resolve request, sending
  \[ S_{P_4}((PCS_{P_1}((m, 3), P_4, T), PCS_{P_3}((m, 3), P_4, T), PCS_{P_4}((m, 3), P_4, T)), S_{P_4}((m, 1)))) \]
  This request overturns the previous aborts, and hence violates fairness as $T$ sends the signed contract back to $P_4$.

In other scenarios, we discovered that protocol is unfair for signers $P_1$, $P_2$ and $P_3$ when $n = 4$. We did not find any scenario that compromised fairness for $P_4$. This is probably because the tests indicate that $P_4$ can never be added to $F_m$, when $n = 4$.

**Correcting the Garay-MacKenzie Protocol.** In order to restore fairness in the Garay-MacKenzie protocol, we had to do major revisions in the recovery protocol. We were unsuccessful to restore fairness with minor changes, and we believe that this is because the meaning of the list $F_m$ is not clear in the protocol. The central idea behind the revision is that $T$, when presented with a recovery request, overturns its abort decision if and only if $T$ can infer dishonesty on the part of each of the signer that contacted $T$ in the past. This is also the main idea behind the recovery protocol in [3].

The main protocol remains the same. Major changes are in the recovery protocol. The recovery messages are designed so that $T$ can infer the promises that an honest signer would have sent when it launched the recovery protocol (note that a signer may have dishonestly sent other promises). For $P_1$ to recover, it sends the message

\[ S_{P_1}((PCS_{P_1}((m, k_1), P_1, T))_{j \in \{1...n\} \setminus \{i\}}, S_{P_1}((m, 1)))) \]

where $k_j$ is computed as following:
Intuitively, we believe that the revised protocol would be fair in a more general setting, and for an arbitrary number of signers.

Let $T$ maintain the set $S_m$ of indices of signers that contacted $T$ in the past and received an abort token. For each signer $P_i$, in the set $S_m$, $T$ also maintains two integer variables $h_i(m)$ and $l_i(m)$. Intuitively, $h_i$ is the highest level promise an honest $P_i$ could have sent to any higher indexed signer before it contacted $T$. $l_i$ is the highest level promise an honest $P_i$ could have sent to a lower indexed signer before it contacted $T$. The protocol for $T$ works as follows:

- If $T$ ever replies with a signed contract for $m$, then $T$ responds with the contract for any further request.
- If the first request to $T$ is a resolve request, then $T$ sends back a signed contract.
- If the first request is an abort request, then $T$ aborts the contract. $T$ may overturn this decision in the future if it can deduce that all the signers in $S_m$ have behaved dishonestly. $T$ deduces that a signer $P_i$ in $S_m$ is dishonest when contacted by $P_j$ if
  1. $j > i$ and $P_j$ presents to $T$ a $k$-level promise from $P_i$ such that $k > h_i(m)$, or
  2. $j < i$ and $P_j$ presents to $T$ a $k$-level promise from $P_i$ such that $k > l_i(m)$.

We describe the abort and recovery protocols in detail in table 6, respectively.

We analyzed the revised protocol for both 3 and 4 signers and MOCHA did not detect any errors in the revised protocol. Please note that this should not be construed as proof of correctness since we are using a restricted communication model and are modeling a single run. Nevertheless, we believe that the revised protocol would be fair in a more general setting, and for an arbitrary number of signers.

### Table 6 Revised GM multi-party contract-signing protocol—Abort

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$P_i \rightarrow T$: $S_T(m, P_i, (P_3, \ldots, P_n), abort)$</td>
</tr>
<tr>
<td></td>
<td>if not validated($m$) then</td>
</tr>
<tr>
<td></td>
<td>$S_m = S_m \cup {i}$;</td>
</tr>
<tr>
<td></td>
<td>otherwise $T$ stores $S_T(m, P_i, (P_3, \ldots, P_n), abort)$;</td>
</tr>
<tr>
<td>2.</td>
<td>$T \rightarrow P_i$: $S_T(m, P_i, (P_3, \ldots, P_n), abort), S_T(m, abort)$</td>
</tr>
<tr>
<td></td>
<td>(validated($m$)=true)</td>
</tr>
<tr>
<td>3.</td>
<td>$T \rightarrow P_i$: ${S_T((m, k_j))}_{j \in {1, \ldots, n} \setminus {i}}$</td>
</tr>
<tr>
<td></td>
<td>where $k_j$ is the level of the promise from $P_j$ that was converted to a universally-verifiable signature during the recovery protocol.</td>
</tr>
</tbody>
</table>

1. If $P_i$ runs the resolve protocol in step 5 of the main protocol (see table 1), then $k_j = 1$ for $j > i$ and $k_j = i - 1$ for $j < i$.
2. In step 6.2 of the main protocol, $k_j = a - 1$ for $1 < j < a - 1, j \neq i$ and $k_j = 1$ for $j > a - 1$.
3. In step 6.4 of the main protocol, $k_j = a - 1$ for $j < i$, $k_j = a$ for $i < j < a$ and $k_j = 1$ for $j > a$.
4. In step 7 of the main protocol, $k_j = n$ for all $j$.
5. In step 9 of the main protocol, $k_j = n$ for all $j < i$ and $k_j = n + 1$ for all $j > i$.

$k_j$ may alternately be computed as:

- If $j < i$, $k_j$ is the maximum level of promises received from all signers $P_j', j' < i$, i.e. the min-max of the promises from signers with lower index. (For example, if the maximum level of the promises received by $P_4$ from $P_3$ and $P_2$ was 6, and the maximum level received by $P_4$ from $P_1$ was 5, then it would send the 5-level promises for $P_1$, $P_2$, and $P_3$.)

- Let $l$ be the maximum value $l'$ such that $P_i$ has $l'$ level promises from $P_j$ for all $i < j < l'$. If no such $l'$ exists then let $l = 0$. If $l = 0$, then let $k_j = 1$ for all $j > i$. If $l \neq 0$, then let $k_j = l$ for all $i < j < l$ and $k_j = 1$ for all $j > l$. (For example, if $P_2$ has received level 1 promise from $P_5$, level 5 and 1 promises from $P_3$, level 5, 4 and 1 promises from $P_4$, and level 4, 3 and 1 promises from $P_3$ then $k_6 = 1, k_5 = 1, k_4 = 4, k_3 = 4$.)

$T$ maintains the set $S_m$ of indices of signers that contacted $T$ in the past and received an abort token. For each signer $P_i$ in the set $S_m$, $T$ also maintains two integer variables $h_i(m)$ and $l_i(m)$.
Table 7 Revised GM multi-party contract-signing protocol—Recovery

1. \( P_i \to T: \{ PCSP_j((m, k_j), P_i(T)) \}_{j \in \{1, \ldots, n\} \setminus \{i\}}, \{ S_{P_j}((m, 1)) \} \)
   if \( i \in S_m \), \( T \) ignores the message
   else if validated(\( m \))
   2. \( T \to P_i: \{ S_{P_j}((m, k_j)) \}_{j \in \{1, \ldots, n\} \setminus \{i\}} \)
      where \( k_j \) is the level of the promise from \( P_j \) that was converted to a universally-verifiable signature.
      else if validated(\( m \))=false and \( S_m = 0 \)
      validated(\( m \)):=true
      3. \( T \to P_i: \{ S_{P_j}((m, k_j)) \}_{j \in \{1, \ldots, n\} \setminus \{i\}} \)
      else (validated(\( m \))=false \& \( S_m \neq 0 \))

1. If there is some \( p < i \) in \( S_m \) such that \( k_p \leq h_p(m) \), or if there is some \( p > i \) in \( S_m \) such that \( k_p \leq l_p(m) \), then \( T \) sends back the stored abort \( S_T(PCS_P(m, P_j, (P_1, \ldots, P_n), \text{abort})) \) to \( P_i \). \( T \)
   adds \( i \) to \( S_m \), and computes \( h_i(m) \) and \( l_i(m) \) as follows
   \[
   (h_i(m), l_i(m)) = (k_{i+1}, 0), \quad \text{if } i = 1 \text{ (intuitively, } P_1 \text{ has contacted } T \text{ in either step 6.2 of the main protocol with } a = k_{i+1} + 1 \text{ or in step 7 of the main protocol)},
   \]
   \[
   = (0, i), \quad \text{if } 1 < i \text{ and } k_{i-1} = i - 1 \text{ (intuitively, } P_i \text{ has contacted } T \text{ in step 5 of the main protocol)},
   \]
   \[
   = (k_{i-1}, k_{i-1}), \quad \text{if } 1 < i < n, i \leq k_{i-1} \leq n \text{ and } k_{i+1} \leq k_{i-1} \text{ (intuitively, } P_i \text{ has contacted } T \text{ in step 6.2 of the main protocol with } a = k_{i-1} + 1),
   \]
   \[
   = (k_{i-1}, k_{i-1} + 1), \quad \text{if } 1 < i < n, i \leq k_{i-1} < n \text{ and } k_{i+1} > k_{i-1} \text{ (intuitively, } P_i \text{ has contacted } T \text{ in step 6.4 of the main protocol with } a = k_{i-1} + 1),
   \]
   \[
   = (n, n), \quad \text{if } 1 < i < n \text{ and } k_{i-1} = k_{i+1} = n. \text{ (intuitively, } P_i \text{ has contacted } T \text{ in step 7 of the main protocol)},
   \]
   \[
   = (n + 1, n + 1), \quad \text{if } 1 < i < n, k_{i-1} = n \text{ and } k_{i+1} = n + 1. \text{ (intuitively, } P_i \text{ has contacted } T \text{ in step 9 of the main protocol)},
   \]
   \[
   = (0, n + 1), \quad \text{if } i = n \text{ and } k_{i-1} = n. \text{ (intuitively, } P_n \text{ has contacted } T \text{ in step 9 of the main protocol)},
   \]

2. Otherwise, \( T \) sends \( \{ S_{P_j}((m, k_j)) \}_{j \in \{1, \ldots, n\} \setminus \{i\}} \) to \( P_i \), stores all the signatures, and sets validated(\( m \)) to true.

**Abuse-freeness.** We now describe the anomaly that we discovered for \( n = 3 \) signers in the GM protocol. The anomaly exploits the fact that when \( T \) replies with an abort decision, it also signs the list \( S_m \) of the signers who have received an abort from \( T \). Recall that an optimistic signer [6] is one that prefers to wait for "some time" before contacting the trusted party. Following [6], we say that a protocol is abuse-free for a signer \( P_i \) if the protocol does not provide provable advantage to the remaining signers. A coalition of signers is said to have provable advantage against \( P_i \) at a point in the protocol if (i) they have a strategy to abort the contract against an optimistic \( P_i \), (ii) they have a strategy to get optimistic \( P_i \)'s contract, and (iii) they can prove to an outside challenger, Charlie, that \( P_i \) is participating in the protocol.

Now consider the protocol instance with three signers \( P_1, P_2 \) and \( P_3 \). Assume that \( P_3 \) is optimistic and \( P_1 \) and \( P_2 \) are colluding to cheat \( P_3 \). \( P_3 \) starts the protocol by sending its level 1 promises to \( P_1 \) and \( P_2 \), and waits for level 2 promises from them. \( P_2 \) on receiving this sends its level 1 promise to \( P_1 \), and then sends an abort request to \( T \) which aborts the protocol. Now, \( P_1 \) has received level 1 promises from \( P_2 \) and \( P_3 \). Using these first level promises, \( P_1 \) sends a recovery request to \( T \). Note that, in the protocol, \( P_1 \)'s recovery request is refused and \( T \) sends
\[
S_T(PCS_P(m, P_2, (P_1, P_2, P_3), \text{abort}))
\]
and
\[
S_T(m, S_m = \{1, 2\}, \text{abort})
\]

At this point, we make the following observations:
the abort reply contains the set \( S_m = \{1, 2\} \) and is different from the one \( P_2 \) received,
- if \( P_1 \) receives an abort reply from \( T \), it is always the answer to a recovery request,
- a recovery request always includes a promise from each signer which is verified by \( T \).

From these remarks, we can conclude that if \( P_1 \) shows the abort reply to Charlie, then Charlie will be convinced that \( P_3 \) has started the protocol even though Charlie is unable to verify the PCS from \( P_3 \). In other words, we can say that \( T \) has verified the PCS for Charlie. Also note that at this point \( P_1 \) and \( P_2 \) can force the exchange to abort by simply quitting the protocol: \( P_3 \) has no promises from \( P_1 \) and \( P_2 \). \( P_1 \) and \( P_2 \) can also force a successful completion of the contract exchange by simply (dishonestly) engaging \( P_3 \) in the main protocol. Hence the protocol is not abuse-free for \( P_3 \).

This vulnerability can be easily addressed by excluding the set \( S_m \) from the abort reply. In this case, the abort messages from \( P_3 \) and \( P_2 \) are exactly similar and can be obtained by \( P_2 \) without \( P_3 \)’s participation. Hence, an abort reply does not prove \( P_3 \)’s participation in the protocol. This rather amusing scenario illustrates that sometimes additional information may be harmful. While explicitness is often considered a good engineering practice (and we do not attempt to criticize such thumb rules), care should be taken when applying these principles. In personal communication with the authors of the protocol, they propose a different fix in letting \( P_3 \) abort the protocol rather than just quitting.

Abuse-freeness for \( P_3 \) is naturally expressed in ATL as follows:

\[
\neg\exists\diamond(T.\text{send}(\text{abort}) \rightarrow P_1) \\
\langle\langle P_1, P_2 \rangle\square(\neg P_3.\text{S}_P(m) \lor \neg P_3.\text{S}_P(m)) \land \\
\langle\langle P_1, P_2 \rangle\square(P_3.\text{stop} \rightarrow (P_1.\text{S}_P(m) \land P_2.\text{S}_P(m)))
\]

The boolean variable \( T.\text{send}(\text{abort}) \rightarrow P_1 \) is set to true when \( P_1 \) receives the abort token \( S_T(S_P(m, P_2, (P_1, P_2, P_3), \text{abort})) \) with \( S_m = \{1, 2\} \). As discussed before, this serves as a proof of \( P_3 \)’s participation. The variables \( P_i.\text{S}_P(m) \) reflect that player \( i \) has received player \( j \)’s signature on the contract. More precisely, the formula requires that it is possible to reach a point where

1. \( P_1 \) and \( P_2 \) can prove to Charlie that the protocol was started by \( P_3 \) (\( T.\text{send}(\text{abort}) \rightarrow P_1 \) is true),
2. \( P_1 \) and \( P_2 \) have a strategy to choose an unsuccessful outcome, i.e., \( P_3 \) cannot get some signer’s contract (\( \langle\langle P_1, P_2 \rangle\square(\neg P_3.\text{S}_P(m) \lor \neg P_3.\text{S}_P(m)) \) is true), and
3. \( P_1 \) and \( P_2 \) have a strategy to choose a successful outcome, i.e., when honest \( P_3 \) stops, the dishonest players must have obtained \( P_3 \)’s contract (\( \langle\langle P_1, P_2 \rangle\square(P_3.\text{stop} \rightarrow (P_1.\text{S}_P(m) \land P_2.\text{S}_P(m))) \) is true).

The requirement of \( P_3 \) stopping in condition 3 is to prevent it from idling forever. Even though as discussed before, the protocol is not abuse-free if \( P_3 \) is optimistic, the above formula is validated if \( P_3 \) is honest. An honest \( P_3 \) may contact \( T \) non-deterministically as permitted by the protocol. Indeed, in the scenario discussed above, an honest \( P_3 \) could prevent \( P_1 \) and \( P_2 \) from getting \( P_3 \)’s signature if it contacts \( T \). Therefore, in order to capture the scenario described above, we needed to model optimistic signers.

Following [6], we implement an optimistic signer by adding signals that the signer uses to decide when to quit waiting for messages from other signers and contact \( T \). \( P_3 \) uses 3 signals for this: one to decide when to ask \( T \) to abort and 2 to decide when to contact \( T \) for the two recovery protocols that \( P_3 \) can launch. These signals are controlled by a new player, \( P_3 \text{TimeOuts} \), that is added to the model.

The decision to abort is modeled by 2 boolean variables: \( \text{setTimeOutAbort} \) and \( \text{expiredTimeOutAbort} \). While \( P_3 \) changes the value of \( \text{setTimeOutAbort} \), \( \text{expiredTimeOutAbort} \) is changed by \( P_3 \text{TimeOuts} \). When \( P_3 \) sends level 1 promise to \( P_1 \) and \( P_2 \), it sets the value of \( \text{setTimeOutAbort} \) to true, and then waits for level 2 promises from them. \( P_3 \text{TimeOuts} \) may set \( \text{expiredTimeOutAbort} \) to true once \( \text{setTimeOutAbort} \) is set to true by \( P_3 \). If the promises arrive before \( \text{expiredTimeOutAbort} \)
is true, then $P_3$ continues with the main protocol, otherwise $P_3$ may contact $T$ with an abort request. The decision to send recovery requests are modeled similarly.

Following [6], abuse-freeness is modeled by having a coalition of $P_3$TimeOuts, $P_1$ and $P_2$. This coalition can choose a sufficiently "long time" to keep $P_3$ from contacting $T$, while allowing $P_1$ and $P_2$ to schedule its messages in order to get the desired result. Abuse-freeness can then be expressed as

$$\neg\exists\diamond(T.\ send(abort)\ to\ P_1\land
\langle\langle P_1, P_2, P_3\rangle\rangle (\neg P_3.S_{P_1}(m) \lor \neg P_3.S_{P_2}(m))\land
\langle\langle P_1, P_2, P_3\rangle\rangle (P_3.stop \rightarrow (P_1.S_{P_3}(m) \land P_2.S_{P_3}(m)))$$

Please note that even an optimistic $P_3$ should eventually be allowed to contact $T$, otherwise $P_3$ may be stuck forever. Hence, $P_3$TimeOuts must eventually set expiredTimeOutAbort and other signals to true. Ideally, this should be set non-deterministically. However, ensuring that a variable changes its value in MOCHA slows down verification considerably. In order to make the verification feasible, we put a maximum limit, tick, on the number of computation steps after which the value must change, and vary this limit manually. Please note that the signals may change before this limit is reached. This modeling is sound in the sense that if the formula is violated for some value of tick then abuse-freeness must be violated: $P_3$ just needs to wait for sufficiently "long time" to allow $P_1$ and $P_2$ to schedule its messages. Indeed, the above property is violated when tick is set to 3 giving us the attack on abuse-freeness. As expected, if we drop the player $P_3$TimeOuts in the formula, then the property is not violated: $P_1$ and $P_2$ are not able to schedule their messages ahead of $P_3$.

However, if $P_3$ is not optimistic, the above given formulation of abuse-freeness is not violated. In a non-optimistic setting, the vulnerability can be detected by weakening the third requirement of the formula in order to say that there is a trace in which $P_3$ does not contact $T$, and $P_1$ and $P_2$ get $P_3$’s signed contract. The ATL formula which captures this is

$$\neg\exists\diamond(T.\ send(abort)\ to\ P_1\land
\langle\langle P_1, P_2\rangle\rangle (\neg P_3.S_{P_1}(m) \lor \neg P_3.S_{P_2}(m))\land
\exists\diamond(P_3.stop \rightarrow (P_1.S_{P_3}(m) \land P_2.S_{P_3}(m)))$$

This property is violated even in the original non-optimistic model. MOCHA also detected the violation of a stronger version of abuse-freeness proposed in [12]. This formulation requires that as soon as $P_2$ and $P_3$ can prove to Charlie that $P_1$ started the protocol, then they may not achieve an unsuccessful outcome anymore.

5 Conclusions and Future Work

We have studied two multi-party contract-signing protocols [10,3] using a finite-state tool, MOCHA, that allows specification of properties in a branching-time temporal logic with game semantics. In order to make this analysis feasible, we model single runs and assume a restricted communication model. Our analysis did not find any errors in the BW protocol [3]. We did encounter problems with fairness in the case of four signers in the GM protocol [10]. It appears that fairness cannot be restored without completely rewriting the subprotocols. The revised subprotocols are inspired by the BW protocol. We also discovered a rather amusing problem with abuse-freeness in the GM protocol with three signers that occurs because abort messages from the trusted party reveal who have contacted it in the past. This problem is easily addressed by ensuring that the trusted party does not send this extra information. We had to implement optimistic signers to demonstrate this problem using MOCHA.

We plan to verify the protocols without fixing the number of signers. One major challenge in such a parametric verification is that the protocol descriptions change fundamentally with the
number of signers in that the protocol for \( n \) signers is not merely putting \( n \) identical processes in parallel. We hope to prove the correctness of these protocols in a more general setting which accounts for cryptography, multiple concurrent sessions, and relaxes the communication model. We plan to use, at least partially, abstraction techniques such as proposed by Das and Dill [7] to achieve this.

References